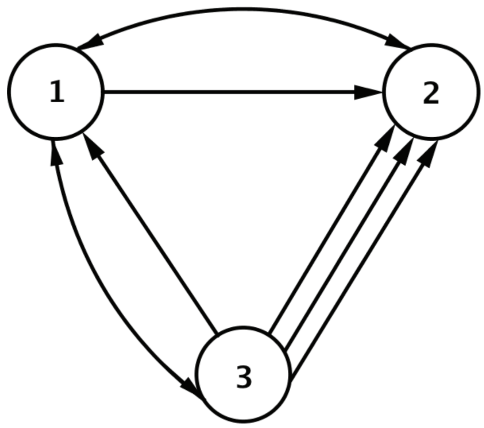
NAME: ____________________ PERIOD: ____________________ DATE: ________________

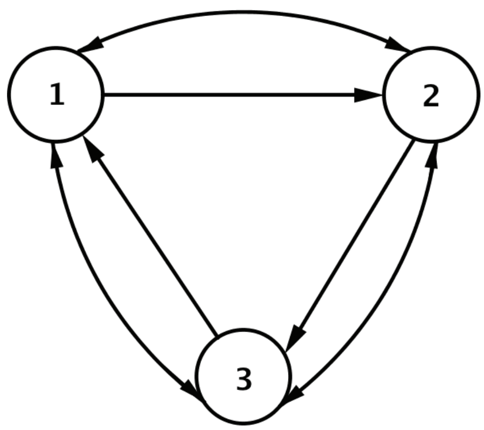
Homework Problem Set

1. Consider the railroad map between Cities 1, 2, and 3, as shown. Create a matrix R to represent the railroad map between Cities 1, 2, and 3.



$$R = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 0 \\ 2 & 3 & 0 \end{bmatrix}$$

2. Consider the subway map between stations 1, 2, and 3, as shown. Create a matrix S to represent the subway map between stations 1, 2, and 3.



$$S = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix}$$

3. Suppose the matrix R represents a railroad map between cities 1, 2, 3, 4, and 5.

$$R = \begin{bmatrix} 0 & 1 & 2 & 1 & 1 \\ 2 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 2 & 2 \\ 1 & 1 & 0 & 0 & 2 \\ 1 & 1 & 3 & 0 & 0 \end{bmatrix}$$

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A. How many different ways can you travel from City 1 to City 3 with exactly one connection?

$1 \to 2 \to 3$ $\quad$ $1 \to 4 \to 3$ $\quad$ $1 \to 5 \to 3$

$r_{1,2} \cdot r_{2,3} + r_{1,4} \cdot r_{4,3} + r_{1,5} \cdot r_{5,3} = \boxed{4 \text{ total ways}}$

$1 \cdot 1$ $\quad$ $1 \cdot 0$ $\quad$ $1 \cdot 3$

$\boxed{1}$ $\quad$ $\boxed{0}$ $\quad$ $\boxed{3}$

B. How many different ways can you travel from City 1 to City 5 with exactly one connection?

$1 \to 2 \to 5$ $\quad$ $1 \to 3 \to 5$ $\quad$ $1 \to 4 \to 5$

$r_{1,2} \cdot r_{2,5} + r_{1,3} \cdot r_{3,5} + r_{1,4} \cdot r_{4,5} = \boxed{6 \text{ total ways}}$

$1 \cdot 0$ $\quad$ $2 \cdot 2$ $\quad$ $1 \cdot 2$

$\boxed{0}$ $\quad$ $\boxed{4}$ $\quad$ $\boxed{2}$

C. How many different ways can you travel from City 2 to City 5 with exactly one connection?

$2 \to 1 \to 5$ $\quad$ $2 \to 3 \to 5$ $\quad$ $2 \to 4 \to 5$

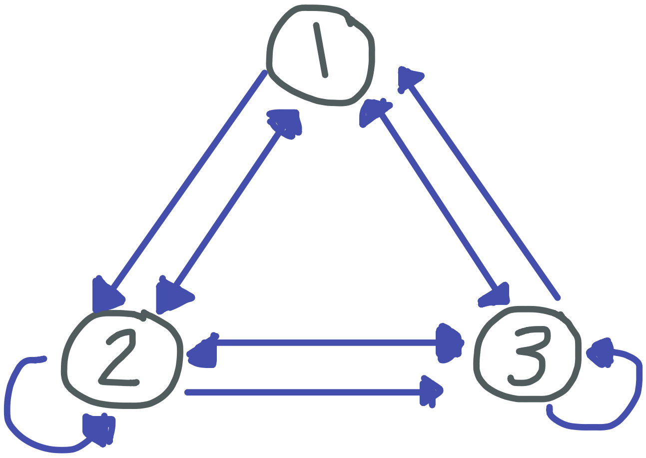
$r_{2,1} \cdot r_{1,5} + r_{2,3} \cdot r_{3,5} + r_{2,4} \cdot r_{4,5} = \boxed{6 \text{ total ways}}$

$2 \cdot 1$ $\quad$ $1 \cdot 2$ $\quad$ $1 \cdot 2$

$\boxed{2}$ $\quad$ $\boxed{2}$ $\quad$ $\boxed{2}$

4. Let $B = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{bmatrix}$ represent the bus routes between 3 cities.

A. Draw an example of a network diagram represented by this matrix.



B. How many routes are there between City 1 and City 2 with one stop in between?

$1 \to 3 \to 2$ + $1 \to 2 \to 2$ = $\boxed{3 \text{ total routes}}$

$b_{1,3} \cdot b_{3,2}$ + $b_{1,2} \cdot b_{2,2}$

$1 \cdot 1$ = $\boxed{1}$ $\quad$ $2 \cdot 1$ = $\boxed{2}$

C. How many routes are there between City 2 and City 2 with one stop in between?

$2 \to 3 \to 2$ + $2 \to 1 \to 2$ + $2 - 2 - 2$ = $\boxed{5 \text{ Total routes}}$

$b_{2,3} \cdot b_{3,2}$ + $b_{2,1} \cdot b_{1,2}$ + $b_{2,2} \cdot b_{2,2}$

$2 \cdot 1$ = $\boxed{2}$ $\quad$ $1 \cdot 2$ = $\boxed{2}$ $\quad$ $1 \cdot 1$ = $\boxed{1}$

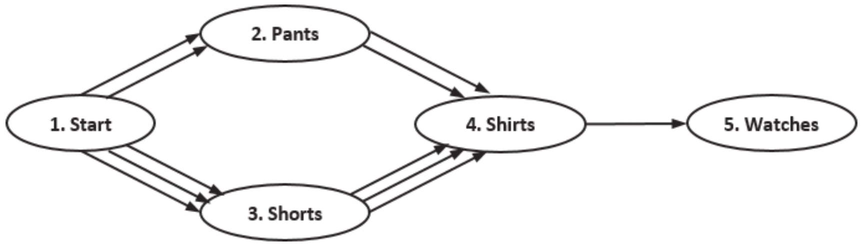
D. How many routes are there between City 3 and City 2 with one stop in between?

$3 \to 1 \to 2$ + $3 \to 2 \to 2$ + $3 \to 3 - 2$ = $\boxed{6 \text{ Total routes}}$

$b_{3,1} \cdot b_{1,2}$ + $b_{3,2} \cdot b_{2,2}$ + $b_{3,3} \cdot b_{3,2}$

$2 \cdot 2$ = $\boxed{4}$ $\quad$ $1 \cdot 1$ = $\boxed{1}$ $\quad$ $1 \cdot 1$ = $\boxed{1}$

5. Consider the following directed graph representing the number of ways Trenton can get dressed in the morning (only visible options are shown):



A. What reasons could there be for there to be three choices for shirts after "traveling" to shorts but only two after traveling to pants?

maybe one of his shirts doesn't match his pants.

B. What could the order of the vertices mean in this situation?

The order Trenton dresses

C. Write a matrix A representing this directed graph.

$$A = \begin{bmatrix} 0 & 2 & 3 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

D. Delete any rows of zeros in matrix A, and write the new matrix as matrix B. Does deleting this row change the meaning of any of the entries of B? If you had deleted the first column, would the meaning of the entries change? Explain.

$$B = \begin{bmatrix} 0 & 2 & 3 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

* Deleting a row didn't change meanings (sequence remains same).
* Deleting first column changes starting point of sequence.

E. Calculate $b_{1,2} \cdot b_{2,4} \cdot b_{4,5}$. What does this product represent?

$2 \cdot 2 \cdot 1 = \boxed{5}$

Trenton can wear 5 outfits assuming he wears pants instead of shorts

F. How many different outfits can Trenton wear assuming he always wears a watch?

$1 \rightarrow 2 \rightarrow 4 \rightarrow 5 \quad + \quad 1 \rightarrow 3 \rightarrow 4 \rightarrow 5 \quad = \boxed{13}$ Different Outfits

$b_{1,2} \cdot b_{2,4} \cdot b_{4,5} \qquad b_{1,3} \cdot b_{3,4} \cdot b_{4,5}$

$2 \cdot 2 \cdot 1 \qquad 3 \cdot 3 \cdot 1$

$\boxed{4} \quad + \quad \boxed{9}$