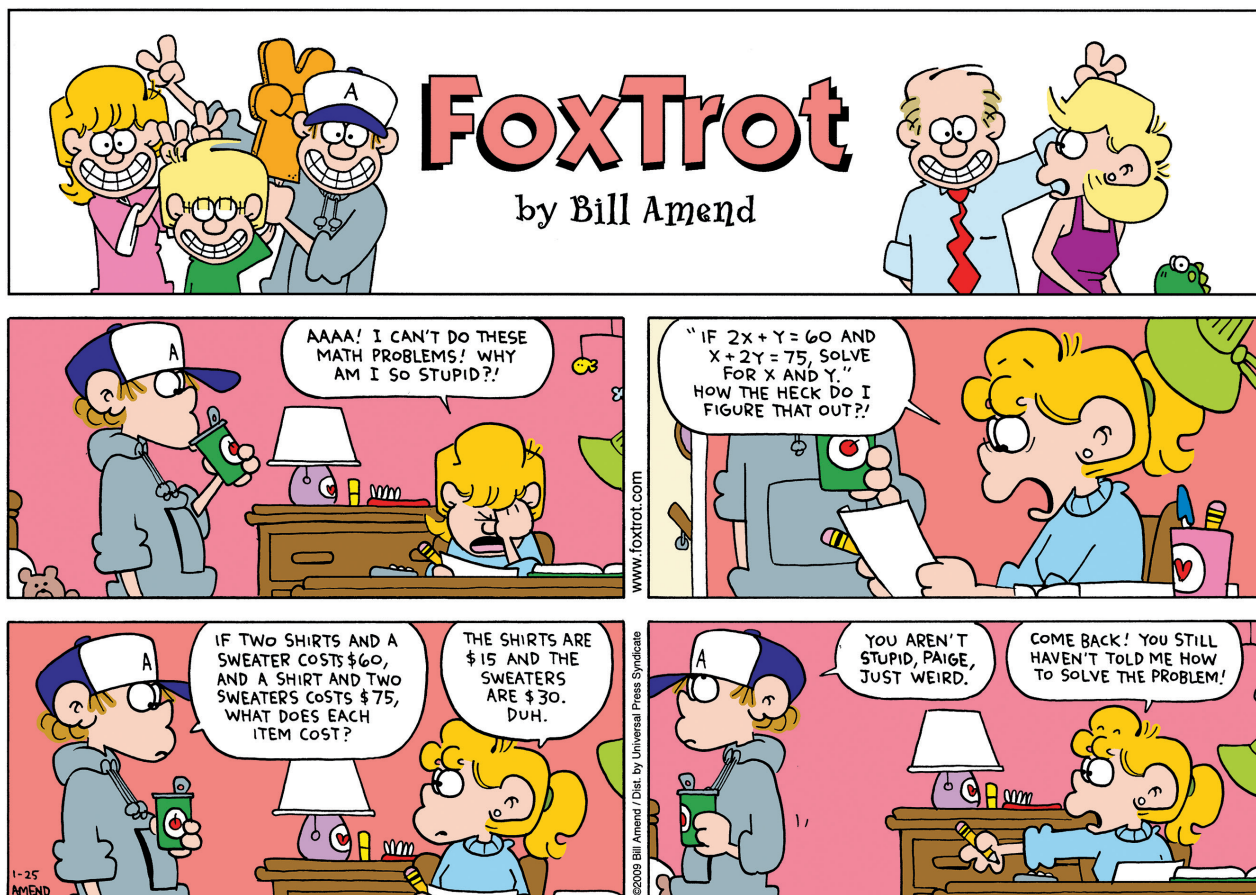# Solving Systems with Substitution

LEARNING OBJECTIVES

➤ Today I am: solving systems with substitution.

➤ So that I can: model real—life situations.

➤ I'll know I have it when I can: Isolate one of the variables in one equation to plug on and solve for the other equation.

Opening Exercise

FOXTROT © 2009 Bill Amend. Reprinted with permission of ANDREWS MCMEEL SYNDICATION. All rights reserved.

1. A. Read over Peter's restatement of the problem. What does *x* represent?

$x =$ the cost of a shirt

   B. What does *y* represent?

$y =$ the cost of a sweater.

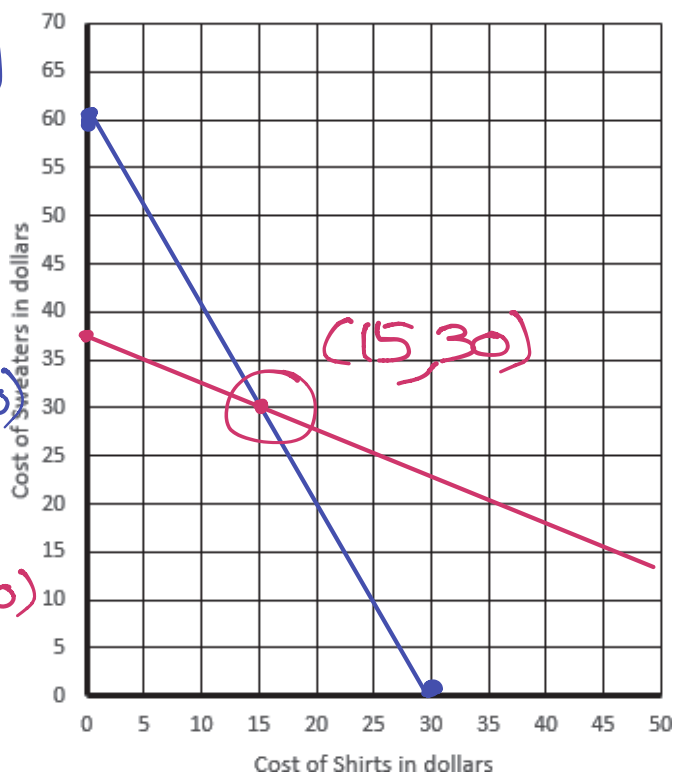2. A. Graph the two equations from the comic strip on the grid below.

$$2x + y = 60 \qquad x + 2y = 75$$

$\longrightarrow$ x-inter y=0

$X + 2(0) = 75$

$X = 75$

$(75,0)$

y-inter. x=0

$0 + 2y = 75$

$y = 37.5$

$(0, 37.5)$

standard

$2x+y = 60$

using intercepts

x-inter $\boxed{y=0}$

$2x+0 = 60$

$x = 30$ $(30,0)$

y-inter $\boxed{x=0}$

$2(0)+y = 60$ $(0,60)$

$y = 60$

Paige's Problem

Cost of Sweaters in dollars (y-axis, 0 to 70)

(15, 30)

Cost of Shirts in dollars (x-axis, 0 to 50)

   B. Where do you see Paige's answer of shirts are $15 and sweaters are $30 on the graph?

The intersection point.

   C. Why was Paige able to solve the problem her brother gave her but she couldn't solve the problem on her worksheet?

Her brother changed the context.

The Substitution Method for Solving Systems of Equations

You may have learned the substitution method for solving systems of equations in an earlier grade. With this method you replace a variable by its equivalent expression.

To solve Paige's problem with substitution, we would need to isolate one of the variables in one of the equations. Each step is broken down for you in Exercise 3.

3. A. Discuss with your partner which variable you'd isolate in the equation $2x + y = 60$. Then isolate that variable.

$$2x + y = 60 \xrightarrow{\text{solve } y} y = -2x + 60$$

$$x + 2y = 75$$

B. Now substitute the expression that represents your variable into the second equation, $x + 2y = 75$.

$$x + 2(-2x + 60) = 75$$

$$x - 4x + 120 = 75$$

C. Solve for the variable.

$$-3x + 120 = 75$$
$$ -120 -120$$

$$\frac{-3x}{-3} = \frac{-45}{-3}$$

D. Find the value of the other variable.

$$x = \boxed{15}$$

Back substitute

$$2x + y = 60$$
$$2(15) + y = 60$$
$$30 + y = 60$$
$$y = 30$$

E. Check your work with both equations.

$$(15, 30) \enspace ?$$
$$2(15) + 30 \overset{?}{=} 60$$
$$60 = 60 \checkmark$$
$$15 + 2(30) \overset{?}{=} 75$$
$$15 + 60 = 75 \checkmark$$

(handwritten) $(15, 30)$

Let's look at a problem from Lewis Carroll's *Through the Looking Glass* when Alice encounters Tweedledum and Tweedledee.

4. Tweedledum says to Tweedledee, "The sum of your weight and twice mine is 361 pounds." Tweedledee replies, "The sum of your weight and twice mine is 362 pounds." Help Alice find both of their weights.

(Illustration with speech bubbles) "The sum of your weight and twice mine is 361 pounds." "The sum of your weight and twice mine is 362 pounds."
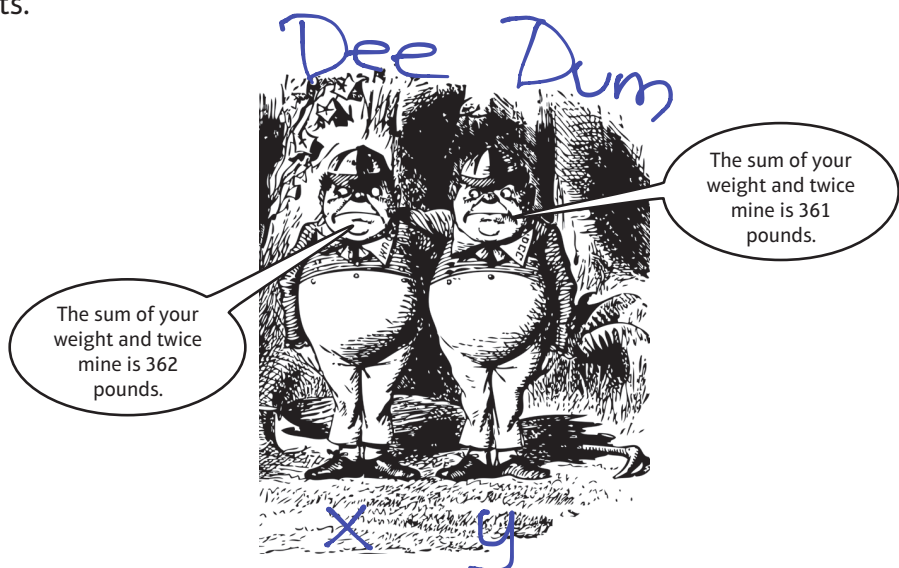
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A. First define your variables: Let $x =$ ___*Dee's*___ weight and $y =$ ___*Dum's*___ weight.

B. Write an equation for each statement.

(handwritten)
$$2x + y = 362$$
$$x + 2y = 361$$
$$x = 361 - 2y$$

C. Solve the system of equations using substitution. You'll need to get one variable isolated in one of the equations.

(handwritten)
$$2(361 - 2y) + y = 362$$
$$722 - 4y + y = 362$$
$$722 - 3y = 362$$
$$-722 \qquad -722$$
$$-3y = -360$$
$$y = 120$$

D. Check that your answer makes sense.

(handwritten) Back sub
$$x = 361 - 2(120)$$
$$x = 361 - 240$$

$$X = 121$$

5. Lulu tells her little brother, Jack, that she is holding 20 coins, all of which are either dimes or quarters. They have a value of $4.10. She says she will give him the coins if he can tell her how many of each she is holding. Solve this problem for Jack.

A. First define your variables:
Let D = __# of dimes__ (0.10)
and Q = __# of quarters__ (0.25)

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B. Write an equation for each statement.

coins
$$D + Q = 20$$

solve Q
$$Q = 20 - D$$

Value $\times 100$
$$(0.1D + 0.25Q = 4.10)$$
$$10D + 25Q = 410$$

C. Solve the system of equations by substitution.

$$10D + 25(20 - D) = 410$$
$$10D + 500 - 25D = 410$$
$$-15D + 500 = 410$$
$$ - 500 \quad -500$$
$$\frac{-15D}{-15} = \frac{-90}{-15}$$
$$D = 6$$

Back Sub
$$Q = 20 - 6$$
$$= 14$$

D. Check that your answer makes sense.

6. Tickets to a school play cost $5 for adults and $3 for children. One day they sold 175 tickets and brought in $675. How many children tickets were sold that day?

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$$A = \text{# of adult tix}$$
$$C = \text{# of children tix}$$

$$A + C = 175 \longrightarrow A = 175 - C$$
$$5A + 3C = 675$$

$$5(175 - C) + 3C = 675$$
$$875 - 5C + 3C = 675$$

$$875 - 2C = 675$$
$$-875 \quad\quad -875$$
$$2C = -200$$
$$C = 100$$

Reflection

7. A. What are the advantages of finding the solution by graphing?

~ Visual

~ Good for whole numbers

B. What are the advantages of finding the solution algebraically with substitution?

— Good for decimals

— large numbers

C. If a system of linear equations had the same slope and the same *y*-intercept, what would the solution be?

same line
infinite solutions
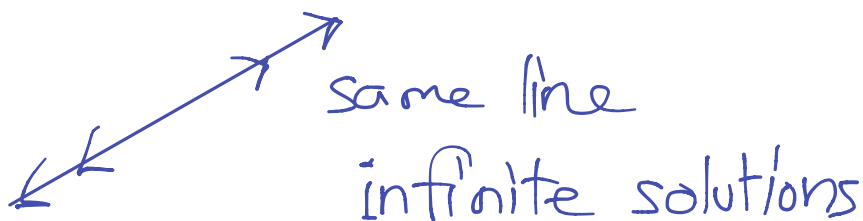
D. If a system of linear equations had the same slope and different *y*-intercepts, what would the solution be?

no solution

parallel
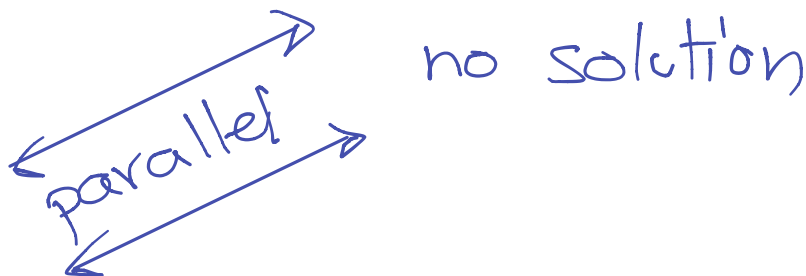
Lesson Summary

Solve the system of equations: $y = \overset{m}{2}x - \overset{b}{6}$

$\qquad\qquad\qquad x + 2y = -2.$

$\dfrac{2y}{2} = \dfrac{-x}{2} - \dfrac{2}{2}$

$y = \dfrac{-x}{2} - 1$

Graphically:

The solution to the system of equations is $\underline{(2,-2)}$.

$y = \boxed{2x - 6}$

$x + 2y = -2$
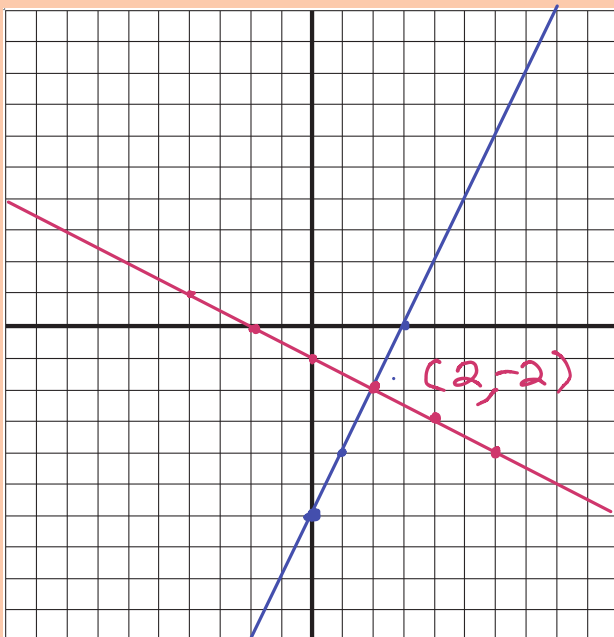
Algebraically with Substitution:

$x + 2(2x - 6) = -2$

$x + 4x - 12 = -2$

$5x - 12 = -2$

$5x = 10$

$x = 2$

$\underline{\text{Back sub}}$

$\qquad y = 2(2) - 6$

$(2, -2) \qquad = -2$

8. Finish the Lesson Summary problem and find the solution to the problem.

NAME: _____   PERIOD: _____   DATE: _____

Homework Problem Set

1. Kristin spent $131 on shirts. Fancy shirts cost $28 and plain shirts cost $15. If she bought a total of 7 shirts, how many of each kind did she buy?

   Let f = the number of fancy shirts

   Let p = the number of plain shirts

   A. What is the cost of 1 fancy shirt? Of 2 fancy shirts? Of f fancy shirts?

   B. What is the cost of 1 plain shirt? Of 2 plain shirts? Of p plain shirts?

   C. Write an equation that describes the amount of money spent by Kristin.

   D. Write an equation that describes the number of shirts Kristin bought using f and p.

   E. Determine the number of each type of shirt Kristin bought.

2. There are 13 animals in the barn. Some are chickens and some are pigs. There are 40 legs in all. How many of each animal are in the barn?

Let c = the number of chickens

Let p = the number of pigs
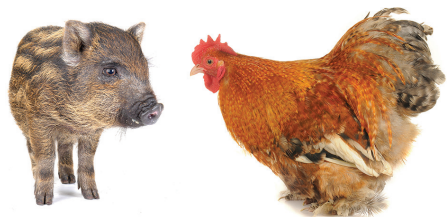
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A. Write an equation that describes the number of animals using c and p.

B. Write an equation that describes the number of legs in the barn.

C. Determine the number of chickens and pigs in the barn.

3. Pam has two part time jobs. At one job, she works as a cashier and makes $8 per hour. At the second job, she works as a tutor and makes $12 per hour. One week she worked 30 hours and made $268. How many hours did she spend at each job? Be sure to show your work and explain your thinking.

X = # hours cashier

Y = # hours tutoring

$-8(X + Y = 30)$

$8X + 12y = 268$

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4. At a state fair, there is a game where you throw a ball at a pyramid of cans. If you knock over all of the cans, you win a prize. The cost is 3 throws for $1, but if you have an armband, you get 6 throws for $1. The armband costs $10.
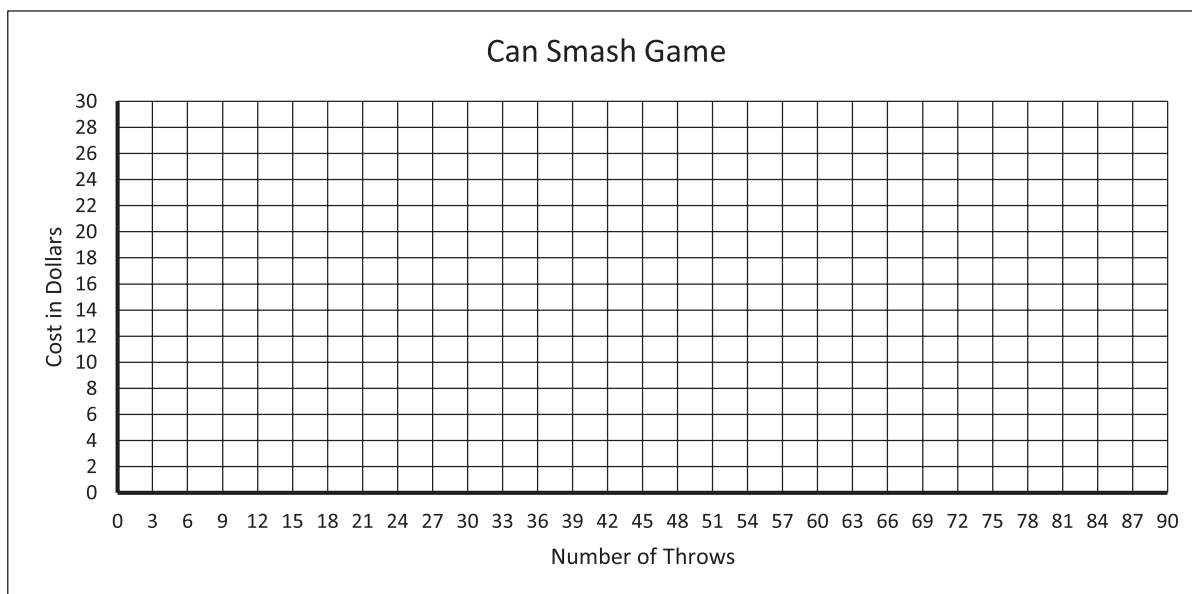

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A. **Challenge**   Write two cost equations for the game in terms of the number of throws purchased, one without an armband and one with.

Let t = number of throws

Let c = total cost

B. Graph the two cost equations on the same graph. Think about whether this data is discrete or continuous.

Can Smash Game

Cost in Dollars (y-axis: 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30)

Number of Throws (x-axis: 0, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60, 63, 66, 69, 72, 75, 78, 81, 84, 87, 90)

Source: texasentertainmentgroup.com

C. Does it make sense to buy the armband? Explain your thinking.