LESSON 14

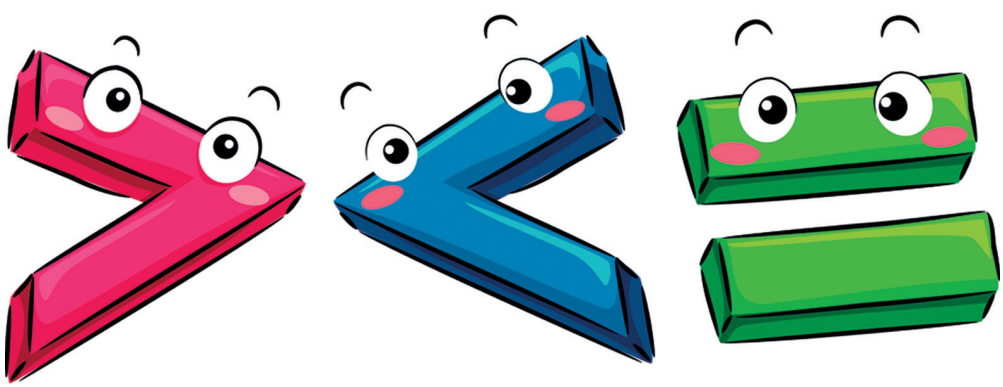
Solving Inequalities

LEARNING OBJECTIVES

- Today I am: exploring solutions to inequalities.
- So that I can: apply equation rules to inequalities.
- I'll know I have it when I can: write a rule for multiplying or dividing by a negative number in an inequality.

Opening Exercise

Solving inequalities is very similar to solving equations. But there is a situation that doesn't work for inequalities. Can you find it?



Determine the solution for each equation and inequality. Write the rule you used to solve the problems. One example has been done for you. Remember, one of these will cause problems in the inequality.

	Equation	Rule Used	Inequality	Rule Used
Example	$x + 4 = 10$ $x = 6$ Check: $6 + 4 = 10$	Subtraction Property	$x + 4 < 10$ $x < 6$ Check: If $x = -2$, then $-2 + 4 < 10$.	Subtraction Property
1.	$x - 4 = 10$ $x = 14$	Add prop	$x - 4 \geq 10$ +4 +4 $x \geq 14$	
2.	$2x = 10$ $x = 5$	divide prop	$2x > 10$ $x > 5$	
3.	$-5x = 10$ $x = -2$	division prop	$-5x \leq 10$ $x \leq -2$ check $-5(-3) \leq 10$ $15 \leq 10$	?
4.	$\frac{x}{5} = 10$ $x = 50$	mult. prop.	$\frac{x}{5} < 10$ $x < 50$	

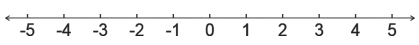
5. **Discussion** Which inequality didn't follow the equation rules?

#3

You can use various properties to solve multi-step inequalities, just like you can with solving equations. For now, we'll skip those that require division or multiplication by negative values like Exercise 3 in the Opening Exercise.

Find the solution set to each inequality. Express the solution graphically on the number line and give the solution in interval notation.

6. $x + 4 \leq 7$

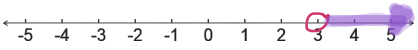


7. $\frac{m}{3} + 8 > 9$

$\frac{m}{3} > 1$

$m > 3$

$(3, \infty)$



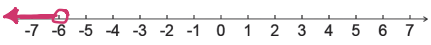
8. $8y + 4 < 7y - 2$

$-7y \quad -7y$

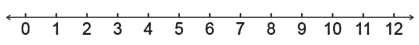
$y + 4 < -2$

$y < -6$

$(-\infty, -6)$



9. $6(x - 5) \geq 30$



10. $4(x - 3) > 2(x - 2)$

$4x - 12 > 2x - 4$

$-2x \quad -2x$

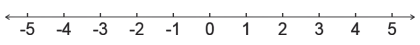
$2x - 12 > -4$

$2x > 8$

$x > 4$

$(4, \infty)$

11. $\frac{x - 2}{3} > 1$



Multiplying or Dividing by a Negative Value

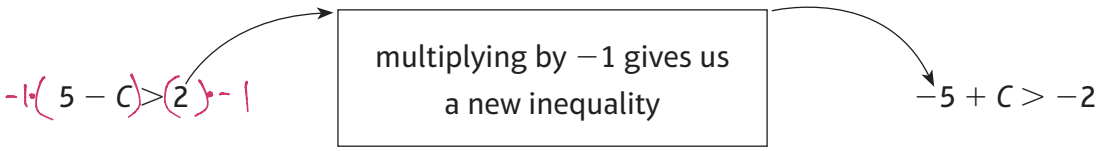
12. Stephanie says, **"So far we have the following rules for inequalities:**

If $A > B$, then $A + c > B + c$ for any real number c.

If $A > B$, then $kA > kB$ for any positive real number k."

Explain to your partner what Stephanie means by these statements. Be prepared to share out loud with the class.

Stephanie is quite clear in her rules that you cannot multiply by a negative number. Let's see what happens if we do multiply by -1 with the inequality $5 - C > 2$. This will help us understand Exercise 3.



13. Find one number that works for the first inequality. Does it work for the second inequality?

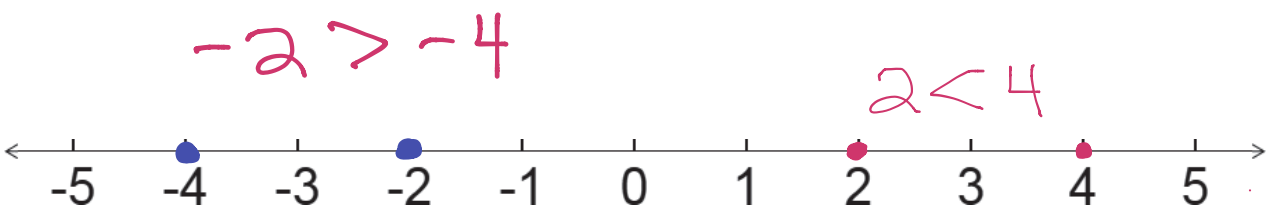
$5 - C > 2$

C=1 5-1>2
4>2 ✓

$-5 + C > -2$

C=1 -5+1>-2
-4>-2 ?
False

14. A. Let's look at why this happens. If we choose two numbers on the number line, let's say 2 and 4 and mark them. We can see that $2 < 4$.



B. Now multiply our two numbers by -1 and mark these new numbers on the number line.

(-1)2 < 4(-1)
-2 > -4

You should now see that if we don't change the inequality symbol we would have $-4 > -2$. But we know that $-4 < -2$! Therefore, we need a new rule. What does multiplying by -1 do to our inequality?

The Properties of Inequalities

- Addition property of inequality: If $A > B$, then $A + c > B + c$ for any real number c.
- Multiplication property of inequality: If $A > B$, then $kA > kB$ for any positive real number k.

15. Use the properties of inequality to show that each of the following is true for any real numbers p and q.

A. If $p \geq q$, then $-p \leq -q$.

B. If $p < q$, then $-5p > -5q$.

16. Based on the results from Exercises 14 and 15, how might we expand the multiplication property of inequality?

- Multiplication property of inequality: If $A > B$, then $kA > kB$ for any positive real number k.
- Multiplication property of inequality: If $A > B$, then $kA < kB$ for any negative real number k.

flip the inequality

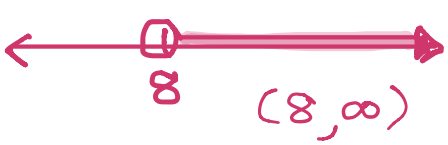
* Flip the inequality when mult/div by a negative number.

17. Find the solution set to each inequality. Express the solution graphically on a number line and in interval notation.

A. $-2f < -16$

$\frac{-2}{-2} \quad \frac{-16}{-2}$

$f > 8$



$(8, \infty)$

B. $-\frac{x}{12} \leq \frac{1}{4}$

$(-12) \quad (-12)$

$\frac{-3}{1}$

$x \geq -3$

$[-3, \infty)$

C. $6 - a \geq 15$

$-6 \quad -6$

$\frac{-a}{-1} \geq \frac{9}{-1}$

$a \leq -9$

$(-\infty, -9]$

D. $-3(2x + 4) > 0$

$-6x - 12 > 0$

$\frac{-6x}{-6} > \frac{12}{-6}$

$x < -2$

$(-\infty, -2)$

You will need: Inequality dice

18. With your partner, roll the inequality dice to get two expressions and an inequality symbol. Record your inequality in the table below. Then solve it, graph it on the number line, and give the solution in interval notation.

1st Expression	Inequality Symbol	2nd Expression	Graph of Solution	Solution in Interval Notation
Work to solve the inequality				

Lesson Summary

The Properties of Inequalities

- Addition property of inequality: If $A > B$, then $A + c > B + c$ for any real number c.
- Multiplication property of inequality: If $A > B$, then $kA > kB$ for any positive real number k. If $A > B$, then $kA < kB$ for any negative real number k.

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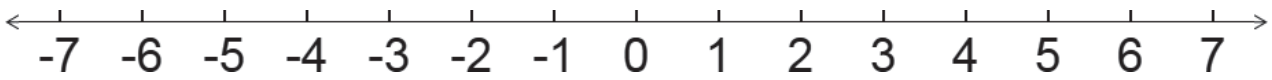
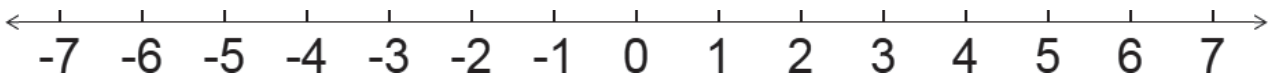
Homework Problem Set

Find the solution set to each inequality. Express the solution graphically on the number line and give the solution in interval notation.

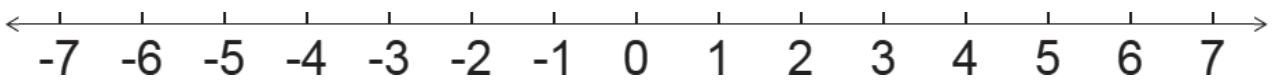
1. $2x < 10$



2. $-15x \geq -45$

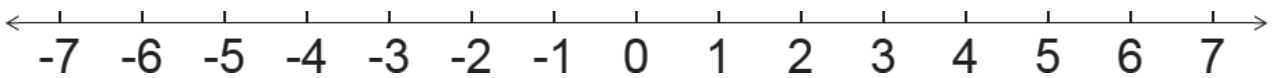


3. $6\left(\frac{2}{3}x\right) < \left(\frac{1}{2} + 2\right)6$

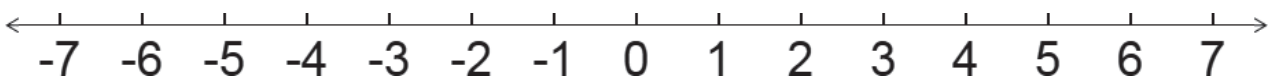


$4x < 3 + 12$

4. $-5(x-1) \geq 10$



5. $13x < 9(1-x)$



6. Solve $-\frac{x}{16}+1 \geq -\frac{5x}{2}$, for x without multiplying by a negative number. Then, solve by multiplying on both sides by -16.

Solving without multiplying by a negative number	**Solving by multiplying by -16**
$-\frac{x}{16}+1 \geq -\frac{5x}{2}$	$-\frac{x}{16}+1 \geq -\frac{5x}{2}$

7. Lisa brought half of her savings to the bakery and bought 12 croissants for $14.20. The amount of money she brings home with her is more than $2.00. Use an inequality to find how much money she had in her savings before going to the bakery. (Write the inequality that represents the situation, and solve it.)

8. Solve $-18 - 16t > -6 - 100t$, for t in two different ways: first without ever multiplying on both sides by a negative number and then by first multiplying on both sides by $-\frac{1}{2}$ or dividing by -2.

Solving without multiplying by a negative number	**Solving by multiplying by $-\frac{1}{2}$ or dividing by -2**
$-18 - 16t > -6 - 100t$	$-18 - 16t > -6 - 100t$

Find the solution set to each inequality. Express the solution in interval notation.

9. $2x + 4 \geq 24$

10. $\frac{m}{3} - 3 \leq -6$

11. $-3(p + 1) < 18$

12. $-4(-4 + x) > 56$

13. $-b - 2 > 8$

14. $-4(3 + n) > -32$

15. $4 + \frac{n}{3} < 6$

16. $-3(r - 4) \geq 0$

17. $3(p - 7) > -21$

18. $7x - 7 < -56$

19. $\frac{-9 + a}{15} > 1$

20. $-11x - 4 > -15$

Spiral REVIEW—Solving Absolute Value Equations

Solve each absolute value equation for the variable. Be sure to check your solution.

21. $|3x| = 9$

22. $|-3r| = 9$

23. $\left|\frac{b}{5}\right| = 1$

24. $|-6m| = 30$

25. $\left|\frac{n}{3}\right| = 2$

26. $|-4 + 5x| = 16$

27. $|-2r - 1| = 11$

28. $|1 - 5a| = 29$

29. $3|-8x| + 8 = 80$