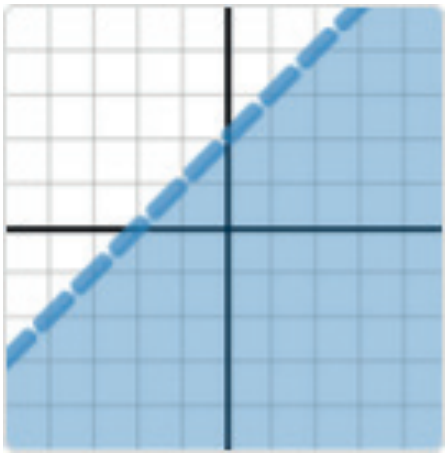
LESSON 24 Inequalities in Real Life

LEARNING OBJECTIVES

- Today I am: playing *Polygraph: Linear Inequalities* on Desmos.
- So that I can: think about the precise vocabulary needed when describing inequalities.
- I'll know I have it when I can: analyze student work with inequalities.

Opening Exploration—Polygraph with a Twist

You will need: a Desmos class code



1. Go to student.desmos.com and type in your class code: ______________________ to play *Polygraph: Linear Inequalities*. You played a game similar to this one in Lesson 18 with linear equations. Now you'll have linear inequalities to identify.

Discussion

2. What are some of the words you or your partner(s) used to determine the correct graph?

3. How was this game the same as the one in Lesson 18? Was it more difficult? Explain.

Inequalities are often used to look at maximizing or minimizing costs, materials, profit or time.

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4. The Student Council is selling tickets to the Winter Dance. Tickets cost \$5 per person or \$8 per couple. To cover expenses, at least \$1200 worth of tickets must be sold. No more than 500 students can fit in the gym where the dance is being held.

   A. Write and graph a system of inequalities to find possible solutions to this problem.

   Let x = the number of \$5 tickets

   Let y = the number of \$8 tickets

   \# of people ① $x + 2y \le 500$

   \$ ② $5x + 8y \ge 1200$

   B. Give three possible combinations of tickets that could be sold, so that the Student Council makes a profit, yet stays under the room capacity. Mark these on your graph.

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   C. Is there a solution where the Student Council brings in the most money? Explain.

$(500, 0)$

\$5 \$8

$5(500) + 8(0) = 2500$

$0 \leq 500$ T

$0 + 2(0) \leq 500$

$x + 2y \leq 500$

Use intercepts

x-inter, $y = 0$

$x + 2(0) = 500$

$x = 500$

y-inter, $x = 0$

$0 + 2y = 500$

$y = 250$

$0 \geq 1200$ F

$5(0) + 8(0) \geq 1200$

$5x + 8y \geq 1200$

Use intercepts

x-inter, let $y = 0$

$5x + 8(0) = 1200$

$5x = 1200$

$x = 240$

$(240, 0)$

y-inter, let $x = 0$

$5(0) + 8y = 1200$

$8y = 1200$

$y = 150$

y = 150
(0, 150)

Below is a maximization problem with hand-made boomerangs.

Boomerangs

Phil and Cath make and sell boomerangs for a school event.
The money they raise will go to charity.

The plan to make them in two sizes: small and large.

Phil will carve them from wood.
The small boomerang takes 2 hours to carve and the large one takes 3 hours to carve.
Phil has a total of 24 hours available for carving.

Cath will decorate them.
She only has time to decorate 10 boomerangs of either size.

The small boomerang will make \$8 for charity.
The large boomerang will make \$10 for charity.
They want to make as much money for charity as they can.

How many small and large boomerangs should they make?

How much money will they then make?

Source: http://map.mathshell.org/download.php?fileid=1718

Understanding the Problem

5. Let's break this problem down a little before you solve it.

A. What facts do you know?	B. What do you need to find out?
C. What can vary or change? What limitations are there?	D. How many large boomerangs are definitely too many? Why?
E. How can you organize the number of large and small boomerangs in a systematic way?	F. How many small boomerangs are definitely too many? Why?

Error Analysis

Exercises 6–9 show the work of four different students. Read over each problem and determine any errors the student has made.

6. Alex's solution →

Phil can only make 12 small or 8 large boomerangs in 24 hours
12 small makes \$96
8 large makes \$80
Cath only has time to make 10, so \$96 is impossible.
She could make 10 small boomerangs which will make \$80.
So she either makes 8 large or 10 small boomerangs
and makes \$80.

Alex's error(s):

7. Danny's solution →

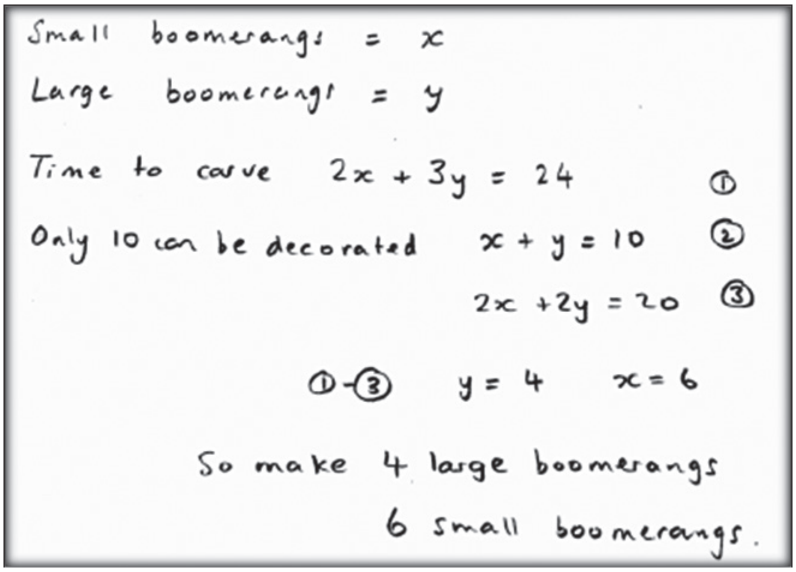
No of Small s	S×8	No of large	ℓ×10	Profit
0	0	8	80	80
1	8	~~8~~ 7	70	78
2	16	6	60	76
3	24	5	50	74
4	32	5	50	82 ←
5	40	4	40	80
6	48	3	30	78

The most Profit is \$82

Danny's error(s):

8. Jeremiah's solution →

   Jeremiah's error(s):



Small boomerangs = x

Large boomerangs = y

Time to carve $2x + 3y = 24$ ①

Only 10 can be decorated $x + y = 10$ ②

$2x + 2y = 20$ ③

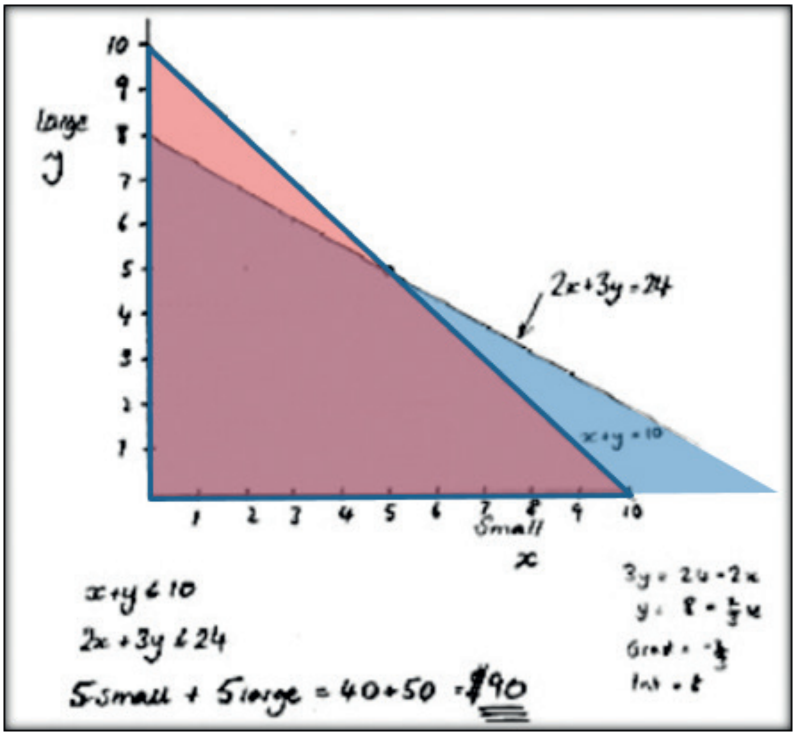
① - ③ $y = 4$ $x = 6$

So make 4 large boomerangs

6 small boomerangs.

9. Tanya's solution →

   Tanya's error(s):



On Your Own

10. The cross country team is going to a state competition. There are no more than 32 people going on the trip, but only 5 of them can drive. They have cars and vans that can be used. A van seats 8 people, including the driver and a car seats 4 people including the driver.

Let v = the number of vans and c = the number of cars

A. Explain what these inequalities represent in the problem.

(x, y)

(c, v)

$v + c \leq 5$ (5, 0) (0, 5)

$8v + 4c \leq 32$ (0, 4) (8, 0)

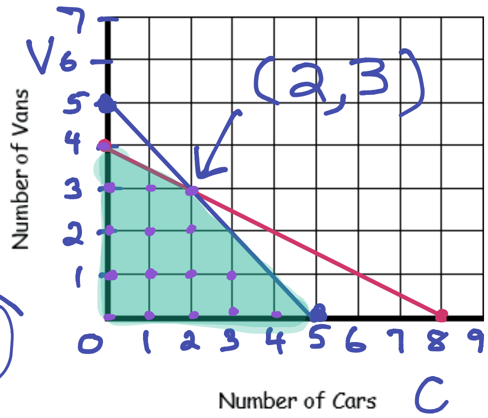
B. Graph the inequalities on the grid at the right.

C. How many vans and cars do you think the team needs for the trip? Explain your reasoning. Where is your solution on the graph?

2 cars, 3 vans

$8(3) + 4(2) = 24 + 8$

32



D. What are the limitations or constraints in this problem?

E. What is another method you can use to find the solutions to this problem?

In the last exercise, we couldn't take ½ of a car on the cross country trip. You were limited to whole number values for the number of cars and the number of vans. We'll now look at more abstract cases where fractional values are allowable.

Test(0,0)
$0 \leq 8$ T

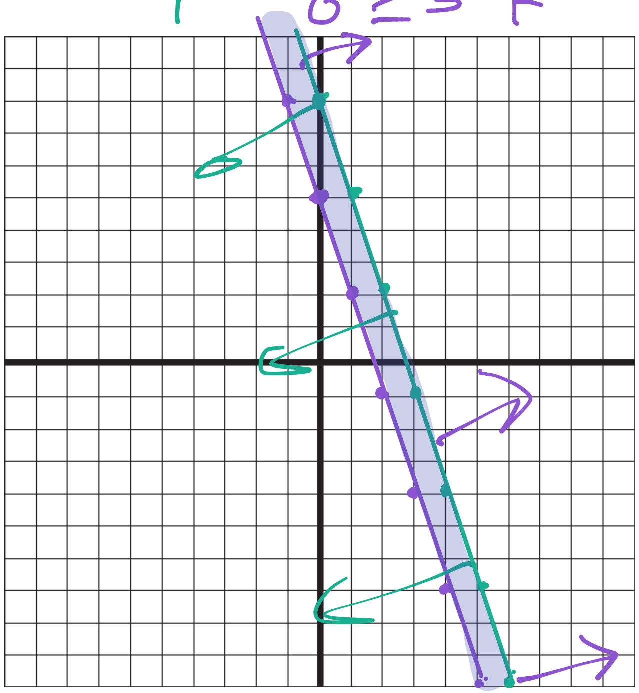
Test(0,0)
$0 \geq -3(0)+5$
$0 \geq 5$ F

11. Consider the systems of inequalities below:

$$\begin{cases} 3x + y \geq 5 \\ 3x + y \leq \cancel{5}\ 8 \end{cases}$$

$y \geq -3x+5$

$y \leq -3x+8$



A. Graph the solution set. The steps are listed below.

B. How is the system of inequalities similar to the system of equations? How is it different?

C. Where are the solutions to the system of inequalities?

The area between the two lines.

Remember: When graphing inequalities in two variables on a coordinate plane, you need to do the following:

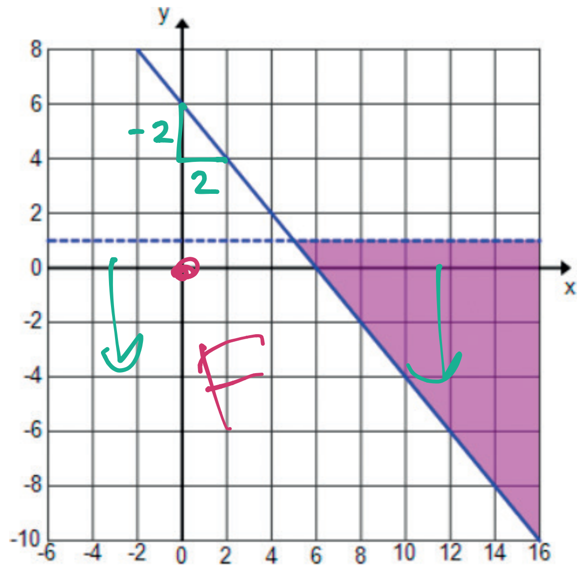
1. Isolate y in the inequality. $y=mx+b$
2. Graph the boundary line with either a solid or dashed line.

   Solid line if $\leq$ or $\geq$

   Dashed line if $<$ or $>$
3. Choose a test point to determine which side of the boundary line to shade. Then shade the appropriate side.

$m = \frac{-2}{2} = -1$

12. Write a system of inequalities that represents the shaded region of the graph shown.



$y < 1$

$y \boxed{\geq} -x + 6$

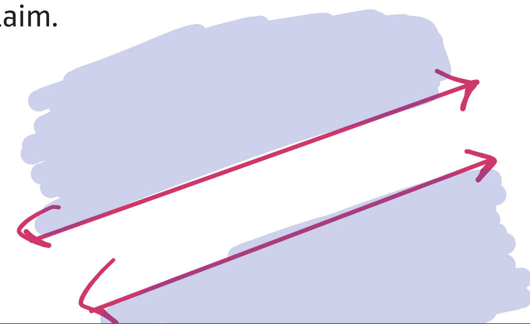
Test (0,0)

$0 \geq -(0) + 6$

F

13. Is it possible to have a system of inequalities that has no solution? Provide an explanation or an example to support your claim.

Yes



Lesson Summary

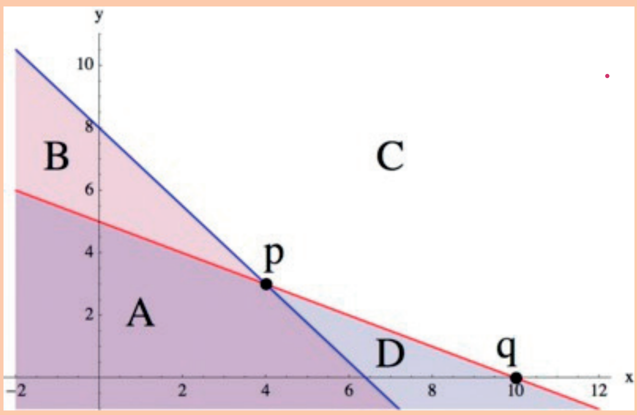
When graphing a ***system of inequalities***, the solution will be in the region that is ______________ to both inequalities.

The system for the graph at the right is

$y \leq -0.5x + 5$ and $y \leq -1.25x + 8$.

For this graph, Region ______ is showing the solution to this system.

To get no solution the inequalities must have ____________ regions in common.



Source: Illustrative Mathematics

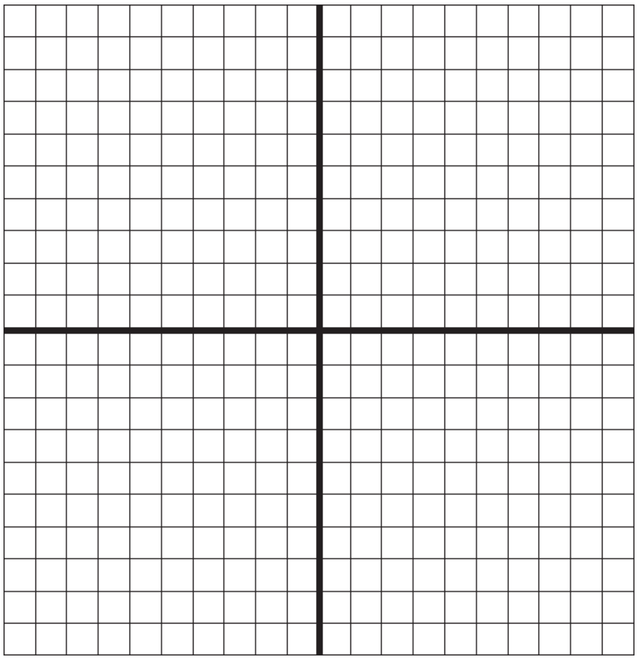
The ***constraints*** of an application problem are the limitations due to size, cost, materials, etc. These constraints can help you determine which quadrant you'll be using and the values on your axes.

NAME: ______________________ PERIOD: ____________________ DATE: _______________

Homework Problem Set

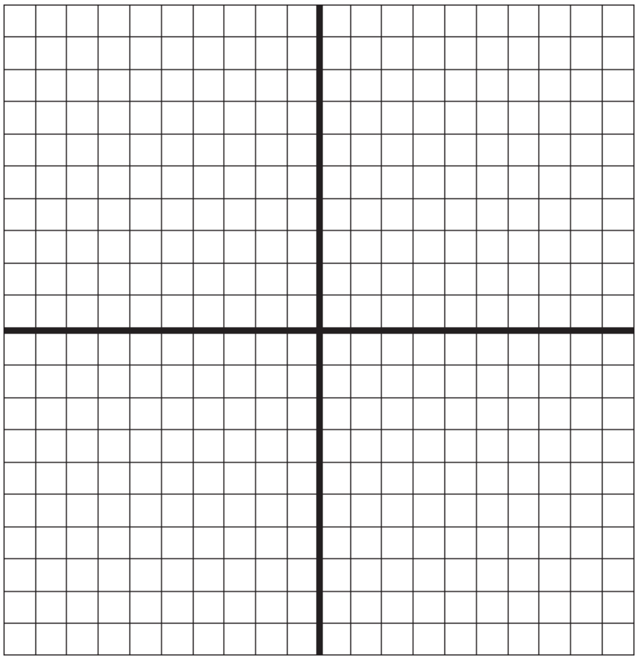
1. Graph the solution to the following system of inequalities:

$$\begin{cases} x \geq 0 \\ y < 2 \\ x + 3y > 0 \end{cases}$$

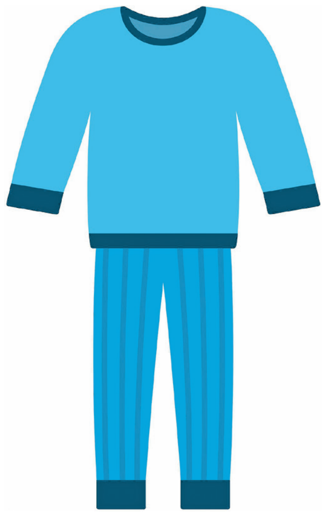


2. Graph the solution set to the system of inequalities.

$2x - y < 3$ and $4x + 3y \geq 0$



3. A clothing manufacturer has 1,000 yds. of cotton to make shirts and pajamas. A shirt requires 1 yd. of fabric, and a pair of pajamas requires 2 yds. of fabric. It takes 2 hr. to make a shirt and 3 hr. to make the pajamas, and there are 1,600 hrs. available to make the clothing.

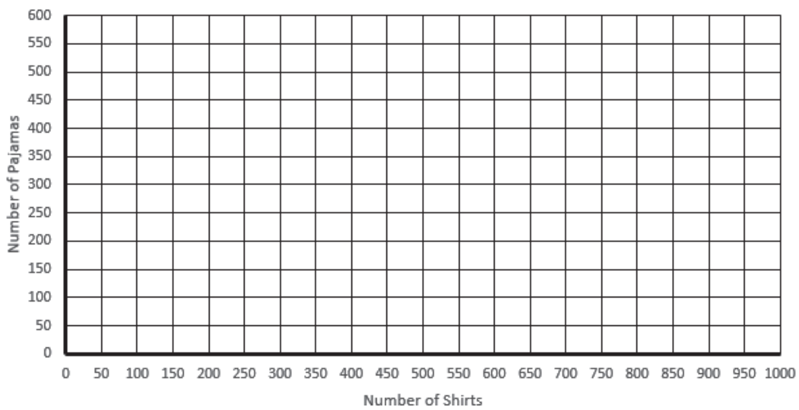


A. What are the variables?

B. What are the constraints?

C. Write inequalities for the constraints.

D. Graph the inequalities and shade the solution set.



E. What does the shaded region represent?

F. Suppose the manufacturer makes a profit of \$10 on shirts and \$18 on pajamas. How would it decide how many of each to make?

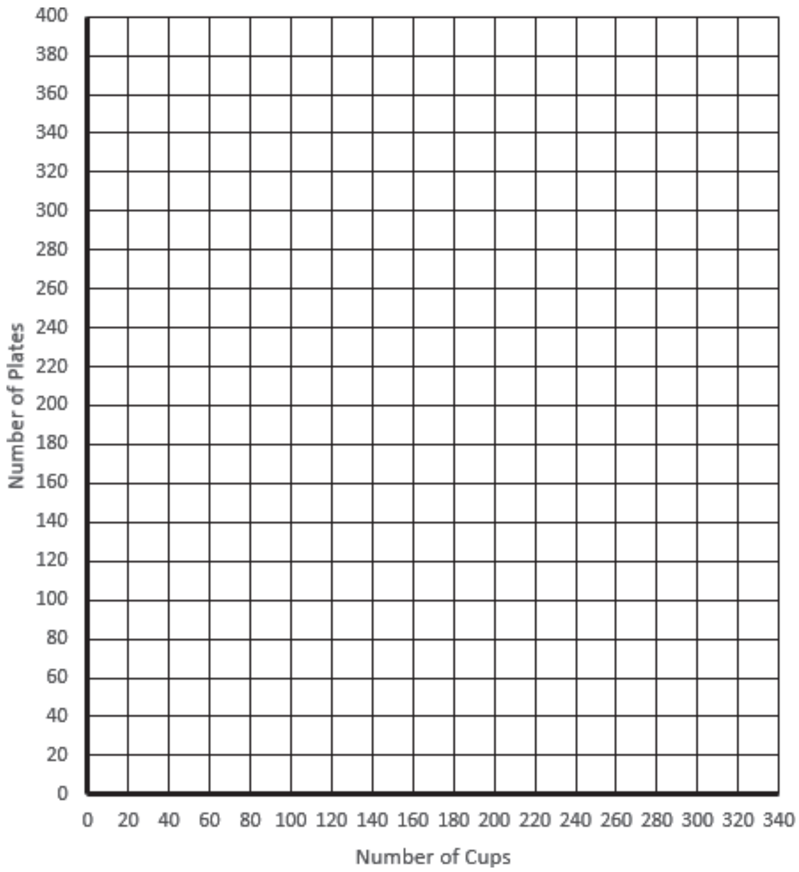
G. How many of each should the manufacturer make, assuming it will sell all the shirts and pajamas it makes?

4. A potter is making cups and plates. It takes her 6 mins. to make a cup and 3 mins. to make a plate. Each cup uses $\frac{3}{4}$ lb. clay, and each plate uses 1 lb. of clay. She has 20 hrs. available to make the cups and plates and has 250 lbs. of clay.

A. What are the variables?

B. Write inequalities for the constraints.

C. Graph and shade the solution set.

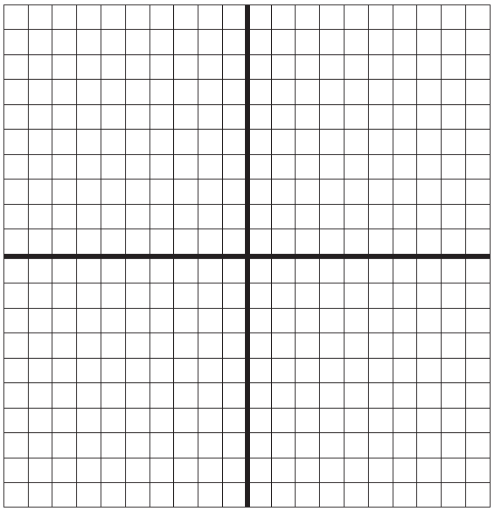


D. If she makes a profit of $2 on each cup and $1.50 on each plate, how many of each should she make in order to maximize her profit?

E. What is her maximum profit?

Graph the solution set to each system of inequalities.

5. $\begin{cases} x - y > 5 \\ x > -1 \end{cases}$



6. $\begin{cases} y \leq x + 4 \\ y \leq 4 - x \\ y \geq 0 \end{cases}$