LESSON 12

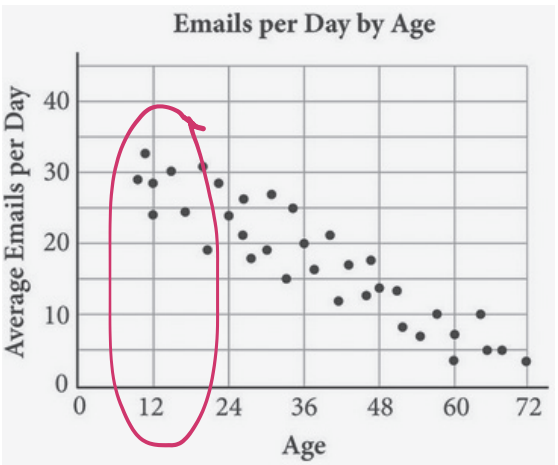
Lines of Best Fit

LEARNING OBJECTIVES

- Today I am: using scatterplots and lines of best fit to make predictions.
- So that I can: learn to write equations of lines of best fit.
- I'll know I have it when I can: write the equation of a line with two points.

In Lesson 11, you graphed and analyzed a variety of scatterplots. In this lesson, you'll further explore how to write the equation of a line to describe the scatterplot. Your equations will allow you to make predictions about the data.

Opening Exercise—Reading Graphs



Source: CrackSAT.net

1. What data is this graph displaying?

The average emails per day.

2. Are you surprised by this data? Explain.

Yes for 12 year olds.

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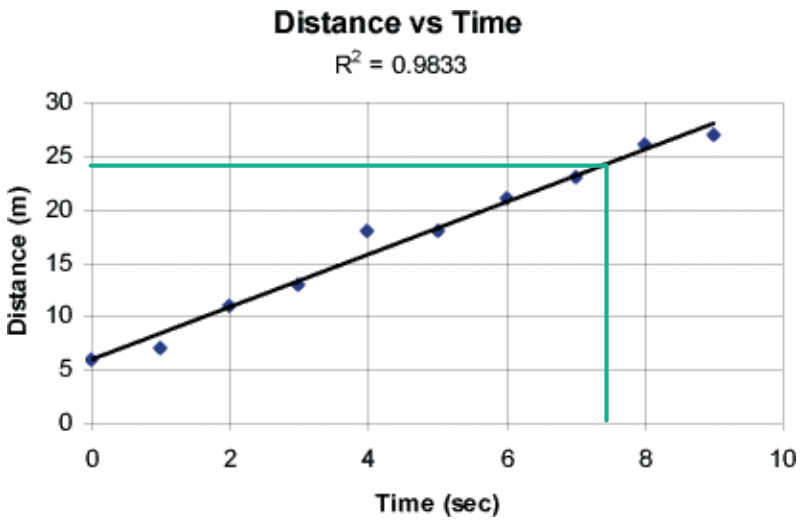
Exploration: Writing an Equation of a Line

The equation of a line can be in the form $y = mx + b$, where m represents the slope of the line and b represents the y-intercept. We'll start with a strong positive linear scatterplot. The line of best fit has been drawn in for you.



Source: http://dev.physicslab.org/Document.aspx?doctype=3&filename=IntroductoryMathematics_DataAnalysisMethods.xml

3. Choose two points on the line of best fit and then determine the slope between these two lines.

   A. My Points: (x_1 0 , y_1 6) and (x_2 2 , y_2 11)

   B. Slope using my two points: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 6}{2 - 0} = \frac{5}{2} = \frac{2.5}{1}$ m/sec

   C. What does this slope mean in terms of the data?

The distance increases by 2.5 m every second.

4. What is the estimated y-intercept for the line?

6

5. Write the equation of the best fit line using $y = mx + b$, where m = slope and b = y-intercept.

$y = 2.5x + 6$

6. The slope of the line shown is 2.5. Did you get that value? If not, how much did your slope differ from this value? Why do you think there might be a difference?

Oh yeah!

7. The *y*-intercept generated using the actual data points and a calculator or computer program was 6. How much did your *y*-intercept differ from this value? Why do you think there might be a difference?

Oh yeah!

8. You can use your equation to find other points on the line.

   A. Use the graph to estimate the distance when time is 7.5 seconds.

$y = 2.5(7.5) + 6$ X

$y = 24.75$ m

   B. Use your equation from Exercise 5 to estimate the distance when time is 7.5 seconds.

   C. Which model (graph or equation) would you use to estimate the distance at 10 seconds? What is that value? Which model is more accurate?

Equation is more accurate.

   D. How much time would you estimate would go by for 15 meters? Which model did you use? Why?

y

$15 = 2.5x + 6$

$-6 \qquad -6$

$\frac{9}{2.5} = \frac{2.5x}{2.5}$

$x = 3.6$ s

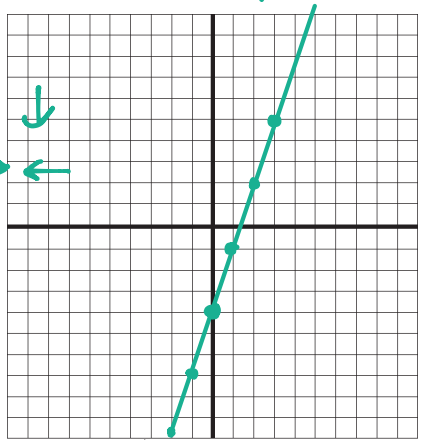
Practice—Writing & Graphing Equations of Lines

For Exercise 9–14, use the given information to write an equation of the line. Then graph the line.

9. Slope = 3; *y*-intercept = −4

Equation:

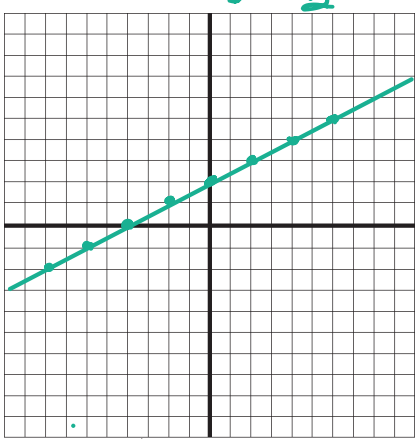
Graph:



10. Slope = ½; *y*-intercept = 2

Equation:

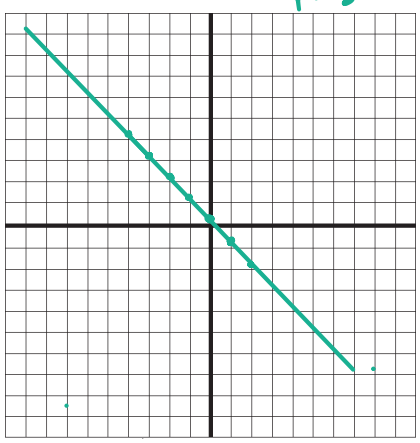
Graph:



11. Slope = −1; *y*-intercept = ¼

Equation:

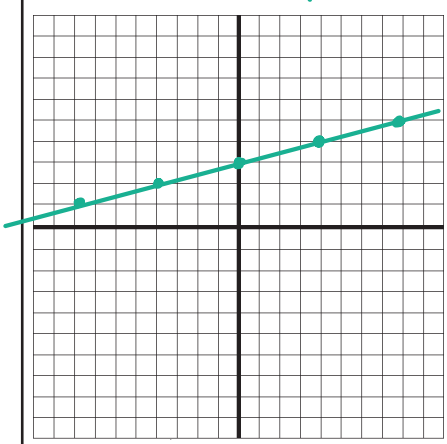
Graph:



12. Two points on the line (4, 4) and (8, 5) with a *y*-intercept of 3.

Equation:

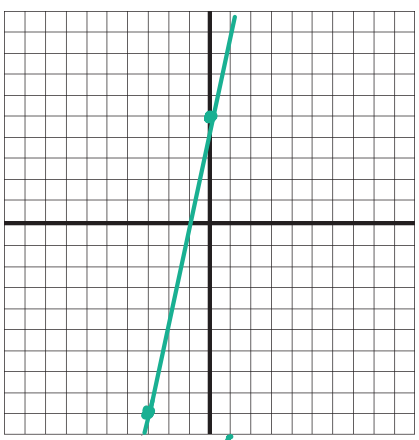
Graph:



13. Two points on the line (−3, −9) and (0, 5) with a *y*-intercept of 5.

Equation:

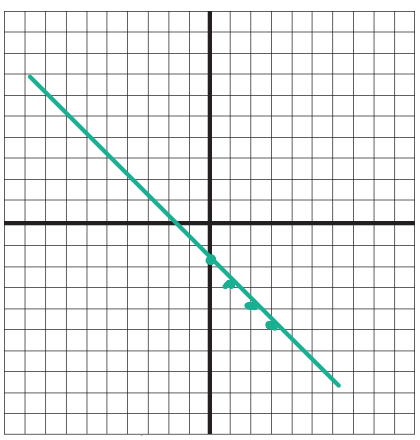
Graph:



14. Two points on the line (−2, 0) and (4, −6) with a *y*-intercept of −2.

Equation:

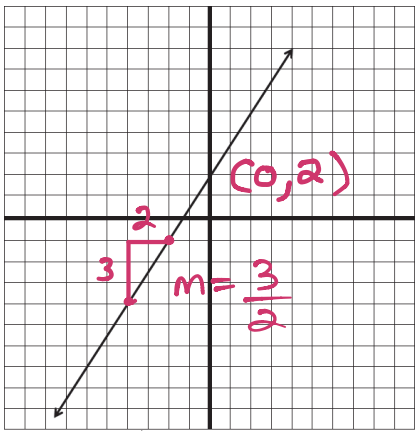
Graph:



Practice—Writing Equations of Lines from Graphs

For Exercise 15–20 determine the slope and *y*-intercept of the line. Then write the equation of the line.

15. Graph:

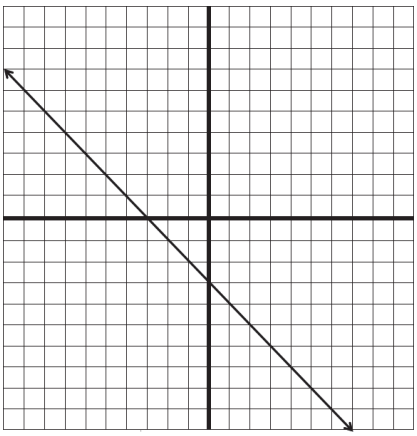


Slope = 3/2

y-intercept = 2

Equation: $y = \frac{3}{2}x + 2$

16. Graph:

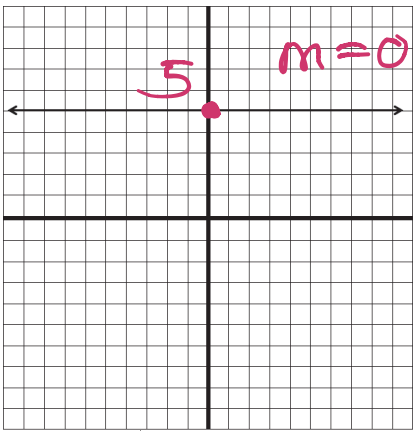


Slope = _____

y-intercept = _____

Equation: _____

17. Graph:

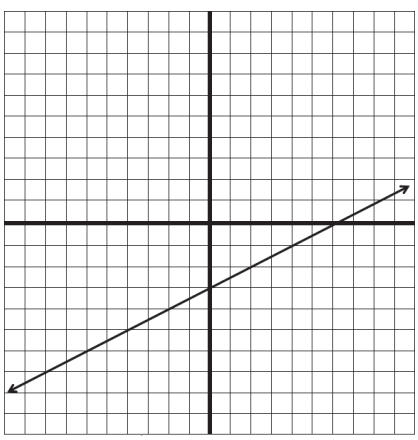


Slope = 0

y-intercept = 5

Equation: $y = 5$

18. Graph:

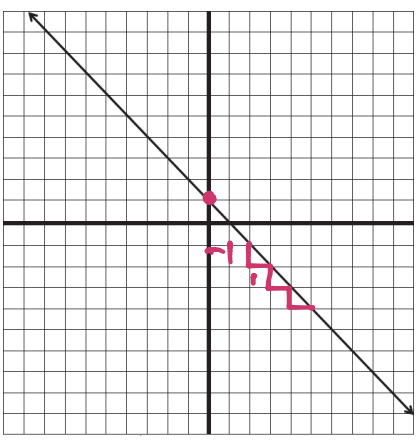


Slope = _____

y-intercept = _____

Equation: _____

19. Graph:

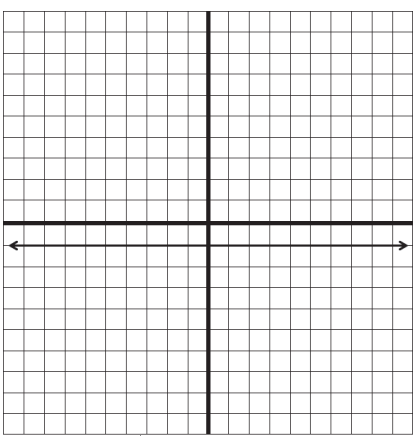


Slope = −1

y-intercept = 1

Equation: $y = -x + 1$

20. Graph:



Slope = _____

y-intercept = _____

Equation: _____

Point-Slope Form of a Line

The equation you wrote in Exercise 3 is in slope-intercept form or $y = mx + b$. Point-slope is another very useful form of a linear equation. For point-slope we need any point on the line and the slope.

When no y-intercept is available, we can use a different form of the equation of a line. This new form comes from the slope formula.

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad m = \frac{y_1 - y_2}{x_1 - x_2}$$

Point-Slope Equation of a Line: $y_2 - y_1 = m(x_2 - x_1)$ **or** $y - y_1 = m(x - x_1)$

21. How did they get from the slope equation to the point-slope equation?

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m(x_2 - x_1) = y_2 - y_1$ $\quad y - y_1 = m(x - x_1)$

22. Suppose two points on the line in Exercise 3 are (1, 8.5) and (7, 23.5). (x_1, y_1) → a point

    A. Use the point-slope equation to write the equation of the line.

    Find the slope: $m = \frac{23.5 - 8.5}{7 - 1} = \frac{15}{6} = \frac{5}{2} = 2.5 = m$ (m → slope)

    Use the formula. You may use either point for the equation: $y - 8.5 = 2.5(x - 1)$

    B. Discuss with your partner how you could rearrange this equation to isolate the y and get it in slope-intercept form.

    C. Rewrite your equation in Part A in slope-intercept form.

    D. How does this equation compare to the one you wrote in Exercise 5?

point-slope → slope-intercept.

$y - 8.5 = 2.5(x - 1)$

$y - 8.5 = 2.5x - 2.5$

$+8.5 \qquad +8.5$

$y = 2.5x + 6$

23. What information do you need to find the equation of a line? List all possibilities.

If I have . . .	Then I can use . . .
• slope m • y-intercept b	$y = mx + b$
• slope and a point or • two points	$y - y_1 = m(x - x_1)$ $(x_1, y_1) \rightarrow$ a point.

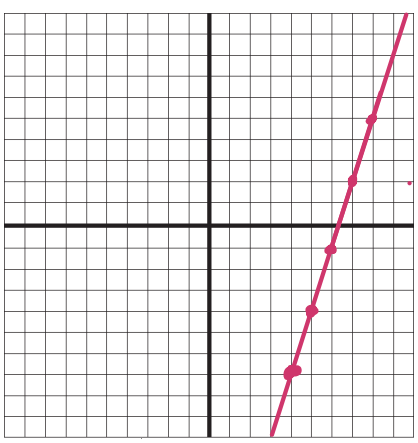
Practice Writing and Graphing Linear Equations

For Exercise 24–26, write the equation of the line in point-slope form. Then graph the line.

24. Slope = 3; Point = (4, −7) x_1, y_1

Equation: $y + 7 = 3(x - 4)$

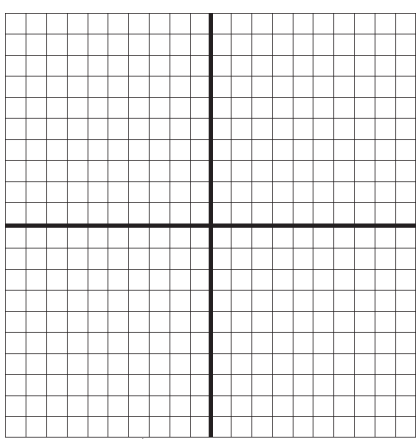
Graph:



25. Slope = ½; Point = (5, 0)

Equation:

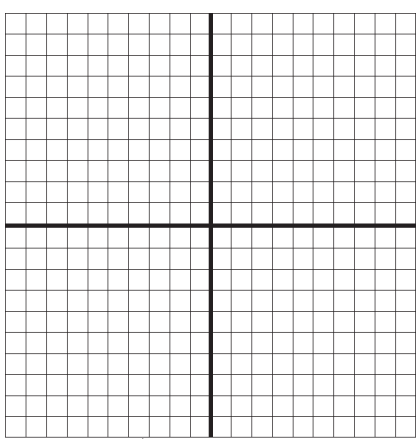
Graph:



26. Slope = −1; Point = (−½ , 4)

Equation:

Graph:



Practice Writing and Graphing Linear Equations

For Exercise 27–29 write the slope and equation for the line. Then graph the line.

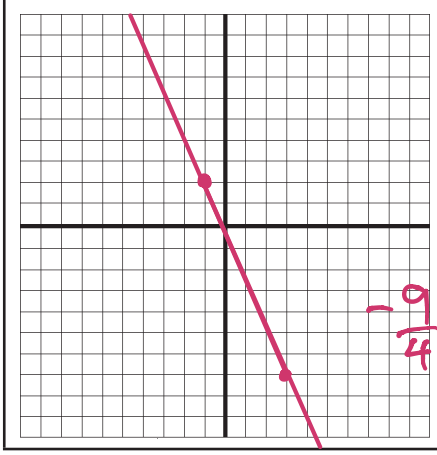
27. Point = (−1, 2);
Point = (3, −7)

Slope: $m = \frac{-7-2}{3+1} = \frac{-9}{4}$

Equation:

$y - 2 = \frac{-9}{4}(x+1)$

Graph:

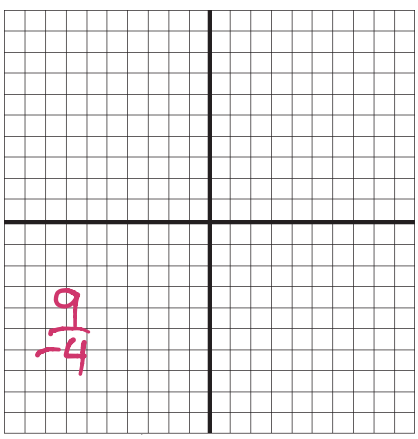


28. Point = (−3, 5);
Point = (5, 0)

Slope:

Equation:

Graph:

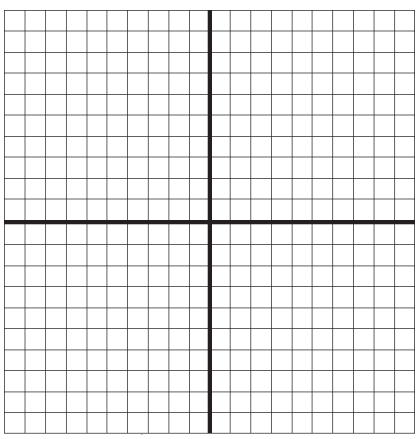


29. Point = (8, 0);
Point = (4 , 4)

Slope:

Equation:

Graph:



Lesson Summary

Writing Equations of Lines

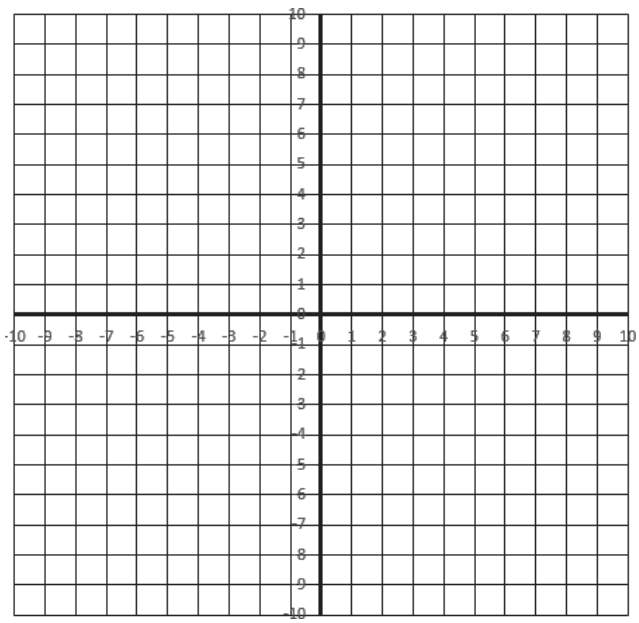
- If you have two points on your line, you can use the **Point-Slope Equation** given by $y - y_1 = m(x - x_1)$, where m = slope and (x_1, y_1) is a point on the line.
- If you have one point and the slope of your line, you can use the **Slope-Intercept Equation** given by $y = mx + b$, where m = slope and b = y-intercept.

NAME: _____ PERIOD: _____ DATE: _____

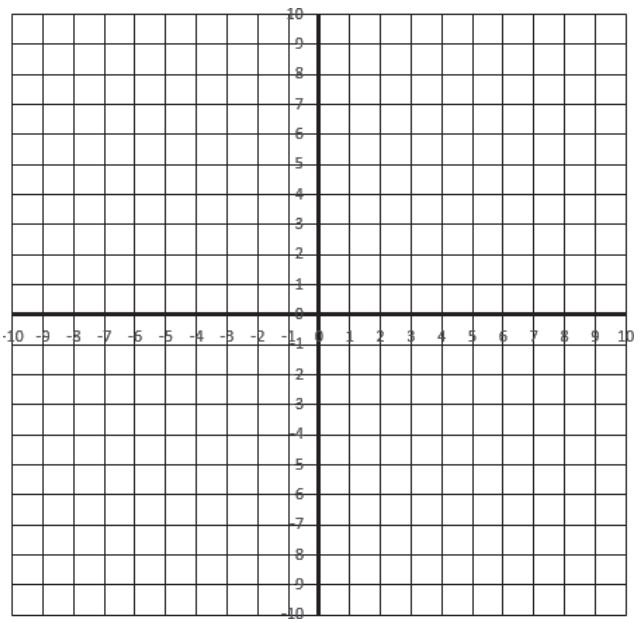
Homework Problem Set

1. Graph the following lines.

   A. $y = \frac{2}{3}x - 4$

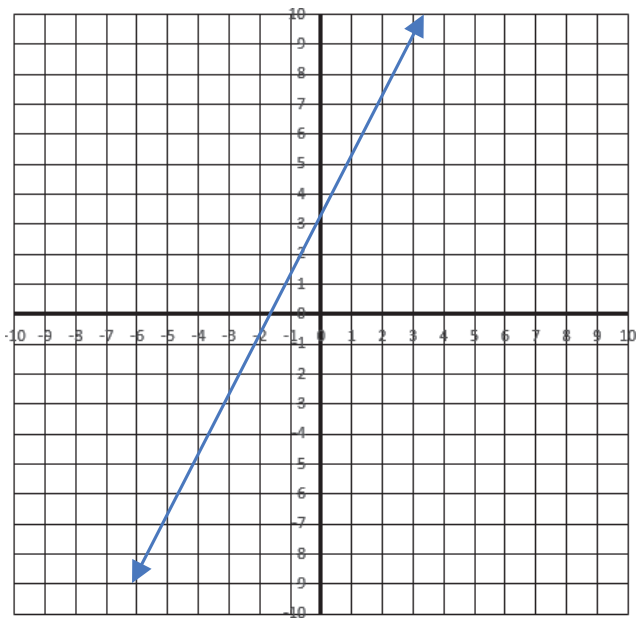
   

   B. $y = -3x + 5$

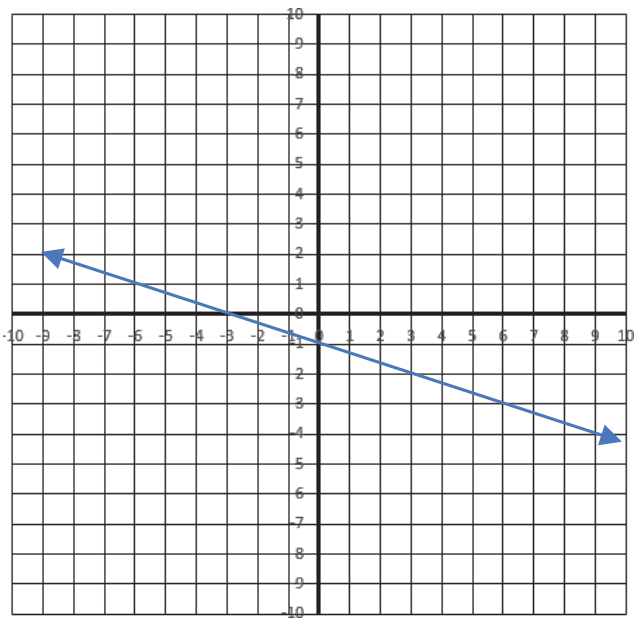
   

2. Write the equation of the line in slope-intercept form.

   A. _____

   B. _____

3. A. Why is $y = mx + b$ called "slope-intercept" form and $y - y_1 = m(x - x_1)$ is called "point-slope" form?

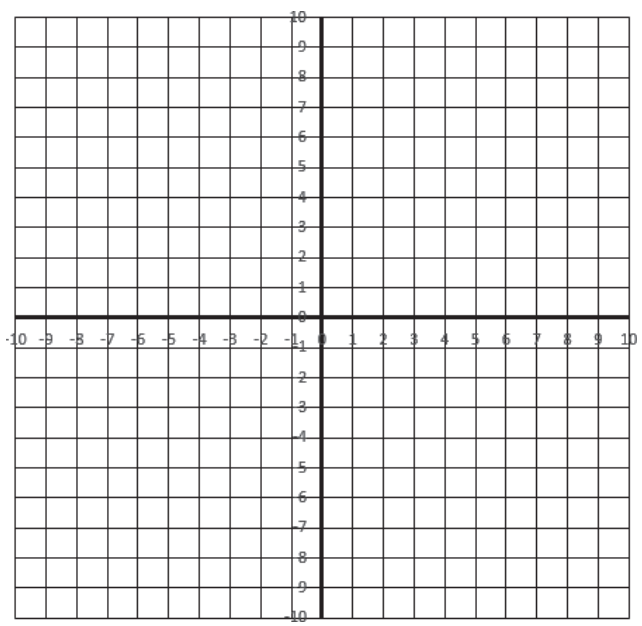
   B. If you wanted to change an equation from point-slope to slope-intercept form, what would you do?

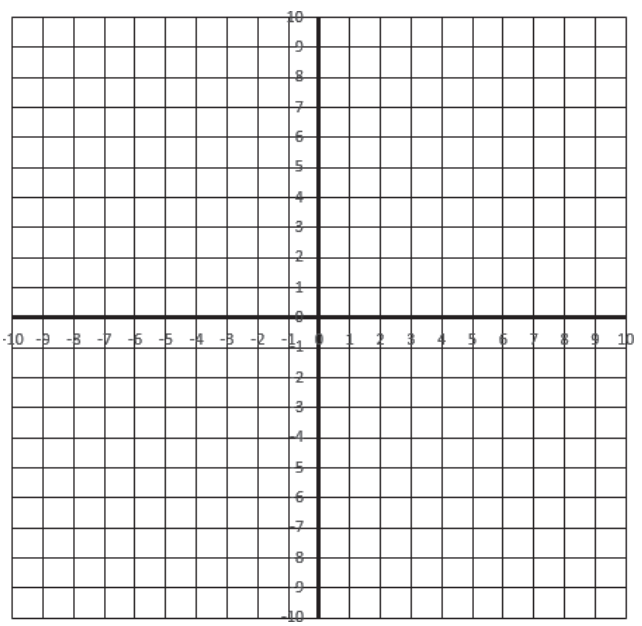
4. Change $y - 3 = -2(x + 1)$ to slope-intercept form.

5. Write the equation of the line in point-slope form. Then change the equation to slope-intercept form and graph the equation to see if your calculations were correct.

   A. (5, 1) and $m = 2$

   B. (2, −2) and (10, 2)





Point-Slope: _____ _____

Slope-Intercept: _____ _____

6. Write the equation in both point-slope and slope-intercept forms.

   A. (2, 5) and (5, −1)

   Point-slope form: _____

   Slope-intercept form: _____

   B. (0, 5) and $m = \frac{1}{3}$

   _____

   _____

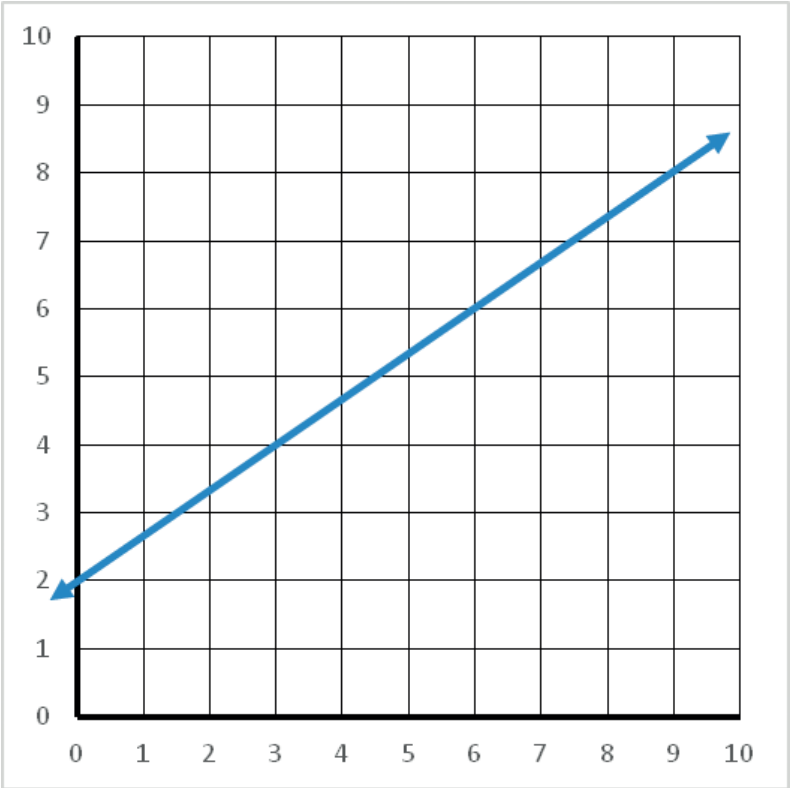
7. Write the equation of the line represented by the data in the table. Write your answer in slope-intercept and point-slope forms.

x	−2	−1	0	1	2	3
y	9	7	5	3	1	−1

   Point-slope form: _____

   Slope-intercept form: _____

8. Write the equation of the line in point-slope and slope-intercept forms.

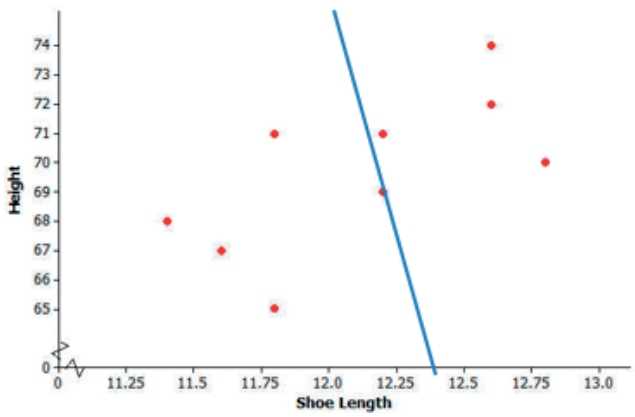


Point-slope form: _____

Slope-intercept form: _____

Spiral REVIEW—Lines of Best Fit

9. Rachel misunderstood how a line of best fit works. Her line is shown below. What might have Rachel been considering when she drew this line?



10. Josh drew the line of best fit as shown below. Explain to Josh why his line is not appropriate for this data.