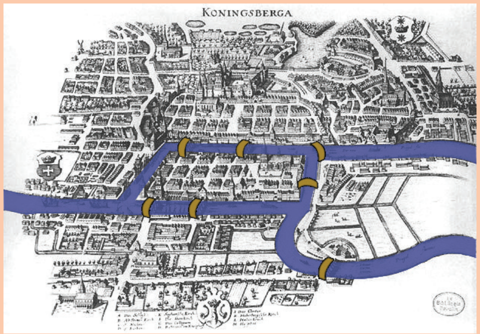
LESSON 1 Introduction to Networks

Exploratory Challenge 1

One classic math puzzle is the Seven Bridges of Königsberg problem which laid the foundation for networks and graph theory.

In the 18th century in the town of Königsberg, Germany, a favorite pastime was walking along the Pregel River and strolling over the town's seven bridges. . . A question arose: Is it possible to take a walk and cross each bridge only once?



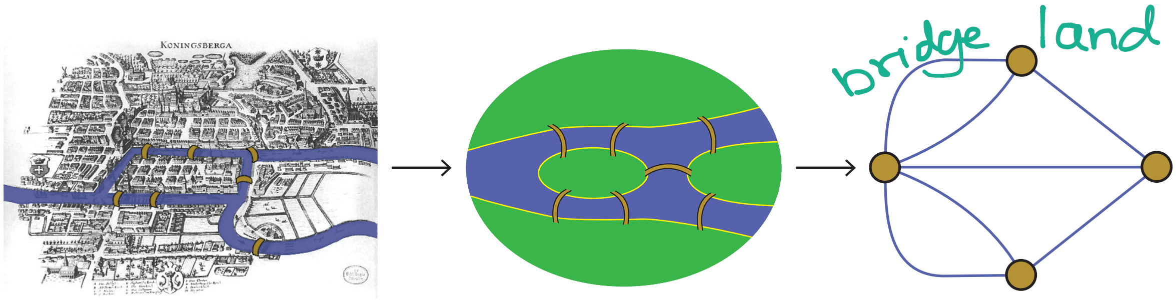
Source: Merian-Erben, 1652

You will need: highlighters

1. Work with your partner to devise a path that crosses each bridge only once. You may draw on the map above.

2. How many solutions did your class find?

Leonhard Euler, a Swiss mathematician, proved in 1736 that it was impossible to cross each bridge exactly once **and** go over every one of the seven bridges. He created a simplified version of the map so that only the bridges and land masses were visible. He then simplified this even more by using dots or **vertices** for the land masses and segments or **arcs** or **edges** for the bridges as shown below. This final "map" is called a **network**.

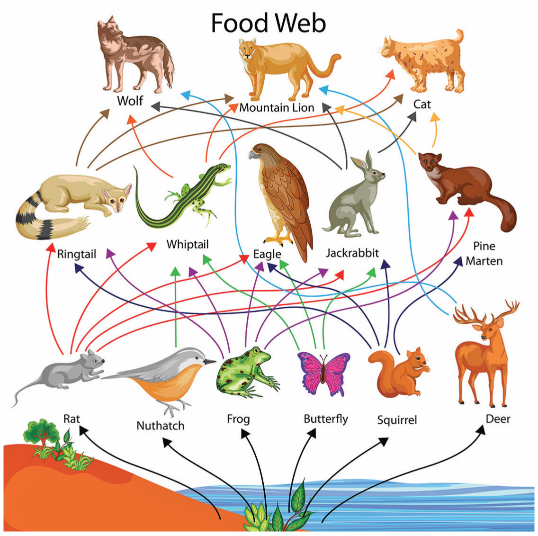


Sources: Merian-Erben, 1652 and https://commons.wikimedia.org/w/index.php?curid=851840 and https://en.wikipedia.org/wiki/Leonhard_Euler

3. Use highlighters to show the connection between each model above. For example, you could highlight one bridge and its corresponding arcs in the same color.

The ideas behind networks are found in many fields and occupations. Two different network examples are shown below.

World Map with Global Technology Network

Food Web Network
 © Vecton/Shutterstock.com

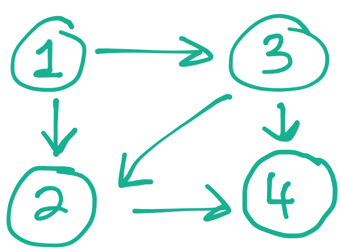
4. Name one other network that you use or know of?

- Sewage system
- City maps
- telephone wires
- Subway map

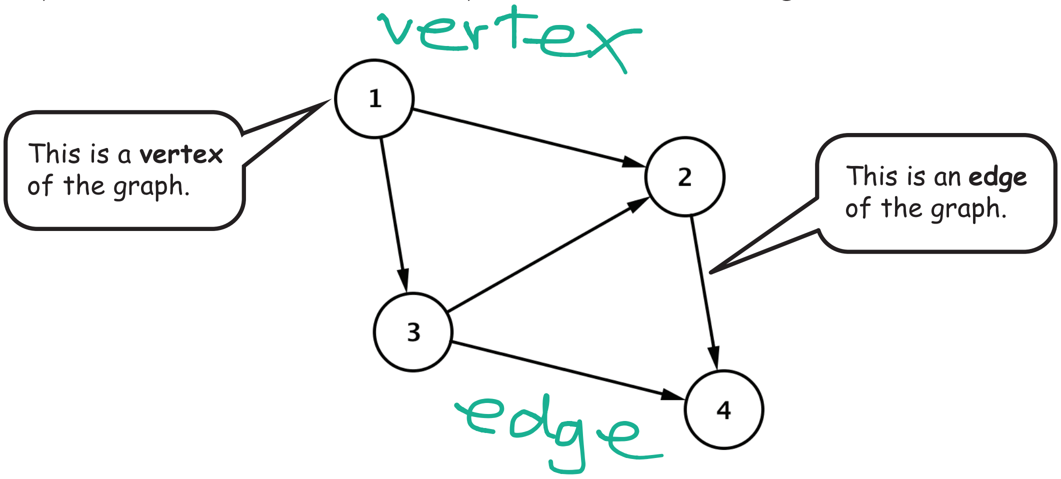
Exploratory Challenge 2

In this exploration, your group will create your own network based on criteria about the bus routes to and from four cities.

5. Work with your group to draw a network for the bus routes in the space below. Be sure your finished network satisfies ALL the conditions for each city.

Network Conditions		**Network Conditions**
The City 1 buses have routes to City 2 and City 3.		The City 3 buses have routes to City 2 and City 4.
The City 2 buses have one route to City 4.		There is no City 4 bus routes.

The diagram you created in the Exercise 5 may have looked something like this:



6. This is a called a **directed** graph. Why do you think it has this name?

Because it has directions and arrows

The routes from one city to another are **edges** on the graph and the cities are **vertices**.

7. How many ways can you travel from City 1 to City 4? Explain how you know.

1 → 2 → 4

1 → 3 → 4

1 → 3 → 2 → 4

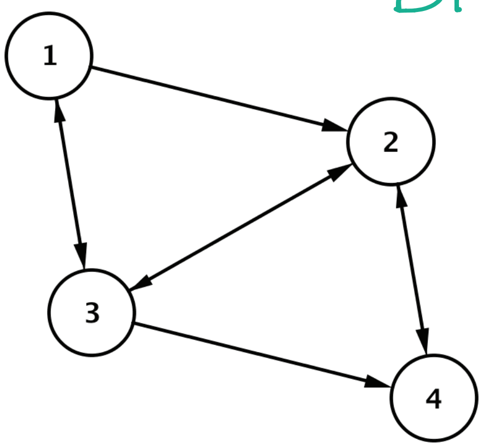
8. What about these bus routes doesn't make sense?

- Can't get out of City 4

- No bus to city 1

It turns out there was an error in printing the first route map. An updated network diagram showing the bus routes that connect the four cities is shown below. Arrows on both ends of an edge indicate that buses travel in both directions (**bidirectional**).

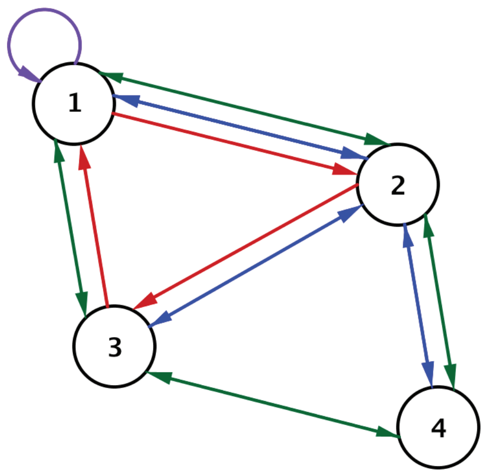
bi → both directions



9. How many ways can you reasonably travel from City 4 to City 1 using the route map above? Explain how you know.

4 → 2 → 3 → 1

A rival bus company offers more routes connecting these four cities as shown in the network diagram in at the right.



Discussion

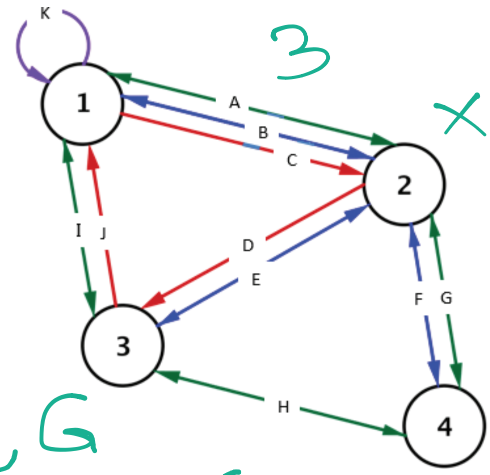
10. What might the loop at City 1 represent?

There's a bus route in City 1.

11. What might be difficult about describing the path from City 1 to City 4 with this diagram?

– There are multiple paths.

To better analyze the different paths from one city to another, mathematicians often label each path. Because there are multiple routes to each city, we'll label the different routes with letters to distinguish one from another. In that way you can distinguish the path from City 1 to City 2 using Route A, Route B or Route C.



3 $x = 6$ 2

12. How many ways can you travel from City 1 to City 4 if you want to stop in City 2 and make no other stops?

A, G B, G C, G
A, F B, F C, F 6

13. How many possible ways are there to travel from City 1 to City 4 without repeating a city?

$1 \to 2 \to 3 \to 4$
$3 \times 2 \times 1 = 6$

$1 \to 2 \to 4$
$3 \times 2 = 6$

$1 \to 3 \to 2 \to 4$
$1 \times 1 \times 2 = 2$

$1 \to 3 \to 4$
$1 \times 1 = 1$

15

$6+2+6+1$

Discussion

As a transportation network grows, these diagrams become more complicated, and keeping track of all of the information can be challenging. People that work with complicated networks use computers to manage and manipulate this information.

14. What challenges did you encounter as you tried to answer Exercises 12 and 13?

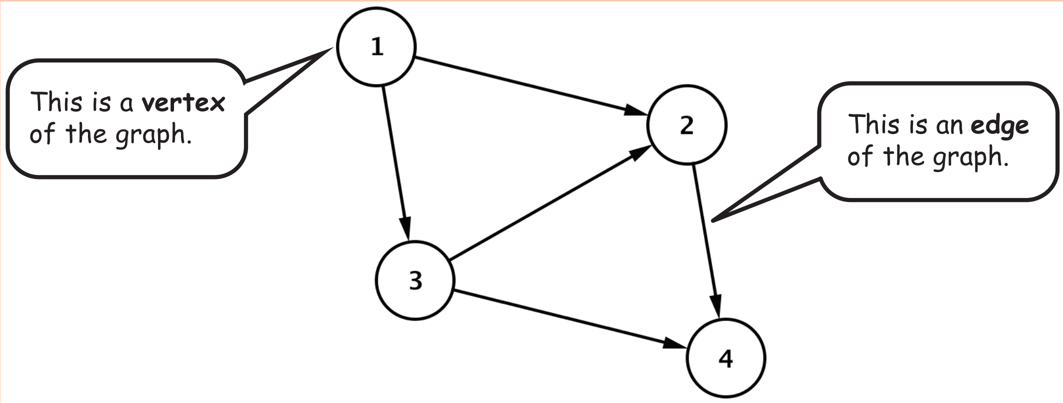
Organizing different routes.

15. How might we present the possible routes in a more organized manner?

Matrices!!! :)

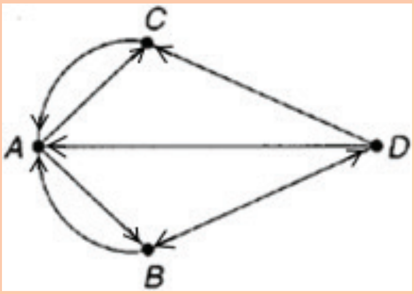
Lesson Summary

A network is a collection of points, called **vertices**, and a collection of lines, called **arcs** or **edges**, connecting these points. A network is a graphical representation of a relationship between objects or ideas.

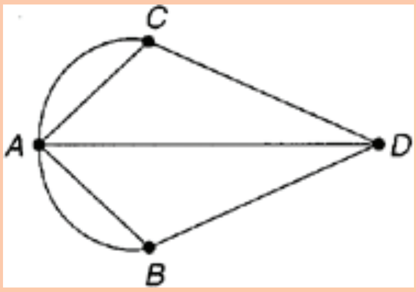


If the edges in a network are shown with arrows, then the network is called a **directed** network or directed graph. If no arrows appear in a network, then it is assumed that all edges are **bidirectional**.

Directed Network



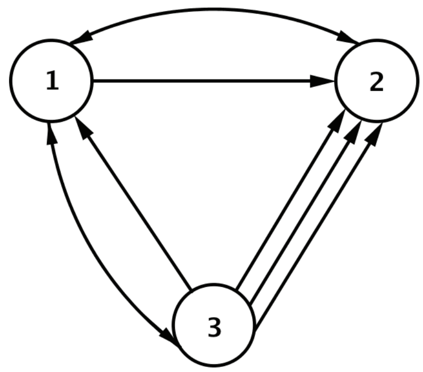
Bidirectional Network



NAME: ____________________ PERIOD: ____________________ DATE: ________________

Homework Problem Set

1. Consider the railroad map between Cities 1, 2, and 3, as shown on the right.



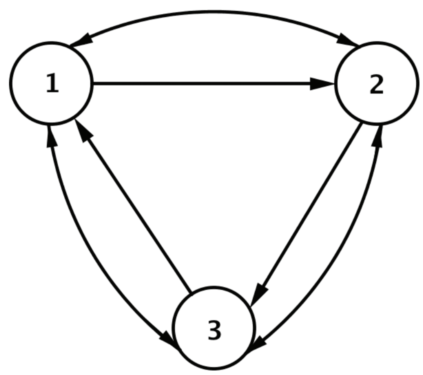
   A. How many different ways can you travel from City 1 to City 3 without passing through the same city twice?

   B. How many different ways can you travel from City 2 to City 3 without passing through the same city twice?

   C. How many different ways can you travel from City 1 to City 2 with exactly one connecting stop?

   D. Why is this not a reasonable network diagram for a railroad?

2. Consider the subway map between stations 1, 2, and 3, as shown.



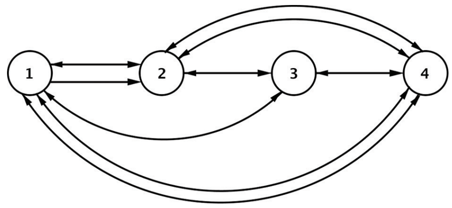
   A. How many different ways can you travel from station 1 to station 3 without passing through the same station twice?

   B. How many different ways can you travel directly from station 1 to station 3 with no stops?

   C. How many different ways can you travel from station 1 to station 3 with exactly one stop?

   D. How many different ways can you travel from station 1 to station 3 with exactly two stops? Allow for stops at repeated stations.

3. Consider the airline flight routes between Cities 1, 2, 3, and 4, as shown.



   A. How many different routes can you take from City 1 to City 4 with no stops?

   B. How many different routes can you take from City 1 to City 4 with exactly one stop?

   C. How many different routes can you take from City 3 to City 4 with exactly one stop?

   D. How many different routes can you take from City 1 to City 4 with exactly two stops? Allow for routes that include repeated cities.

   E. How many different routes can you take from City 2 to City 4 with exactly two stops? Allow for routes that include repeated cities.