LESSON 15

"And" or "Or" or "Not"

LEARNING OBJECTIVES

- Today I am: determining if claims written with "and", "or" or "not" are true or false.
- So that I can: see how this relates to inequalities.
- I'll know I have it when I can: write inequalities based on real-life contexts and then solve them.

Opening Exploration

1. Determine whether each claim given below is true or false.

   A. Right now, I am in math class and English class. F

   B. Right now, I am in math class or English class. T

   C. I am 14 years old or I am not 14 years old. T

   D. I am 14 years old and I am not 14 years old. F

These are all examples of declarative compound sentences.

2. When the two declarations in the sentences above were separated by "and," what had to be true to make the statement true? Both sentences have to be true.

> A compound sentence formed by joining statements with "and" is a <u>conjunction</u>.

3. When the two declarations in the sentences above were separated by "or," what had to be true to make the statement true? At least one of the two sentences has to be true

> A compound sentence formed by joining statements with "or" is a <u>disjunction</u>. not together.

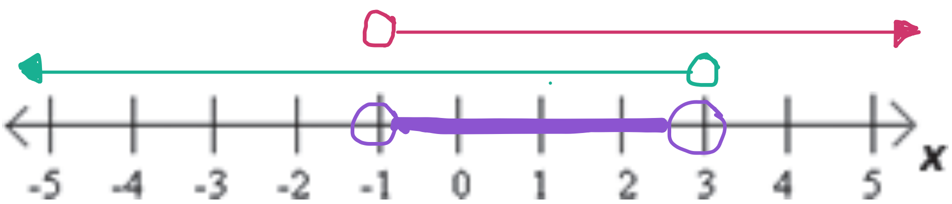
You will need: three colored pencils or highlighters.

4. A. Using a colored pencil, graph the inequality $x < 3$ on the number line below.

   B. Using a different colored pencil, graph the inequality $x > -1$ on the same number line.

   C. Using a third colored pencil, darken the section of the number line where $x < 3$ **<u>and</u>** $x > -1$.

   $-1 < x < 3$ ✓



   D. In order for the compound sentence $x > -1$ and $x < 3$ to be true, what has to be true about x?

   X has to be greater than -1 but less than 3

   E. On the graph, where do the solutions lie?

   Between -1 and 3 not including -1 and 3

F. How many solutions are there to this compound inequality?

Infinitely many

G. Sometimes this is written as $-1 < x < 3$. Explain how this relates to the graph.

small big

x can be any number between -1 and 3.

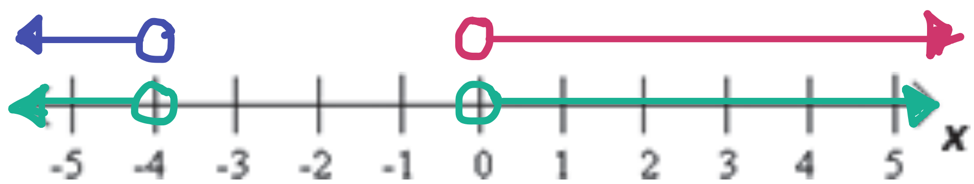
H. **Interval notation** would show the solution as $(-1, 3)$. How does this relate to the graph?

$-1 < x < 3 \rightarrow (-1, 3)$

5. A. Using a colored pencil, graph the inequality $x < -4$ on the number line below.

B. Using a different colored pencil, graph the inequality $x > 0$ on the same number line.

C. Using a third colored pencil, darken the section of the number line where $x < -4$ **or** $x > 0$.



D. In order for the compound sentence $x < -4$ or $x > 0$ to be true, what has to be true about x?

x can be any number less than -4 or greater than 0.

E. On the graph, where do the solutions lie?

To the left of -4 OR to the right of 0.

F. How many solutions are there to this compound inequality?

Infinitely many ∞

x > 0 or x < -4

G. Would it be acceptable to abbreviate this compound sentence as follows: $0 < x < -4$? Explain.

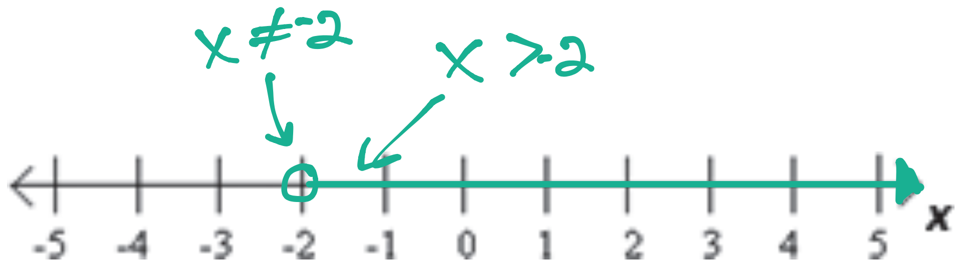
Not possible.

x < -4 x > 0

H. In interval notation we would write $(-\infty, -4)$ or $(0, \infty)$. What does $-\infty$ and ∞ mean?

≠ is the symbol for "not equal to"

6. A. Graph the compound sentence $x \geq -2$ or $x \neq -2$ on the number line below.



B. How could we abbreviate the sentence $x \geq -2$ or $x \neq -2$?

x > -2

C. How could we write this in interval notation?

(-2, ∞)

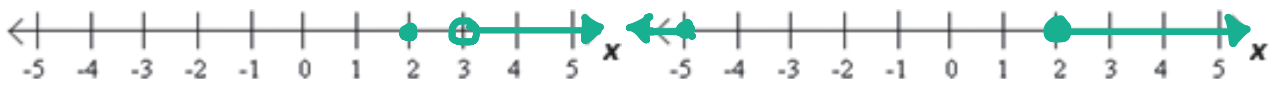
7. Rewrite $x \leq 4$ as a compound sentence, and graph the solutions to the sentence on the number line below. Then write this in interval notation.

x < 4 or x = 4

8. Graph each compound sentence on a number line. Then give the solution in interval notation.

A. $x = 2$ or $x > 3$

B. $x \leq -5$ or $x \geq 2$

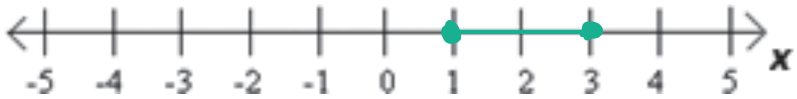


9. A. Rewrite $1 \leq x \leq 3$ as a compound sentence, and graph the sentence on a number line.

$1 \leq x$ $x \leq 3$

$x \geq 1$ and

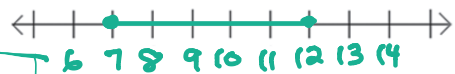
[1, 3]



B. Use interval notation to write the same solution.

[1, 3]

10. Consider the following two scenarios. For each, specify the variable and define what it represents, then write a compound inequality that describes the scenario given. Draw its solution set on a number line.

Scenario	Variable	Inequality Statement	Graph
A. Students are to present a persuasive speech in English class. The guidelines state that the speech must be at least 7 minutes but not exceed 12 minutes.	Let x represent the minutes	$x \geq 7$ and $x \leq 12$; $7 \leq x \leq 12$	 6 7 8 9 10 11 12 13 14
B. Children and senior citizens receive a discount on tickets at the movie theater. To receive a discount, a person must be between the ages of 2 and 12, including 2 and 12, or 60 years of age or older.	Let ___ represent		

11. Zara solved the inequality $18 < 3x - 9$ as shown below. Was she correct?

$$18 < 3x - 9$$
$$+9 \qquad +9$$
$$\frac{27}{3} < \frac{3x}{3}$$
$$9 < x \text{ or } x > 9$$

Yes she was correct $9 < x$ is the same as $x > 9$

12. Consider the compound inequality $-5 < x < 4$.

and

A. Rewrite the inequality as a compound statement of inequality.

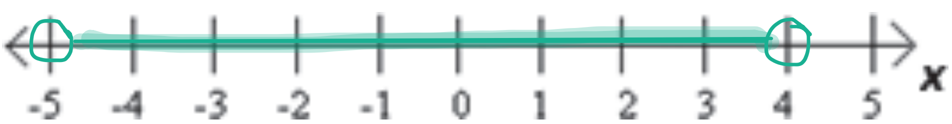
$x < 4$ and $x > -5$

$x > -5$ and $x < 4$

B. Write a sentence describing the possible values of x.

x can be any number between -5 and 4

C. Graph the solution set on the number line below.



D. Rewrite the inequality in interval notation.

$(-5, 4)$

and

13. Consider the compound inequality $-5 < 2x + 1 < 4$.

A. Rewrite the inequality as a compound statement of inequality.

$-5 < 2x + 1$ and $2x + 1 < 4$

$-1 \quad -1 \qquad\qquad -1 \quad -1$

$-6 < 2x \qquad\qquad 2x < 3$

$-3 < x$ and $x < 3/2$

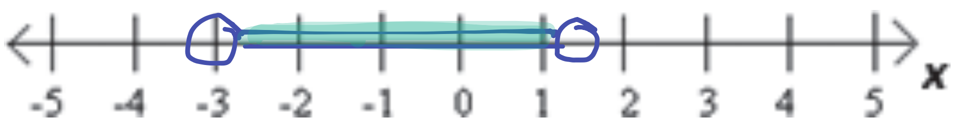
B. Solve each inequality for *x*. Then, write the solution to the compound inequality.

$-3 < x < \frac{3}{2}$

C. Write a sentence describing the possible values of *x*.

x can be any number between -3 and $\frac{3}{2}$ but not -3 or $\frac{3}{2}$

D. Graph the solution set on the number line.



E. Write the solution in interval notation.

$\left(-3, \frac{3}{2}\right)$

14. A friend of mine suggested I could solve the inequality $-5 < 2x + 1 < 4$ as follows. Is she right? Explain your reasoning.

$$-5 < 2x + 1 < 4$$

$$-5 - 1 < 2x + 1 - 1 < 4 - 1$$

$$-6 < 2x < 3$$

$$-3 < x < \frac{3}{2}$$

$-5 < 2x + 1 < 4$

$-1 \quad -1 \quad -1$

$\frac{-6}{2} < \frac{2x}{2} < \frac{3}{2}$

$-3 < x < \frac{3}{2}$

15. Given $x < -3$ or $x > -1$:

A. What must be true in order for the compound inequality to be a true statement?

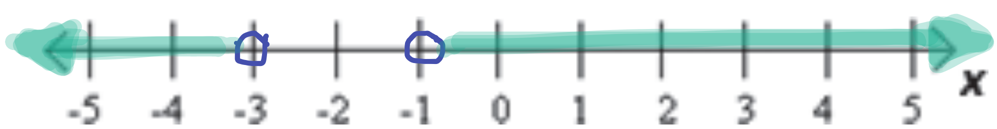
x can be any number less than -3 OR greater than -1

B. Write a sentence describing the possible values of x.

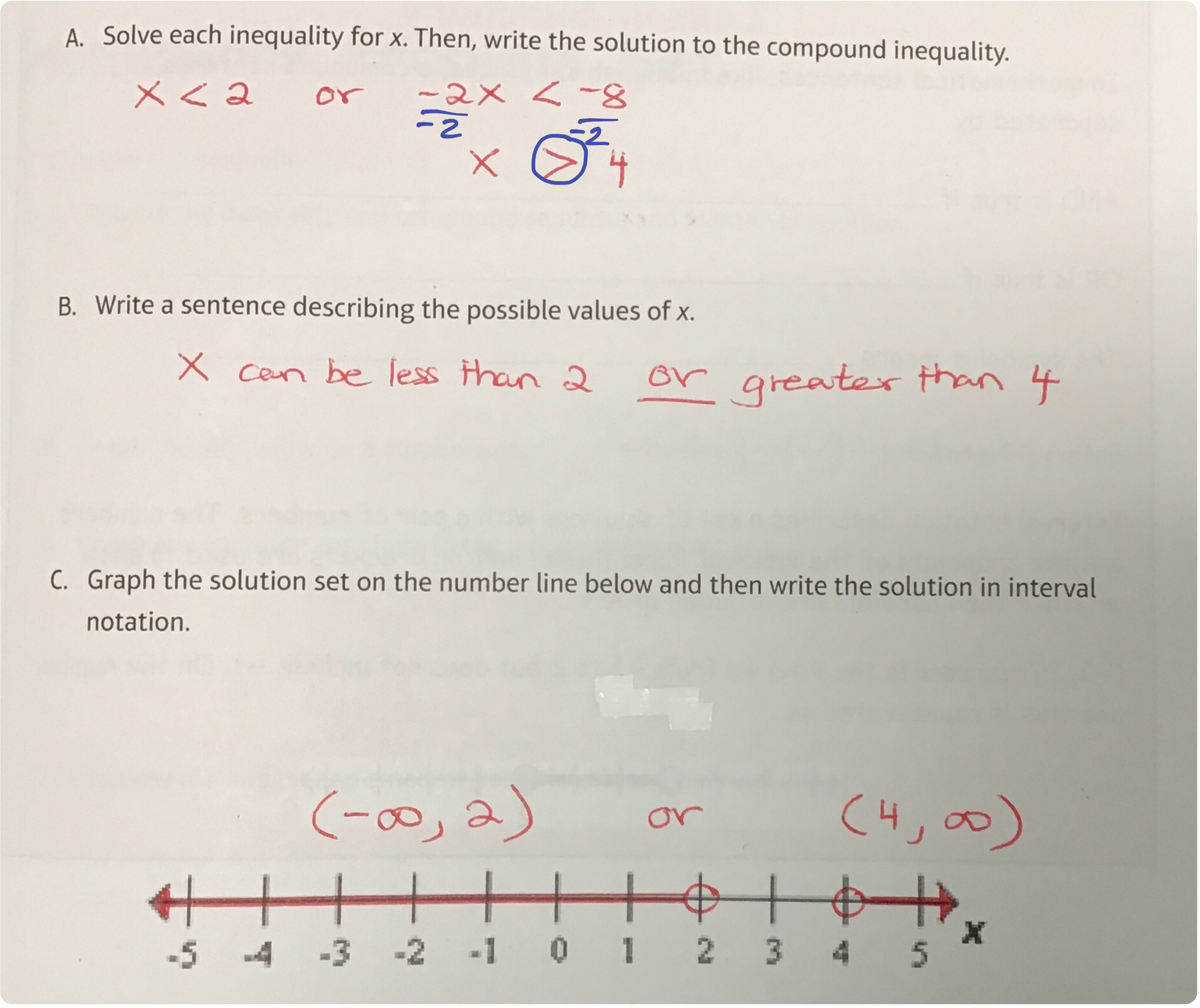
C. Graph the solution set on the number line below and then write the solution in interval notation.

$(-\infty, -3) \cup (-1, \infty)$

or



16. Given $x + 4 < 6$ or $-2x + 2 < -6$:



A. Solve each inequality for x. Then, write the solution to the compound inequality.

$x < 2$ or $-2x < -8$

$\frac{-2x}{-2} < \frac{-8}{-2}$

$x > 4$

B. Write a sentence describing the possible values of x.

x can be less than 2 or greater than 4

C. Graph the solution set on the number line below and then write the solution in interval notation.

$(-\infty, 2)$ or $(4, \infty)$

Lesson Summary

In mathematical sentences, like in English sentences, a compound sentence separated by

AND is true if ____.

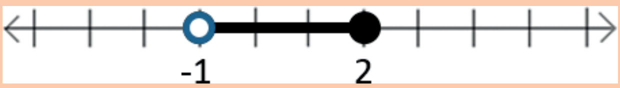
OR is true if ____.

The symbol ≠ means _______________ ______________________.

Interval Notation

Interval notation describes a set of solutions with a pair of numbers. The numbers are the endpoints of the interval. Parentheses and/or brackets are used to show whether the endpoints are included or not.

$(-1, 2]$ represents the interval from -1 to 2 but does not include -1. On the number line this is represented as



NAME: ____________ PERIOD: ____________ DATE: ____________

Homework Problem Set

1. Consider the inequality $0 < x < 3$.

   A. Rewrite the inequality as a compound sentence and in interval notation.

   B. Graph the inequality on a number line.

   C. How many solutions are there to the inequality? Explain.

   D. What are the largest and smallest possible values for x? Explain.

   E. If the inequality is changed to $0 \leq x \leq 3$, then what are the largest and smallest possible values for x?

Write a compound inequality for each graph. Then write it in interval notation.

2.

3.

Write a single or compound inequality for each scenario. Then write it in interval notation.

4. The scores on the last test ranged from 65% to 100%.

5. To ride the roller coaster, one must be at least 4 feet tall.

6. Unsafe body temperatures are those lower than 96°F or above 104°F.

Graph the solution(s) to each of the following on a number line.

7. $x \leq -8$ or $x \geq -1$

8. $3(x - 6) = 3$ or $5 - x \neq 2$

9. $x < 9$ and $x > 7$

10. $x + 5 < 7$ or $x = 2$

11. $x - 4 = 0$ and $3x + 6 = 18$

12. $x < 5$ and $x \neq 0$

Solve each compound inequality for *x*, and graph the solution on a number line. Then write the solution in interval notation.

13. $x + 6 < 8$ and $x - 1 > -1$

14. $-1 \leq 3 - 2x \leq 10$

15. $5x + 1 < 0$ or $8 \leq x - 5$

16. $10 > 3x - 2$ or $x = 4$

Solve each compound inequality for *x*, and graph the solution on a number line.

17. $x - 2 < 4$ or $x - 2 > 4$

18. $x - 2 \leq 4$ and $x - 2 \geq 4$

Solve each compound inequality for *x*, and graph the solution on a number line. Pay careful attention to the inequality symbols and the "and" or "or" statements as you work.

19. $1 + x > -4$ or $3x - 6 > -12$

20. $1 + x > -4$ or $3x - 6 < -12$

21. $1 + x > 4$ and $3x - 6 < -12$

22. A. Solve the inequality $4x + 8 > 2x - 10$ or $\frac{1}{3}x - 3 < 2$ for x, and graph the solution on a number line.

    B. If the inequalities in Part A were joined by "and" instead of "or," what would the solution set become?

23. A. Solve the inequality $7 - 3x < 16$ and $x + 12 < -8$ for x, and graph the solution on a number line.

    B. If the inequalities in Part A were joined by "or" instead of "and," what would the solution set become?

24. A. Is it possible to write a problem separated by "or" that has no solution? Explain or give an example.

B. Is it possible to have a problem separated by "and" that has a solution set consisting of all real numbers? Explain or give an example.

Determine if each sentence is true or false. Explain your reasoning.

25. $8 + 6 \leq 14$ and $\frac{1}{3} < \frac{1}{2}$

26. $5 - 8 < 0$ or $10 + 13 \neq 23$

Solve each system, and graph the solution on a number line.

27. $x - 9 = 0$ or $x + 15 = 0$

28. $5x - 8 = -23$ or $x + 1 = -10$

Graph the solution set to each compound inequality on a number line.

29. $x < -8$ or $x > -8$

30. $0 < x \leq 10$

Write a compound inequality for each graph.

31.

32.

33. A poll shows that a candidate is projected to receive 57% of the votes. If the margin for error is plus or minus 3%, write a compound inequality for the percentage of votes the candidate can expect to get.

34. Mercury is one of only two elements that are liquid at room temperature. Mercury is non-liquid for temperatures less than −38.0°F or greater than 673.8°F. Write a compound inequality for the temperatures at which mercury is nonliquid.

Spiral REVIEW—Solving Absolute Value Equations

Solve the two related equations below. Think about the differences between the two equations and their solutions. Do the solutions make sense?

35. $2\|x + 7\| - 3 = -9$	36. $2\|x + 7\| - 3 = 9$
37. $-7 < x - 4 < -3$	38. $7 < x - 4 < -3$