LESSON 17

An Introduction to Systems of Equations

LEARNING OBJECTIVES

- Today I am: completing the Desmos activity *Systems of Two Linear Equations.*
- So that I can: write and solve a system of two linear equations to explore the numerical and graphical meaning of "solution."
- I'll know I have it when I can: write an equation of a line to describe Duke and Shirley's walk on a ramp.

Opening Exercise

1. Go to student.desmos.com and type in your teacher's code: _____

   You'll be completing the activity, *Systems of Two Linear Equations.* Complete the questions below as you go through the activity.

2. On Screen 18, you wrote two equations where the lines do not intersect. What is true about these two lines?

© alekleks/Shutterstock.com

3. Write a new systems of equations where the lines will not meet.

4. In general, what can we say about lines that don't meet? What must be true about them?

5. For each set of lines below, determine which ones will have a solution and which will not.

A. $x + y = 18$ $x - y = 0$	B. $x - y = -3$ $x - y = 0$	C. $2x + 2y = 8$ $x + y = 0$	D. $3x - 3y = 18$ $x + y = 0$

One Solution No solution No solution One Solution

Mental Math—In many of these problems, the solution can be determined by thinking about the relationship between the numbers. Often you can use mental math to determine the solution. Try it out in Exercises 6 and 7.

6. Write a system of equations where the sum of two numbers is 10 and the difference is 6. Can you determine the solution without graphing? Explain your reasoning.

$x+y=10$ $x=8$
$x-y=6$ $y=2$

7. Write a system of equations where the sum of the two numbers is 10 and the difference is 3. Can you determine the solution without graphing? Explain your reasoning.

$x+y=10$ $x=6.5$
$x-y=3$ $y=3.5$

Continuous and Discrete Data Points—Discrete data points can be numeric—like the number of dogs—but it can also be categorical—like colors or gender. Continuous data are not restricted to defined separate values, but can take on ANY value over a continuous range. Think about which type of data points are given in Exercises 8 and 9.

continuous data

8. Richard thinks of two numbers. The only clue he gives is "The sum of two numbers is 10." What are the numbers Richard could be thinking of?

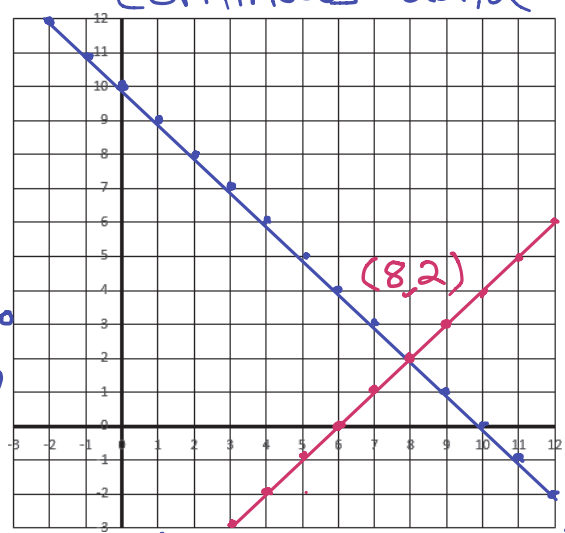
   A. Create an equation using two variables to represent this situation. Be sure to explain the meaning of each variable.

$m=\frac{-1}{1}$ $b=10$

$x+y=10 \rightarrow y=-x+10$

   B. List at least six solutions to the equation you created in Part A.

(5,5) (7,3) (-1,11) (-12,22) (0.5,9.5)



   C. Create a graph that represents the solution set to the equation.

9. Gia had 10 songs in a playlist composed of songs from her two favorite artists, Beyoncé and Jennifer Lopez. How many songs did she have by each one in the playlist?

   A. Create an equation using two variables to represent this situation. Be sure to explain the meaning of each variable.

x = # of Beyonce's
y = # of JLo's

x + y = 10

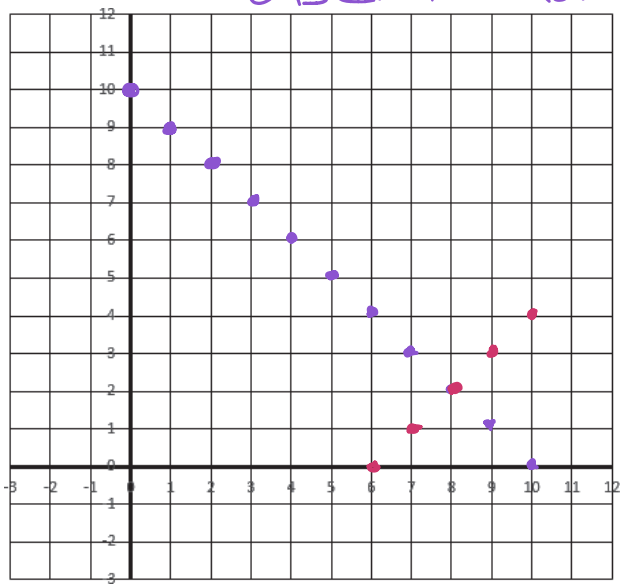
y = −x + 10

   B. List at least three solutions to the equation you created in Part A.

(7,3) (9,1)
(6,4) (10,0)

   C. Create a graph that represents the solution set to the equation.

discrete data



10. Compare your solutions to Exercises 8 and 9. How are they alike? How are they different?

As they stand, Exercises 8 and 9 are not systems of equations, since there is only one equation for each.

11. For Exercise 8, graph an additional clue from Richard, "The difference of my two numbers is 6." Use the graph to determine Richard's two numbers.

x − y = 6

−y = −x + 6

y = x − 6

12. Gia says that the difference in the number of songs is 6. Use the graph in Exercise 9 to find the number of Beyoncé songs and the number of Jennifer Lopez songs Gia has on her playlist.

Throughout this unit, you'll be writing systems of equations or inequalities with different real-world problems.

Let's look at a story of two people walking along a ramp.

Duke starts at the base of a ramp and walks up it at a constant rate. His elevation increases by 3 ft. every second. Just as Duke starts walking up the ramp, Shirley starts at the top of the same 25 ft. high ramp and begins walking down the ramp at a constant rate. Her elevation decreases 2 ft. every second.

© Q Wang/Shutterstock.com

Understanding the Problem

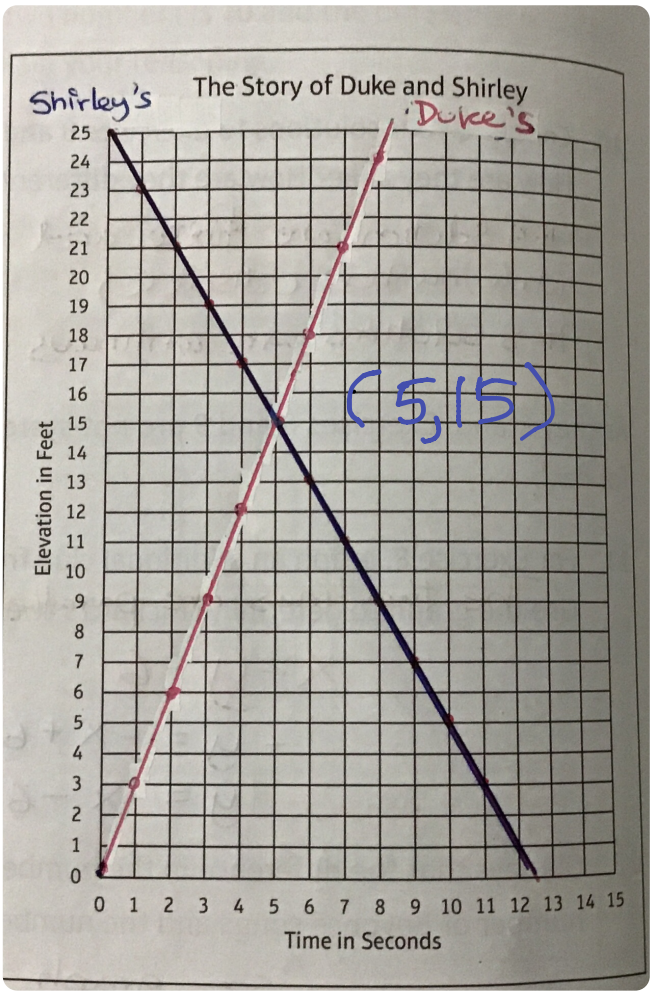
13. Fill in the chart below to help you understand how quickly Duke and Shirley move and where they start.

Duke's Path	
Time in Seconds	**Elevation in Feet**
0	0
1	3
2	6
3	9
4	12
5	15
6	18
7	21
8	24
x	3x

Shirley's Path	
Time in Seconds	**Elevation in Feet**
0	25
1	23
2	21
3	19
4	17
5	15
6	13
7	11
8	9
9	7
10	5
11	3
12	1
x	-2x+25

$m = -2$

$b = 25$



14. Use the extra row in each table to write an expression that describes the elevation pattern.

15. Use the grid to graph the data for Duke and Shirley. Be sure to add a legend to help the reader understand your graph.

Writing an Equation of a Line

The equation of a line can be in the form $y = mx + b$, where m represents the slope of the line and b represents the y-intercept.

16. Determine the slope of Duke's line and Shirley's line. Why is one of the slopes negative?

    Duke's slope: 3 Shirley's slope: −2

17. What is the y-intercept for each line?

    Duke's y-intercept: 0 Shirley's y-intercept: 25

18. Write the equation of Duke's line and Shirley's line.

    Duke's line: $y = 3x$ Shirley's line: $y = -2x + 25$

19. Where do the two lines intersect? (5 sec, 15 dist) This is the **point of intersection**. What time do Duke and Shirley pass each other?

20. How could you use the equations to find the point of intersection?

$y = 3x$
$y = -2x + 25$
set equal $3x = -2x + 25$

The equations you wrote in Exercise 18 are in slope-intercept form or $y = mx + b$.

> ***Point-slope*** is another very useful form of a linear equation. For point-slope we need any point on the line, not just the y-intercept, and the slope of the line. The general form looks like $y - y_1 = m(x - x_1)$ where (x_1, y_1) is any point on the line and m is the slope of the line.

21. A. Use the points when the time is 3 seconds and the slopes from Exercise 16 with the point-slope equation to get equations for Duke and Shirley.

D: (3, 9) $m = 3$
$y - 9 = 3(x - 3)$
$y - 9 = 3x - 9$
$+9 \quad +9$
$y = 3x$

S: (3, 19) $m = -2$
$y - 19 = -2(x - 3)$
$y - 19 = -2x + 6$
$+19 \quad +19$
$y = -2x + 25$

B. How do these equations compare to the ones you wrote in Exercise 18?

Same.

Lesson Summary

A **system of equations** is a set of equations that describe a situation.

Example: Two numbers have a sum of 12 and a difference of 4. What are those two numbers?

Define your variables.	Let x = the first number Let y = the second number
Use the information in the problem to write two equations.	$x + y = 12$ $x - y = 4$

In later lessons, you'll solve systems of equations using graphing, substitution and a new method, elimination.

Linear Equations: If you know the slope and y-intercept, you can use the equation $y = mx + b$ to write an equation for the line. If you have the slope and any point on line you can use the point-slope equation, $y - y_1 = m(x - x_1)$ where (x_1, y_1) is any point on the line and m is the slope of the line.

Example 1: The slope of a line is -3 and the y-intercept is 5.

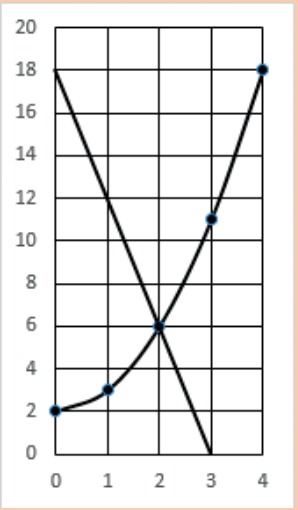
An equation of this line is _____.

Example 2: The slope of a line is $\frac{1}{2}$ and the point (2, 4) is on the line.

The equation of this line is _____.

Point of Intersection: The point of intersection is an ordered pair that is a solution to both equations. On a graph, this is where the two graphs cross each other.

Example: Determine the point of intersection for the graph at the right.



Point of intersection: _____

NAME: _____ PERIOD: _____ DATE: _____

Homework Problem Set

Write a system of equations for Problems 1 and 2. Be sure to define your variables. Then create a table and a graph of each problem and find a solution.

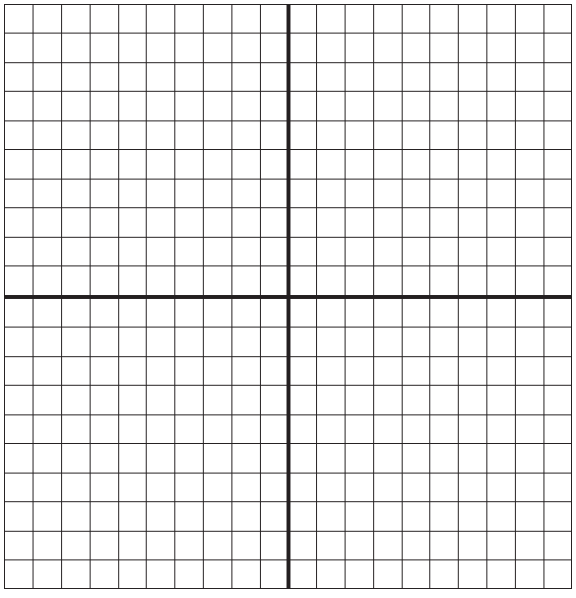
1. The difference of two numbers is 3 and their sum is 13. What are the two numbers?

Let ___ = _____

Let ___ = _____

System of Equations:

1st number	2nd number	Sum of numbers	1st number	2nd number	Difference of numbers
		13			3
		13			3
		13			3
		13			3



Solution: _____

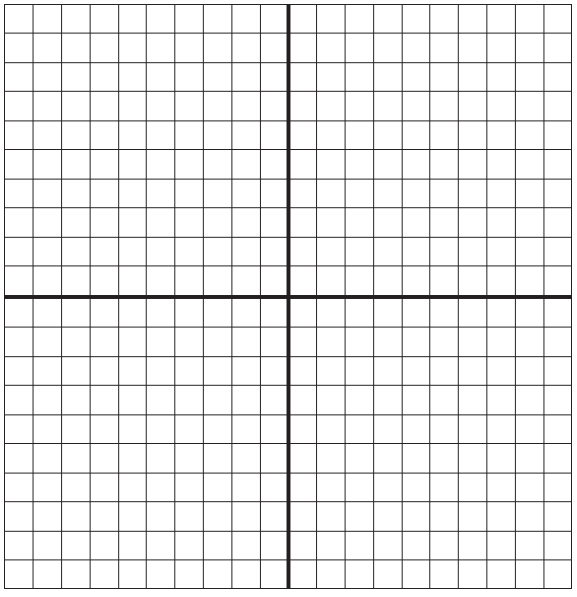
2. The difference of two numbers is −10 and the sum is −4. What are the two numbers?

Let ___ = _____

Let ___ = _____

System of Equations:

1st number	2nd number	Sum of numbers	1st number	2nd number	Difference of numbers
		−4			−10
		−4			−10
		−4			−10
		−4			−10



Solution: _____

Write a system of equations for Problems 3 and 4. Be sure to define your variables. You do NOT need to solve these problems.

3. Jack and his sister, Malonie, are 4 years apart in age. The sum of their ages is 28. What are their ages?

Let ___ = _____

Let ___ = _____

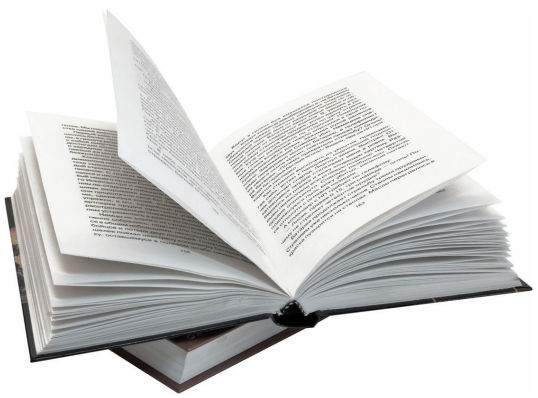
System of Equations:

4. Two of Julie's textbooks are a total of $65. The difference in price between the two books is $9. What is the cost of each book?

Let ___ = _____

Let ___ = _____

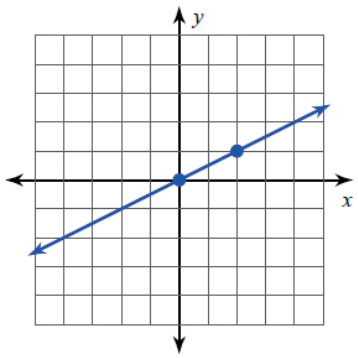
System of Equations:



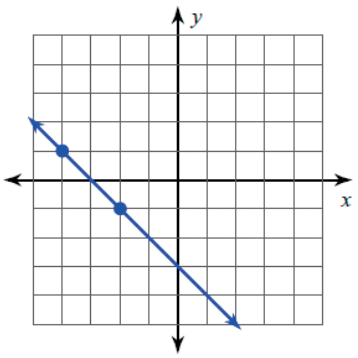
Spiral REVIEW—Determining Slope and *y*-intercept from Graphs

Find the slope and *y*-intercept of each line.

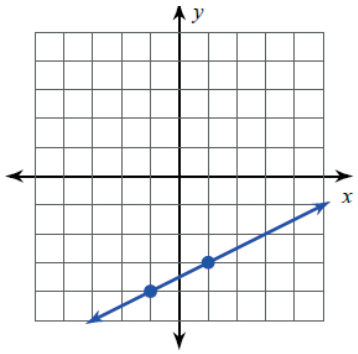
5. slope = ___ *y*-intercept = ___



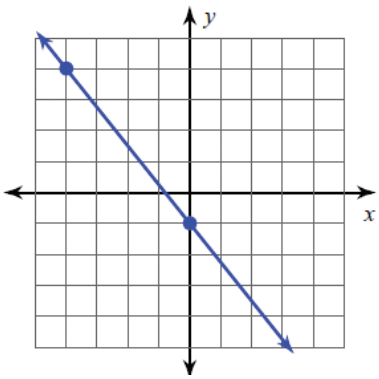
6. slope = ___ *y*-intercept = ___



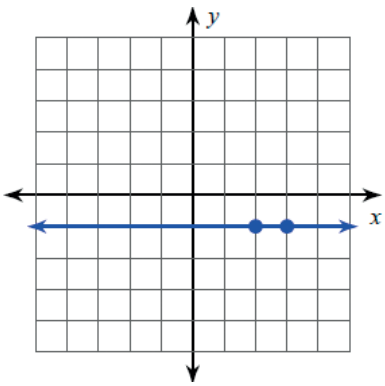
7. slope = ___ *y*-intercept = ___



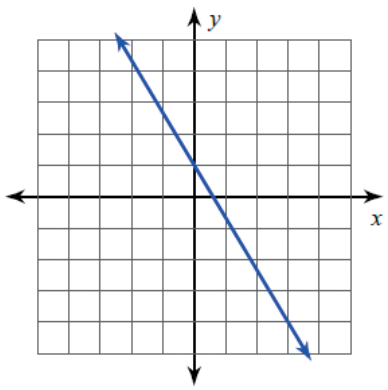
8. slope = ___ *y*-intercept = ___



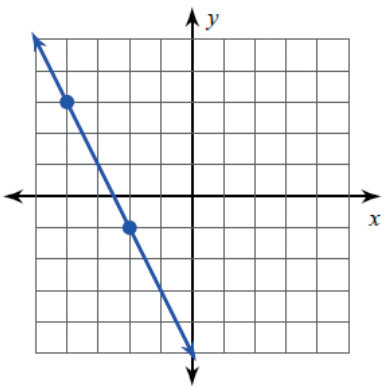
9. slope = ___ *y*-intercept = ___



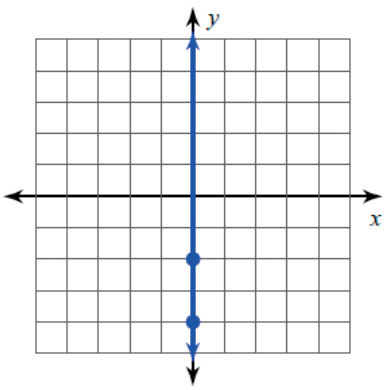
10. slope = ___ *y*-intercept = ___



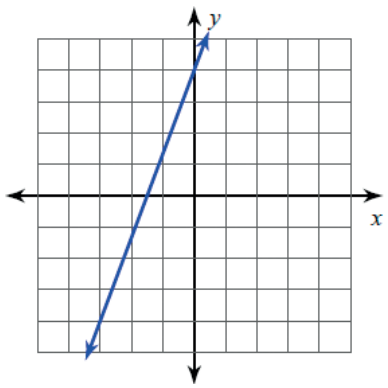
11. slope = ___ *y*-intercept = ___



12. slope = ___ *y*-intercept = ___



13. slope = ___ *y*-intercept = ___



Spiral REVIEW—Solving Equations

Solve each equation.

14. $-16 - 6v = -2(8v - 7)$

15. $2(6b + 8) = 4 + 6b$

Spiral REVIEW—Graphing Lines

16. Match each equation with its graph. Explain your reasoning.

A. $y = 5x - 6$

B. $x + 2y = -12$

C. $2x + y = 4$

D. $y = 3x - 6$

E. $x = -y - 4$