LESSON 16

Absolute Value Inequalities

LEARNING OBJECTIVES

➤ Today I am: reviewing solving and graphing inequalities.

➤ So that I can: determine the steps to solve absolute value inequalities.

➤ I'll know I have it when I can: solve an absolute value inequality like $|2x + 3| + 6 \geq 5$.
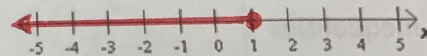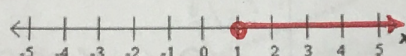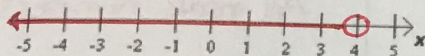
Warm-Up Exercises

1. Use the number lines below to graph each inequality.

   A. $x < 4$ B. $x \geq 1$ C. $2 + x \leq 3$

   $x \leq 1$

2. Determine the solution to each absolute value equation.

   A. $|x| = 4$ B. $|x| = 1$ C. $|2 + x| = 3$

   $x = 4, -4$ $x = 1, -1$ $2 + x = 3$ or $2 + x = -3$

   $x = 1$ or $x = -5$

$\leq$ "and" $\geq$ "or"

Exploratory Exercise

Next, we'll combine two ideas—inequalities and absolute value.

3. For each inequality below, think about all the points that would make the inequality true and plot those points. What type of endpoint should each inequality have? Are there other points that would also make the inequality true?
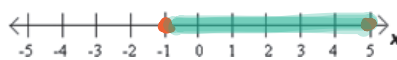
A. $|x| < 4$ *= 4*

All the values that are less than 4 units away from zero.

and <
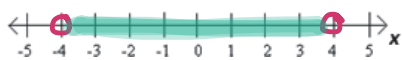
B. $|x| \geq 1$ *→ or*

All the values that are greater than or equal to 1 away from O.
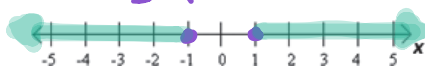
C. $|2 + x| \leq 3$

All the values less than or equal to 3 units away from 2.

4. If we think about the meaning of absolute value as a distance from zero, it leads us to a strategy that doesn't involve guessing. Consider these examples:

	$|x| > 2$	$|x| < 2$	$|x| > -2$
Describe the meaning	All the numbers which are greater than 2 units away from zero.	All the numbers which are _____ than ____ units away from zero.	All the numbers which are _____ than ____ units away from zero.
Represent the solution set on a number line			
Write inequalities to describe the solution set	$x >$ ____ or $x <$ ____		

5.   Work with your partner to decide where the following examples and solutions fit into Tanya's flow chart below.

| No solution. | $x > 2$ or $x < -2$ | All real numbers. |

| $-2 < x < 2$ | | |

| $|x| > 2$ | $|x| > -2$ | $|x| < 2$ | $|x| < -2$ |

| $|x| > 2$ | $|x| > -2$ | $|x| < 2$ | $|x| < -2$ |

Tanja says that there is a better way to get the solutions, rather than just guessing. Below are her steps.

get the abs value by itself

Isolate the absolute value expression on the **left** side of the inequality.

Right side is negative   *Right side is positive*

$|x| > -2$ $|x| < -2$ $|x| > 2$ $|x| < 2$

Your equation either has no solution or all real numbers as solutions. Use the sign of each side of your inequality to decide which of these cases holds.

$|x| > -2$ $|x| < -2$

All real numbers   No solution

Remove the absolute value bars by setting up a compound inequality. The type of inequality sign in the problem will tell us how to set up the compound inequality.

"or"
$|x| > 2$

"and"
$|x| < 2$

Greater than   *Less than*

(stuff inside absolute value) > (number)
OR
(stuff inside absolute value) < – (number)

$X > 2$ or $X < -2$

– (number) < (stuff inside absolute value) < (number)

$x > -2$ and $x < 2$

$-2 < X < 2$

Think "greatOR"!   Think "less thAND"!

and

6. A.   Follow Tanja's steps for the absolute value inequality $|x + 2| \leq 3$. Be sure to show each step.

$-3 \leq x+2 \leq 3$
$-2 \quad -2 \quad -2$

$-5 \leq x \leq 1$

$x+2 \leq 3$ and $x+2 \geq -3$
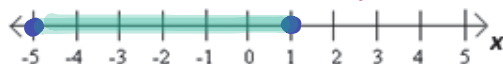
$x \leq 1$ and $x \geq -5$

  B. Graph the solution set on the number line.

Same

  C. Do Tanya's steps work for $\leq$ or $\geq$ absolute value problems? Explain.

7. A.   Follow Tanja's steps for the absolute value inequality $2|3x - 1| < 20$. Be sure to show each step.

  B. Graph the solution set on the number line.

8. A.   Follow Tanja's steps for the absolute value inequality $|2x + 3| + 6 < 5$. Be sure to show each step.

$-6 \quad -6$

$|2x+3| < -1$

No solution

  B. Graph the solution set on the number line.

$1 < 2x+3 < -1$

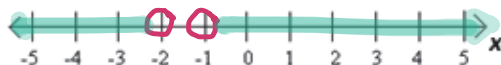9. A. Follow Tanja's steps for the absolute value inequality $|2x + 3| + 6 > 7$. Be sure to show each step.

$$|2x+3| \overset{-6\ -6}{>} 1 \qquad OR$$

$$2x+3 > 1 \text{ or } 2x+3 < -1$$
$$2x > -2 \qquad\qquad 2x < -4$$
$$x > -1 \text{ or } x < -2$$

   B. Graph the solution set on the number line.

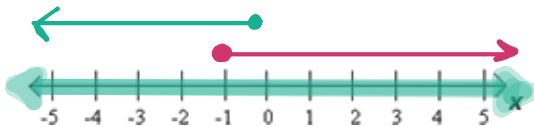10. A. Follow Tanja's steps for the absolute value inequality $|3x + 2| + 6 \geq 5$. Be sure to show each step.

$$|3x+2| \geq -1$$

$$3x+2 \geq -1 \text{ or } 3x+2 \leq 1$$
$$3x \geq -3 \qquad\qquad 3x \leq -1$$
$$x \geq -1 \text{ or } x \leq -\frac{1}{3}$$

   B. Graph the solution set on the number line.

$$(-\infty, \infty)$$

Lesson Summary

Solving Absolute Value Inequalities

$9|m-8|-10>26$

> Begin isolating the absolute value by adding 10 to both sides.

$9|m-8|>36$

> Isolate the absolute value by dividing by 9 on both sides.

$|m-8|>4$

> Create a compound inequality.

$m-8>4$ or $m-8<-4$

> Solve the inequalities.

$$m-8 > 4 \text{ or } m-8 < -4$$
$$\underline{+8 \quad +8} \qquad \underline{+8 \quad +8}$$
$$m \quad > 12 \text{ or } m \quad < 4$$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

$9|m-8|-10<26$

> Begin isolating the absolute value by adding 10 to both sides.

$9|m-8|<36$

> Isolate the absolute value by dividing by 9 on both sides.

$|m-8|<4$

> Create a compound inequality.

$-4 < m-8 < 4$

> Solve the inequality.

$$-4 < m-8 < 4$$
$$\underline{+8 \qquad +8 \quad +8}$$
$$4 < m < 12$$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
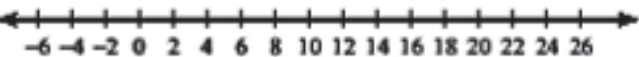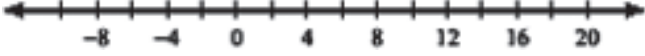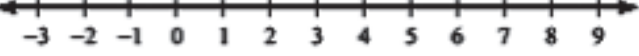
11. A. Draw a number line solution for each example in the Lesson Summary.

B. Give the solutions in interval notation.

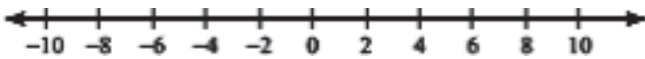NAME: _____ PERIOD: _____ DATE: _____

Homework Problem Set

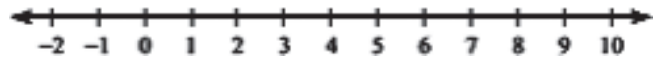**Solve and graph each absolute value inequality.**

1. $|r| < 1$

2. $|p - 10| < 15$

3. $|n| - 9 \geq -5$

4. $-1 + \left|\dfrac{r}{10}\right| > 0$

$+1 \qquad +1$

$\left|\dfrac{r}{10}\right| > 1$

$\dfrac{r}{10} > 1$ or $\dfrac{r}{10} < -1$

5. $|k - 5| > 13$

6. $\dfrac{|4 - 7n|}{3} > 2$

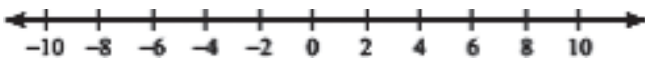| 7. $-10|p| - 7 > -97$	8. $8|-5 + 6m| - 7 \le 49$
9. $1 \le \dfrac{|v - 2|}{8}$	10. $-132 > 12\,|n - 9|$
11. $\left|\dfrac{-8 + n}{2}\right| < 1$	12. $-30 \ge -5|k - 4|$
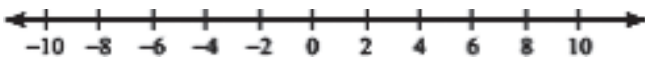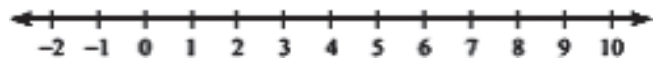
13. Lindsey is making some home-made toffee. The recipe says that she must bring the mixture to a boil at 285 degrees. If she is 7 degrees above or below, the toffee should turn out fine.

    Write, solve, and graph an <u>absolute value inequality</u> to model the range of temperatures that will make yummy toffee.

    <--->

Challenge Problems 14–16

14. Solve for x using the inequality $a|x - b| + c \leq d$. Assume that $d - c > 0$.

15. Write a simple inequality with an absolute value symbol whose solution would be represented by the graph shown below.
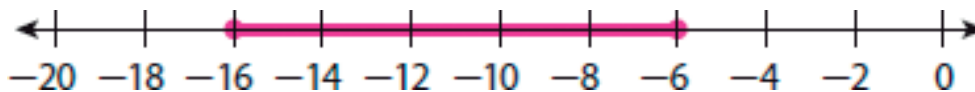
16. A student made an error in the following problem. Determine where the error was made and then complete the problem correctly.

$$|x + 3| + 9 < 5$$

$$|x + 3| < -4$$

$$4 < x + 3 < -4$$

$$1 < x < -7$$

Spiral REVIEW

Combine Like Terms

Source: Open Middle

17. **Open Ended** Using the whole numbers from 1 to 9 in the boxes below, create two expressions that are equivalent to one another. You can use each whole number at most once.

$$\square x + \square x + \square + \square x = \square + \square x + \square$$

Solving Equations

Source: Open Middle

18. **Open Ended** Use the whole numbers 1 through 9, at most one time each, to find the value of *x* closest to 0.

$$\frac{\square}{\square} x \; \boxplus \; \square \; = \; \square$$

19. **Open Ended** Use the digits 1 to 9, at most TWO times each, to fill in the boxes to make an equation with no solutions.

$$\square x \; \boxplus \; \square \; = \; \square x \; \boxplus \; \square$$

Inequalities

Source: Open Middle

20. Using the integers -4 to 4 at most one time each, create an inequality with solutions of $x > \frac{2}{3}$.

$$\square x < \square$$

Spiral REVIEW—Solving Equations

Solve each equation.

21. $10(-6 + n) = -150$

22. $51 = 5x - 9$

23. $6 = 3(8 + m)$

24. $-4 + 2(p - 8) = -36$

25. $2x + 4(5x - 3) = 5(4x + 4)$

26. $-3(4n + 1) = -6(2n - 6)$

Spiral REVIEW—Solving Absolute Value Equations

Solve each absolute value equation. Be sure to check your solution(s).

27. $|x| = 8$

28. $|5 + 8a| = 53$

29. $|9n + 8| = 46$

30. $|3n - 2| = 7$

31. $|3 - x| = 6$

32. $-7|-3 - 3r| = -21$

33. $\dfrac{|-4 - 3n|}{4} = 2$
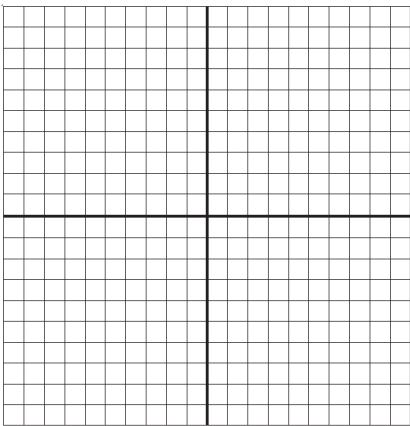
34. $8|x + 7| - 3 = 5$

35. $|x + 2| + 10 = 9$

36. $4|r + 7| + 3 = 59$

Spiral REVIEW—Graphing Lines

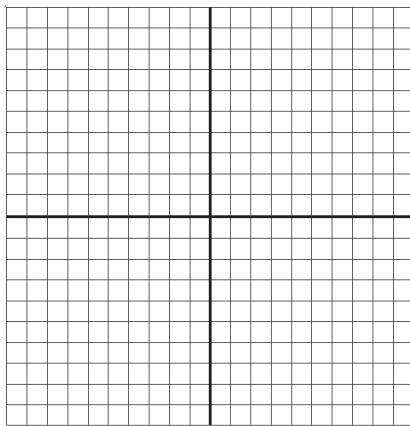**Graph the given points, draw the line connecting the points, and write the equation of the line.**
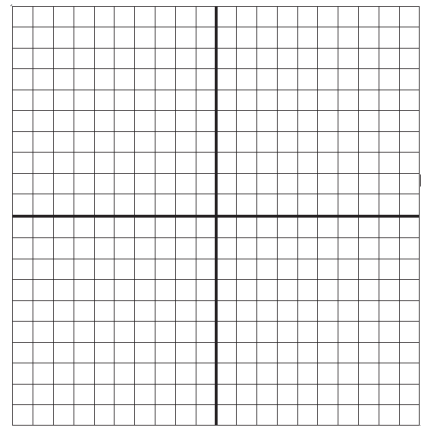
37.

x	y
2	1
4	2
−2	−1

Equation:

38.

x	y
−1	4
0	2
2	−2

Equation:

39.
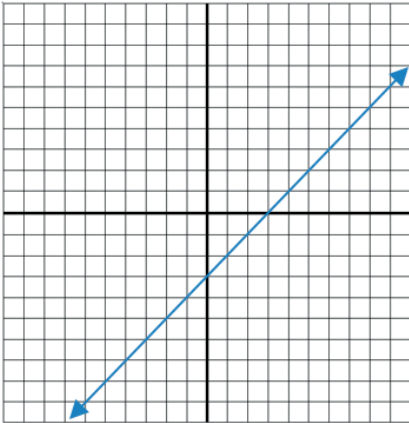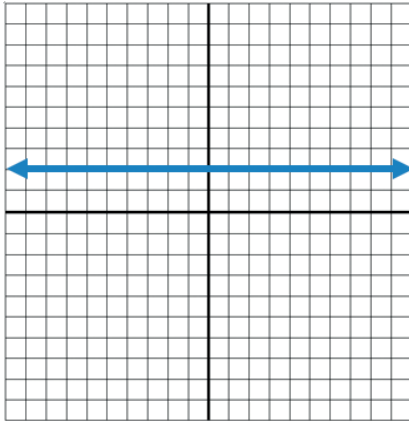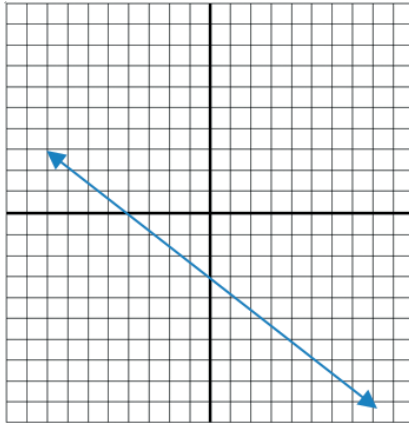
x	y
5	4
2	1
−1	−2

Equation:

Spiral REVIEW—Writing Equations of Lines

Write the equation of each line.

40.	41.	42.
Equation:	Equation:	Equation:

Spiral REVIEW—Combine Like Terms

For each expression, combine like terms to simplify the expression.

43. $7x^3 - 6x^2 - 2x^3 + 10x^2 + 8$

44. $9x - 17 + 10 + x - 3x - 3$

45. $-3x^2 + 2xy - 5xy + 6x^2 + 10xy$

46. $\frac{1}{2}x - 4x^2 - \frac{1}{2}x + 18x^2 + 9 - 1$

Spiral REVIEW—Properties

47. Use algebraic properties to prove that $x + 2x = 30$ results in $x = 10$.

Statements	Reasons
1. $x + 2x = 30$	1. Given
2. $3x = 30$	2.
3. $x = 10$	3.

48. Use algebraic properties to prove that if $\dfrac{2(3(n + 6) - 18)}{6} = 7$, then $n = 7$.

Statements	Reasons
1. $\dfrac{2(3(n + 6) - 18)}{6} = 7$	1. Given
2. $2(3(n + 6) - 18) = 42$	2.
3. $2(3n + 18 - 18) = 42$	3.
4. $2(3n) = 42$	4.
5. $6n = 42$	5.
6. $n = 7$	6.

Spiral REVIEW—Rearranging Formulas

Use algebraic properties to isolate the indicated variable.

49. $\dfrac{w}{138} = p$, for w

50. $\dfrac{4a + 2}{5} = b$, for a

51. $200p + 10 = b$, for p

52. $42x - 5y = x$, for y