LESSON 9

Analyzing Standard Deviation

LEARNING OBJECTIVES

- Today I am: using dot plots to analyze standard deviation.
- So that I can: draw hypothetical dot plots based on the standard deviation.
- I'll know I have it when I can: make predictions about the standard deviation and possible changes to it.

Exploratory Challenge 1—Analyzing Standard Deviation

1. A group of people attended a talk at a conference. At the end of the talk, ten of the attendees were given a questionnaire that consisted of four questions. The questions were optional, so it was possible that some attendees might answer none of the questions, while others might answer 1, 2, 3, or all 4 of the questions. The possible numbers of questions answered are 0, 1, 2, 3, and 4.

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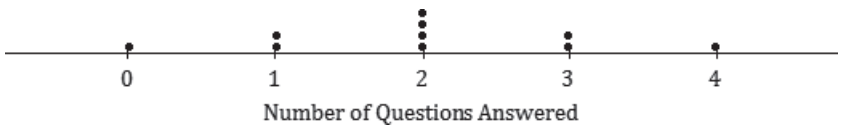
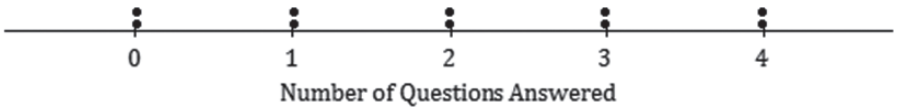
Suppose that the numbers of questions answered by each of the ten people were as shown in the dot plot below. The mean is 2 questions and the standard deviation is 1.15 questions.



Describe the shape of the dot plot. How does this relate to the mean and standard deviation?

Symmetric, bell curve

2. Suppose the dot plot looked like this instead:



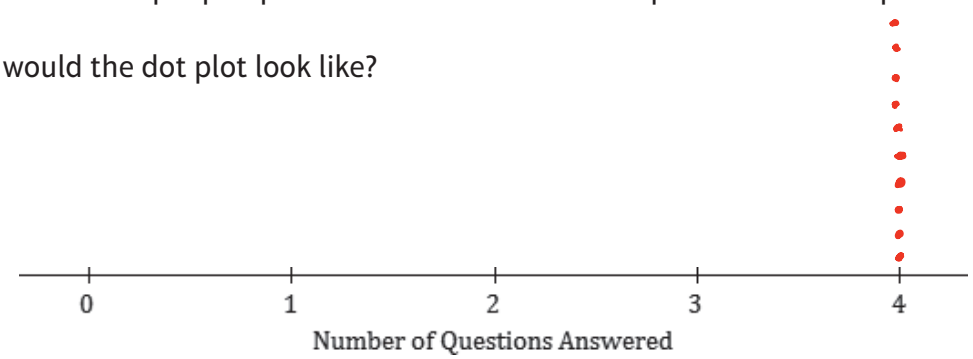
The mean is 2 questions and the standard deviation is 1.49 questions.

Remember that the size of the standard deviation is related to the size of the deviations from the mean. Explain why the standard deviation of this distribution is greater than the standard deviation in Exercise 1.

The standard deviation is greater since the data are spread out.

3. Suppose that all ten people questioned answered all four questions on the questionnaire.

   A. What would the dot plot look like?



   B. What is the mean number of questions answered? (You should be able to answer without doing any calculations!)

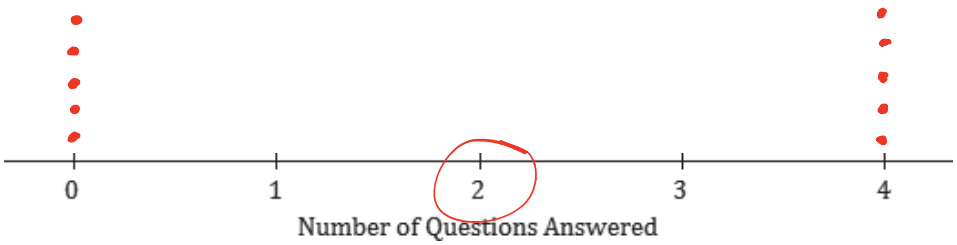
4

   C. What is the standard deviation? (Don't do any calculations!)

0, no variation

4. Continue to think about the situation previously described where the numbers of questions answered by each of ten people was recorded.

   A. Draw the dot plot of the distribution of possible data values that has the largest possible standard deviation. (There were ten people at the talk, so there should be ten dots in your dot plot.) Use the scale given below.

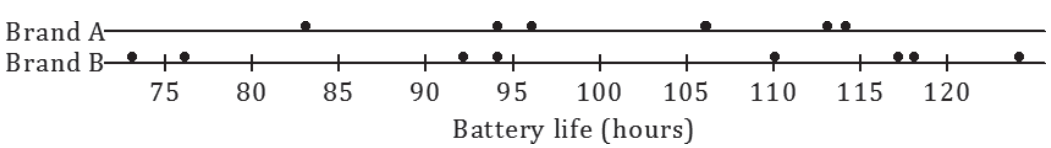


   B. Explain why the distribution you have drawn has a larger standard deviation than the distribution in Exercise 3.

The data points are furthest away from the mean (2).

5. A consumers' organization is planning a study of the various brands of batteries that are available. They are interested in how long a battery can be used before it must be replaced. As part of its planning, it measures lifetime for each of six batteries of Brand A and eight batteries of Brand B. Dot plots showing the battery lives for each brand are shown below.

   How would the standard deviation change if **every** battery tested in Brand A actually ended up lasting 3 hours longer than originally noted? Do NOT calculate the standard deviation. What about Brand B?



* The standard deviation would remain the same why?

6. If they tested 1 extra battery in Brand A and it lasted 200 hours, how would that affect the standard deviation—don't recalculate, but do explain/support your claim.

This would increase the standard deviation.

Lesson Summary

- For any given value in a data set, the deviation from the mean is the value minus the mean. Written algebraically, this is $x - \bar{x}$.
- The greater the variability (spread) of the distribution, the greater the deviations from the mean (ignoring the signs of the deviations).
- The standard deviation measures a typical deviation from the mean.
- To calculate the standard deviation,
  1. Find the mean of the data set;
  2. Calculate the deviations from the mean;
  3. Square the deviations from the mean;
  4. Add up the squared deviations;
  5. Divide by $n - 1$ (if you are working with a data from a sample, which is the most common case);
  6. Take the square root.
- The unit of the standard deviation is always the same as the unit of the original data set.
- The larger the standard deviation, the greater the spread (variability) of the data set.
- The size of the standard deviation is related to the sizes of the deviations from the mean. Therefore, the standard deviation is minimized when all of the numbers in the data set are the same and is maximized when the deviations from the mean are made as large as possible.

NAME: _____ PERIOD: _____ DATE: _____

Homework Problem Set

1. A large city, which we will call City A, holds a marathon. Suppose that the ages of the participants in the marathon that took place in City A were summarized in the histogram below on the left.

   A. Make an estimate of the mean age of the participants in the City A marathon.

   B. Make an *estimate* of the standard deviation of the ages of the participants in the City A marathon. Which has a larger standard deviation?

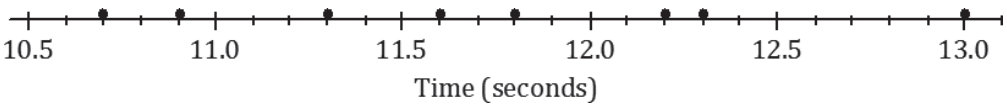
A smaller city, City B, also held a marathon. However, City B restricts the number of people of each age category who can take part to 100. The ages of the participants are summarized in the histogram above to the right.

   C. Approximately what was the mean age of the participants in the City B marathon? Approximately what was the standard deviation of the ages?

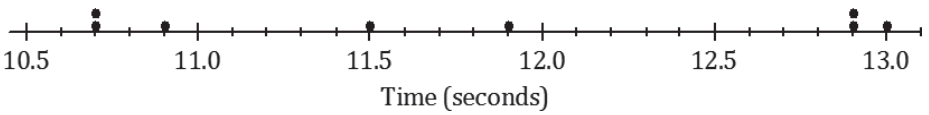
   D. Explain why the standard deviation of the ages in the City B marathon is greater than the standard deviation of the ages for the City A marathon.

2. At a track meet, there are three men's 100-meter races. The times for eight of the sprinters are recorded to the nearest $\frac{1}{10}$ of a second. The results of the three races for these eight sprinters are shown in the dot plots below.

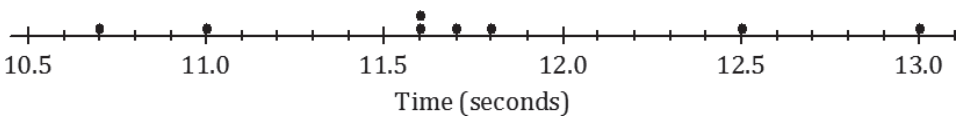
Race 1



Race 2



Race 3



A. Remember that the size of the standard deviation is related to the sizes of the deviations from the mean. Without doing any calculations, indicate which of the three races has the smallest standard deviation of times. Justify your answer.

B. Which race had the largest standard deviation of times? (Again, don't do any calculations!) Justify your answer.

C. Roughly what would be the standard deviation in Race 1? (Remember that the standard deviation is a typical deviation from the mean. So, here you are looking for a typical deviation from the mean, in seconds, for Race 1.)

D. Use your calculator to find the mean and the standard deviation for each of the three races. Write your answers in the table below to the nearest thousandth.

	Mean	**Standard Deviation**
Race 1		
Race 2		
Race 3		

E. How close were your answers for Parts A – C to the actual values?