

LESSON

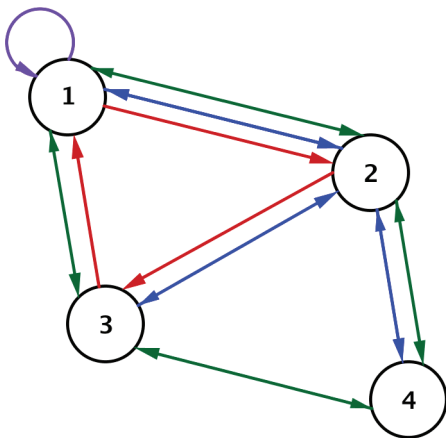
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Organizing Networks with Matrices

Opening Exercise

As we saw in Lesson 1, networks can become complicated and finding a way to organize that data is important.

1. We will consider a “direct route” to be a route from one city to another without going through any other city. Organize the number of direct routes from each city into the table shown below. The first row showing the direct routes between City 1 and the other cities has been completed for you.



DIRECT ROUTES

		Destination Cities			
		1	2	3	4
Cities of Origin	1	1	3	1	0
	2	2	0	2	2
	3	2	1	0	1
	4	0	2	1	0

$$M = 2 \times 3 = \begin{bmatrix} \star & \star & \star \\ \star & \star & \star \end{bmatrix}$$

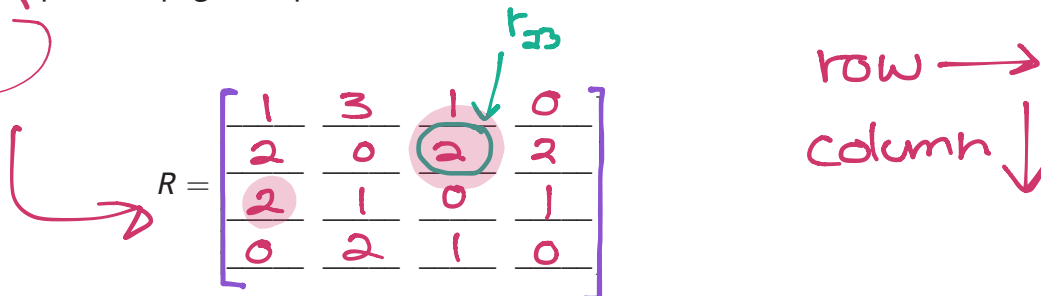
$$M = 3 \times 1 = \begin{bmatrix} \star \\ \star \\ \star \end{bmatrix}$$

A matrix is defined as a rectangular array of numbers arranged in the form shown.

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

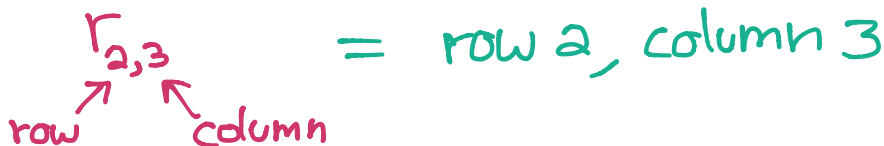
For our table of Direct Routes, a **matrix** would only include the inner cells and not the City labels (1, 2, 3 and 4).

2. Use the table on the previous page to represent the number of direct routes between the four cities in **matrix R**.



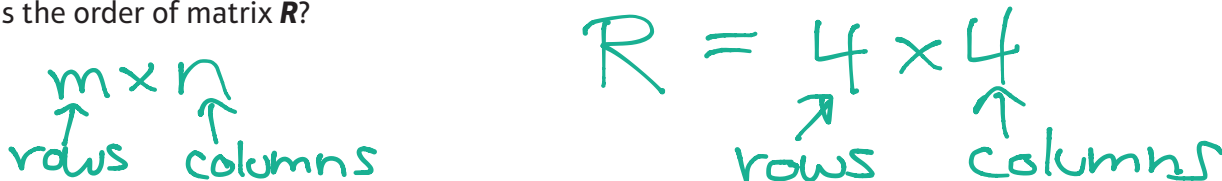
Elements: The numbers in the matrix (e.g. a_{12} , a_{1n} and a_{mn}) are called the elements. The subscript defines where in the matrix the element is located. The first number determines the row and the second determines the column.

3. Circle $r_{2,3}$. What is the value of $r_{2,3}$? What does it represent in this situation?



Order or Dimension of the Matrix: A matrix having m rows and n columns is called a $m \times n$ (m by n) matrix. A $m \times n$ matrix has order $m \times n$.

4. What is the order of matrix **R**?



Square Matrix: An $n \times n$ matrix is a square matrix since the number of rows is equal to the number of columns.

5. Is matrix **R** a square matrix? How can you tell?

Yes because R is 4x4 Matrix



Null or Zero Matrix: A null or zero matrix is a matrix with all elements zero.

6. What would it mean if matrix **R** was a zero matrix? What would that represent in real life?

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{There's no route}$$

Exploratory Challenge

7. What is the value of $r_{2,3} \cdot r_{3,1}$ and what does it represent in this situation?

$$r_{2,3} \cdot r_{3,1} \quad \text{There are 4 routes from City 2 to City 1 stopping at City 3.}$$

$$\downarrow \quad \downarrow$$

$$2 \times 2 = 4$$

8. A. Write an expression for the total number of one-stop routes from City 4 to City 1.

$$r_{4,4} \cdot r_{4,1} \quad r_{4,3} \cdot r_{3,1} \quad r_{4,2} \cdot r_{2,1} \quad r_{4,1} \cdot r_{1,1}$$

$$(0 \cdot 0) + (1 \cdot 2) + (2 \cdot 2) + (0 \cdot 1)$$

B. Determine the total number of one-stop routes from City 4 to City 1.

$$0 + 2 + 4 + 0$$

$$6 \text{ total routes}$$

9. Do you notice any patterns in the expression for the total number of one-stop routes from City 4 to City 1?

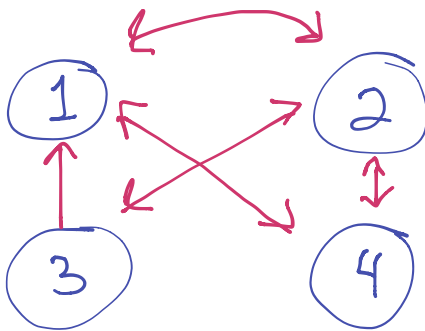
The middle numbers are the stopping cities.

10. How can you find the total number of possible routes between two locations in a network?

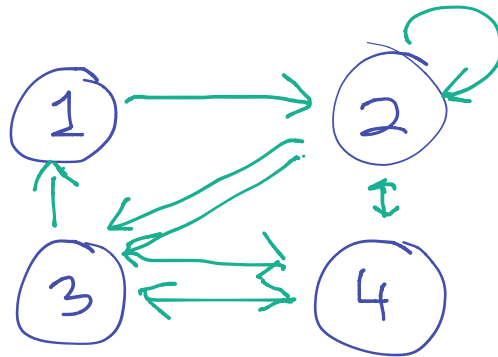
Working Backward—Going from a Matrix to a Network

11. Create a network diagram for the matrices shown below. Each matrix represents the number of transportation routes that connect four cities. The rows are the cities you travel from, and the columns are the cities you travel to.

$$T = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

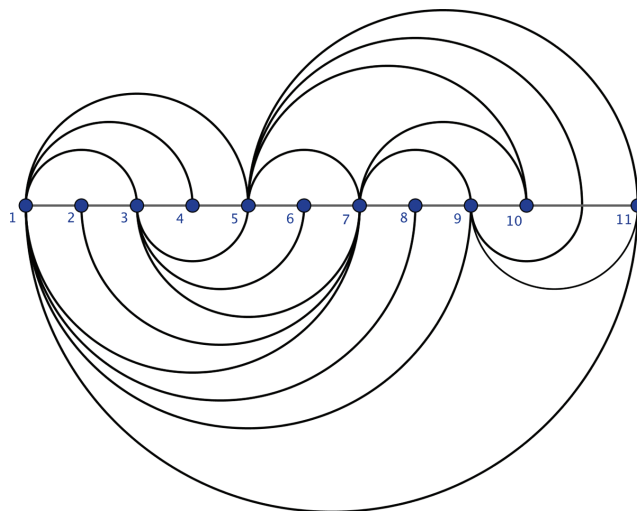


$$U = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$



Arc Diagrams

Here is a type of network diagram called an arc diagram. Notice that there are no arrows on this diagram. When there are no arrows, the arcs are bidirectional.



Suppose the points represent eleven students in your mathematics class, numbered 1 through 11. The arcs above and below the line of vertices 1–11 are the people who are friends on a social network.

12. Complete the matrix that shows which students are friends with each other on this social network. The first row has been completed for you.

Student 1 is friends with ...

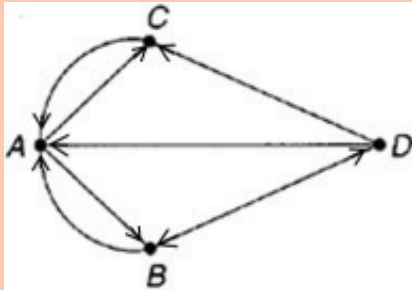
0	0	1	1	1	0	1	1	1	0	1
-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-	-
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-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-	-

13. Student 1 is not friends with Student 10. How many ways could Student 1 get a message to 10 by only going through one other friend?
14. Who has the most friends in this network? Explain how you know.
15. Is everyone in this network connected at least as a friend of a friend? Explain how you know.

Lesson Summary

A matrix is a rectangular array of numbers, symbols or expressions, arranged in rows and columns. The individual items in a matrix are called its elements.

Complete the matrix for this network.

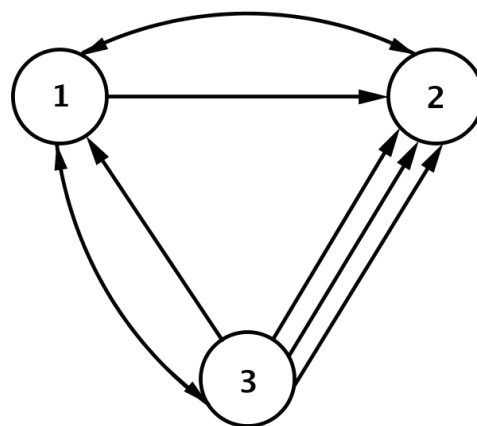


$$\begin{bmatrix} 0 & _ & _ & 0 \\ _ & _ & _ & 1 \\ _ & 0 & _ & _ \\ 1 & 1 & _ & _ \end{bmatrix}$$

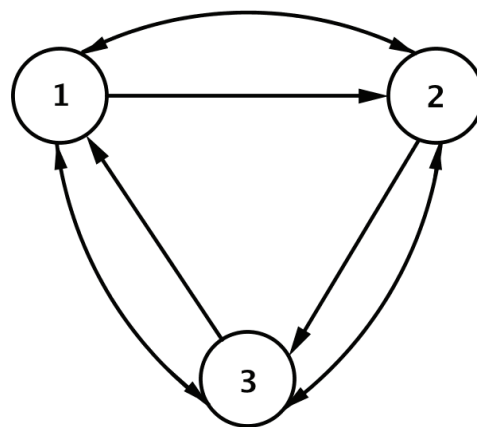
NAME: _____ PERIOD: _____ DATE: _____

Homework Problem Set

1. Consider the railroad map between Cities 1, 2, and 3, as shown. Create a matrix R to represent the railroad map between Cities 1, 2, and 3.



2. Consider the subway map between stations 1, 2, and 3, as shown. Create a matrix S to represent the subway map between stations 1, 2, and 3.



3. Suppose the matrix R represents a railroad map between cities 1, 2, 3, 4, and 5.

$$R = \begin{bmatrix} 0 & 1 & 2 & 1 & 1 \\ 2 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 2 & 2 \\ 1 & 1 & 0 & 0 & 2 \\ 1 & 1 & 3 & 0 & 0 \end{bmatrix}$$



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- A. How many different ways can you travel from City 1 to City 3 with exactly one connection?
- B. How many different ways can you travel from City 1 to City 5 with exactly one connection?
- C. How many different ways can you travel from City 2 to City 5 with exactly one connection?

4. Let $B = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{bmatrix}$ represent the bus routes between 3 cities.

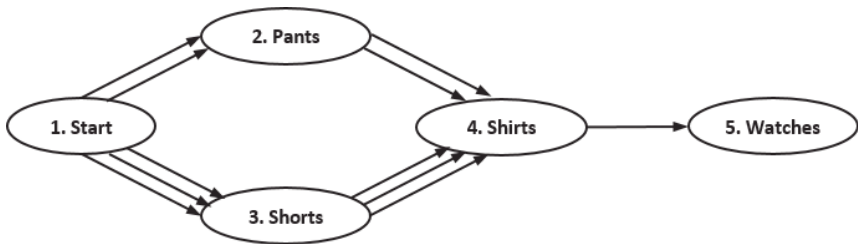
A. Draw an example of a network diagram represented by this matrix.

B. How many routes are there between City 1 and City 2 with one stop in between?

C. How many routes are there between City 2 and City 2 with one stop in between?

D. How many routes are there between City 3 and City 2 with one stop in between?

5. Consider the following directed graph representing the number of ways Trenton can get dressed in the morning (only visible options are shown):



- A. What reasons could there be for there to be three choices for shirts after “traveling” to shorts but only two after traveling to pants?

- B. What could the order of the vertices mean in this situation?

- C. Write a matrix A representing this directed graph.

- D. Delete any rows of zeros in matrix A , and write the new matrix as matrix B . Does deleting this row change the meaning of any of the entries of B ? If you had deleted the first column, would the meaning of the entries change? Explain.

- E. Calculate $b_{1,2} \cdot b_{2,4} \cdot b_{4,5}$. What does this product represent?

- F. How many different outfits can Trenton wear assuming he always wears a watch?