

LESSON

10

Excluded Values and the Zero Product Property

LEARNING OBJECTIVES

- Today I am: watching a YouTube video about why you can't divide by zero.
- So that I can: understand why this is so important in mathematics.
- I'll know I have it when I can: write an equation with 14, 2 and 0 as excluded values.

Opening Exercise

Watch the YouTube video of the TED-Ed cartoon *Why can't you divide by zero?* at <https://www.youtube.com/watch?v=NKmGVE85GUU>.




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Complete each statement.

- | | |
|---|---|
| <p>1. Normally, dividing by smaller numbers</p> <ul style="list-style-type: none">A. Gives you smaller numbers as answersB. Makes you need to use decimal pointsC. Gives you larger numbers as answersD. Results in a negative number as an answer | <p>2. Dividing by a number is essentially the _____.</p> <ul style="list-style-type: none">A. Reverse of subtracting by it.B. Same as multiplying by it.C. Reverse of multiplying by itD. Same as infinity |
|---|---|

<p>3. The product of any number and its multiplicative inverse is always _____.</p> <p>A. The number itself</p> <p>B. 1</p> <p>C. 0</p> <p>D. $\frac{1}{2}$</p>	<p>4. It is impossible to find the multiplicative inverse of zero because _____.</p> <p>A. Anything multiplied by zero is zero</p> <p>B. Anything multiplied by zero is one</p> <p>C. Anything multiplied by zero is infinity</p> <p>D. Anything multiplied by zero is undefined</p>
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5. **Discussion** Why might it be beneficial to break mathematical rules?

 $-\sqrt{-1} \rightarrow i$
 - non Euclidean Geometry (sphere)

6. Consider the equation $\frac{1}{x} = \frac{3}{x-2}$.

A. What values of x would lead to division by 0?
 Why is division by 0 a problem?

0 and 2
 Excluded value
 $x \neq 0, 2$



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B. Thinking back on the work you've done in this module. What could you do to isolate the variable?

$$\frac{1}{x} = \frac{3}{x-2}$$

(The equation above is crossed out with green arrows pointing to the denominators x and x-2, indicating the next step in solving for x.)

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- C. Explain what Avery did in his first two steps and then finish finding the solution.

$$\frac{1}{x} = \frac{3}{x-2}$$

Cross
mult.

$$\frac{1}{x} \cdot \frac{x-2}{x-2} = \frac{3}{x-2} \cdot \frac{x-2}{x-2}$$

$$(x-2)(x)\left(\frac{1}{x}\right) = (x-2)(x)\left(\frac{3}{x-2}\right)$$

Step 1: $\frac{x-2}{-x} = \frac{3x}{-x}$

$$(x-2)(1) = (x)(3)$$

Step 2: $\frac{-2}{-x} = \frac{2x}{-x}$

- D. Are the excluded values solutions to this problem?

No $x \neq 0, 2$

$$\boxed{-1 = x}$$

7. Consider the equation $\frac{3}{x-2} = \frac{5}{x-2}$.

- A. What values of x would lead to division by 0?

- B. Clear the fraction to find the value of x . You can complete the problem in the box on the right.

- C. How could you tell by looking at the original equation that the solution was going to be different?

$$\frac{3}{x-2} = \frac{5}{x-2} \quad x \neq 2$$

$$3(x-2) = 5(x-2)$$

$$\begin{array}{r} 3x - 6 = 5x - 10 \\ -3x \quad -3x \end{array}$$

$$-6 = 2x - 10$$

$$\begin{array}{r} +10 \quad +10 \end{array}$$

$$4 = 2x$$

$$2 = x$$

But $x \neq 2 \rightarrow$ no solution

When solving equations that have a variable in the denominator, it is critical that you **exclude** values that would make the denominator equal to zero.

no-no! value(s)

Determine the excluding the value(s) of x that lead to a denominator of zero for each equation; then, solve the equation for x .

8. $\frac{5}{x} = 1$ $x \neq$ 0

$5 = x$

9. $\frac{1}{x-5} = 3$ $x \neq$ _____

10. $\frac{x+1}{x+1} = 4$ $x \neq$ -1

$x+1 = 4(x+1)$
 $x+1 = 4x+4$
 $-x = -x$
 $1 = 3x+4$
 $-4 = -4$
 $-3 = 3x \rightarrow x = -1$
 no solution

11. $\frac{2}{x} = \frac{3}{x-4}$ $x \neq$ 0, 4

$2(x-4) = 3x$
 $2x-8 = 3x$
 $-8 = x$

12. $\frac{3}{x+6} = -\frac{6}{x+6}$ $x \neq$ -6

$\frac{3}{x+6} = \frac{-6}{x+6}$ No solution

$3x+18 = -6x-36$
 $+6x$ $+6x$
 $9x+18 = -36$
 $9x = -54$
 ~~$x = -6$~~

13. $\frac{x-3}{x+2} = 0$ $x \neq$ -2

$x-3 = 0(x+2)$
 $x-3 = 0$
 $x = 3$

We saw in the TED-Ed video **and** we know from past experience that any number multiplied by zero is zero. This leads us to the Zero Product Property.

Zero Product Property
 If $a \cdot b = 0$, then either $a = 0$, $b = 0$ or $a = b = 0$.

$5 \cdot 0 = 0$ $0 \cdot 0 = 0$
 $0 \cdot 5 = 0$

14. If $(x - 3) = 0$, what do you know must be true about x ?

$x = 3$

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15. If $x(x - 3) = 0$, what do you know must be true about x ?

$3 \cdot 0$
 $3(3-3)$

$x = 0, 3$

16. If $(x + 4)(x - 3) = 0$, what do you know must be true about x ?

$x = -4$ or 3 $(x+4)=0$

$(x-3)=0$

$a \cdot b = 0$
 \uparrow \uparrow
 0 0
 $(x+4)(x-3)=0$
 -4 or 3

Practice Exercises

For each equation below, determine the solution.

17. $(y - 2)(y - 3) = 0$

$y = 2$ or 3

18. $(x + 5)(x + 3) = 0$

$x = -5$ or -3

19. $b(b - 1) = 0$

$b = 0, 1$
 $b = 0$
 $b - 1 = 0$

20. $(2c - 1)(3c + 3) = 0$

$2c - 1 = 0$ $3c + 3 = 0$
 $+1$ $+1$
 $2c = 1$ $3c = -3$
 $c = \frac{1}{2}$ $c = -1$

21. $2p(3p - 7) = 0$

$2p = 0$ $3p - 7 = 0$
 $p = 0$ $p = \frac{7}{3}$

22. $(-x + 5)(x - 5) = 0$

23. $a(2a - 1)(3a + 8) = 0$

$a = 0$ $2a - 1 = 0$ $3a + 8 = 0$
 $2a = 1$ $3a = -8$
 $a = \frac{1}{2}$ $a = -\frac{8}{3}$

24. $(\frac{1}{2}x + 2)(\frac{2}{3}x - 6) = 0$

25. $(8b + 1)(b + 8) = 0$

Lesson Summary

Applying the distributive, commutative, and associative properties and the properties of equality to equations will not change the solution set.

When solving equations, be careful to exclude any solutions that would make the denominator equal to 0.

The equation $\frac{2x}{x-2} = 4$ has an excluded value of 2.

The Zero Product Property states that if $a \cdot b = 0$, then either $a = 0$, $b = 0$ or $a = b = 0$.

NAME: _____ PERIOD: _____ DATE: _____

Homework Problem Set

1. Consider the equation $\frac{10(x^2 - 49)}{3x(x - 4)(x + 1)} = 0$. Is $x = 7$ permissible? Which values of x are excluded? (You do not need to solve this equation.)

Determine the excluding the value(s) of x that lead to a denominator of zero for each equation; then, solve the equation for x .

2. $\frac{2}{x} = \frac{5}{x+1}$ $x \neq$ _____

3. $\frac{1}{5x} = 10$ $x \neq$ _____

4. $\frac{x+3}{x+3} = 5$ $x \neq$ _____

5. $\frac{x+3}{x+3} = 1$ $x \neq$ _____

For each equation, determine the solution(s) using the Zero Product Property.

6. $(b - 4)(3b - 1) = 0$

7. $(n + 3)(6n + 1) = 0$

8. $(r + 5)(r - 3) = 0$

9. $(v + 8)(2v - 5) = 0$

10. $(3p - 5)(p - 2) = 0$

11. $(2x + 1)(7x + 5) = 0$

12. $(3x - 5)(3x + 1) = 0$

13. $(x - 2)(x + 3)(x - 4) = 0$

14. $\left(\frac{2}{5}x + 4\right)\left(\frac{1}{3}x - 5\right) = 0$

Determine the excluded value for each equation. You do NOT need to solve the equation.

15. $\frac{3}{x-7} = 5$

16. $-4 = \frac{3}{x+4}$

17. $\frac{(x-2)(x+1)}{(x-1)(x+1)} = 7$

18. $\frac{(x-3)}{(x-3)(x+4)} = \frac{(x+4)}{(x+4)}$

19. $10 = \frac{(x+3)(x+5)}{(x+5)(x+6)}$

20. $-2 = \frac{4-x}{6}$

21. **Challenge** Write an equation with the restrictions $x \neq 14$, $x \neq 2$, and $x \neq 0$.

22. **Challenge** Use any of the digits 1–9 to create an equation with the smallest solution possible.

$$\frac{\square}{\square} x - \square = \square$$