LESSON 9

Watch Your Step— Proofs in Algebra

In previous lessons, we have looked at techniques for solving equations, a common theme throughout algebra. In this lesson, we'll look at how mathematicians show each step to convince others that their solution is the correct one.
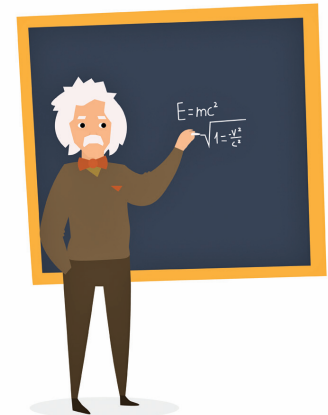
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Opening Exercise

1. Describe the property used to convert the equation from one line to the next:

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$$x(1 - x) + 2x - 4 = 8x - 24 - x^2$$

$$x - x^2 + 2x - 4 = 8x - 24 - x^2$$
$+x^2 \qquad\qquad +x^2$

$$x + 2x - 4 = 8x - 24$$

$$3x - 4 = 8x \ominus 24$$
$+24 \qquad +24$

$$3x + 20 = 8x$$
$-3x \qquad -3x$

$$20 = 5x$$
$\frac{}{5} \quad \frac{}{5}$

$$4 = x$$

Handwritten annotations:
Given
Distributive Prop
Addition Prop of equality
Combine like terms
Addition prop of equality
Subtraction prop of equality
Division prop of equality

In each of the steps above, we applied a property of real numbers or equations to create a new equation.

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2. The initial equation is $x(1 - x) + 2x - 4 = 8x - 24 - x^2$ and the final equation is $20 = 5x$. Do they have the same solution? Explain your thinking.

    Yes, it came from the same equation.

3. What is the common solution set to all these equations?

    $4 = x$

4. You'll be solving the equation for x. The property or action for each step is given, so you'll need to think about how to use that step to solve the equation. It is possible to solve this problem another way, but you'll need to use this set of reasoning steps to solve the equation.

$$3x - [8 - 3(x - 1)] = x + 19$$

Work	Reasoning
1. $3x - [8 - 3(x - 1)] = x + 19$	1. Given statement
2. $3x - (8 - 3x + 3) = x + 19$	2. Distributive property
3. $3x - 8 + 3x - 3 = x + 19$	3. Distributive property
4. $3x + 3x - 8 - 3 = x + 19$	4. Commutative property
5. $6x - 11 = x + 19$	5. Combine like terms
6. $5x - 11 = 19$	6. Subtraction property
7. $5x = 30$	7. Addition property
8. $x = 6$	8. Division property

5. The statement we are trying to prove true is $(xy)z = (zy)x$. You can see that this is a combination of several properties used together. The steps for the proof are given below. Write in the property that was used at each step.

Prove: $(xy)z = (zy)x$	
Work	**Reasoning**
1. $(xy)\underline{z}$	1. Given
2. $z(xy)$	2. Commutative prop of X
3. $z(yx)$	3. Commutative prop of X
4. $(zy)x$	4. Associative prop of X

6. In the next proof, the reasoning is given and you supply the work at each step.

Prove: $(x + y) + z = (z + y) + x$	
Work	**Reasoning**
1. $(x + y) + z$	1. Given
2. $x + (y+z)$	2. Associative Property of Addition
3. $(y+z)+x$	3. Commutative Property of Addition
4. $(z+y)+x$	4. Commutative Property of Addition

7. Finish the mathematical proof to show that $(x + a)(x + b)$ is equivalent to $x^2 + ax + bx + ab$. You'll use the distributive property and the commutative property in this proof.

Statements (Work)	Reasons
1. $(x + a)(x + b)$	1. Given
2. $(x + a)x + (x + a)b$	2. Distributive prop.
3. $x(x + a) + b(x + a)$	3. Commutative prop of $\times$
4. $x^2 + xa + bx + ba$	4. Distributive prop.
5. $x^2 + ax + bx + ab$	5. Commutative prop of $\times$

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Homework Problem Set

1. Solve for *x* and fill in the reasons for each step.

$\frac{1}{5}\left[10 - 5(x - 2)\right] = \frac{1}{10}(x + 1)$	Original statement
$2\left[10 - 5(x - 2)\right] = (x + 1)$	Multiply both sides by ____10____.
$2\left[10 - 5x + 10\right] = (x + 1)$	
$2\left[20 - 5x\right] = (x + 1)$	
$40 - 10x = x + 1$	
$40 = 11x + 1$	
$39 = 11x$	
$\frac{39}{11} = x$	

Solve each equation for *x*. Be sure to show each step, but you do not need to give a reason for each one.

2. $x + 6 - x = 2x + 10$	3. $15 = \frac{3}{5}x$	4. $5(x + 5) = 10$
5. $x + 11 + x = -7$	6. $2x + 7 = 4x - 9$	7. $5x + 4 = 4x + 4$
8. $9(x + 4) = 9x + 4$	9. $3x - 7 + 5 = 2(x - 2)$	10. $2x + 9 + x = 3(x - 2) + 15$

Solve each equation for *x*.

11. $7x - \left[4x - 3(x - 1)\right] = x + 12$

12. $2\left[2(3 - 5x) + 4\right] = 5\left[2(3 - 3x) + 2\right]$

13. $\frac{1}{2}(18 - 5x) = \frac{1}{3}(6 - 4x)$

14. $18 = \frac{2}{3}x$

15. **Challenge** Write an equation that has no solution.

REVIEW—Evaluate Formulas

For each formula, substitute the given value and simplify.

16. Velocity: $v = \dfrac{d}{t}$

If $d = 50$ miles and $t = 2$ hours, what is the velocity, v? Be sure to include units in your answer.

17. Density: $\rho = \dfrac{m}{v}$

If the mass, m, is 50 kg and the volume, V, is 25 cubic centimeters, cm^3, what is the density, ρ?

18. Acceleration: $a = \dfrac{v_f - v_i}{t}$

If the final velocity, V_f, is 20 m/s^2, the initial velocity, V_i, is 10 m/s^2 and the time, t, in which the change occurs is 5 seconds, what is the acceleration?

19. Momentum: $p = mv$

If the mass, m, is 10 kg and the velocity, V, is 10 m/s^2, what is the momentum, p?