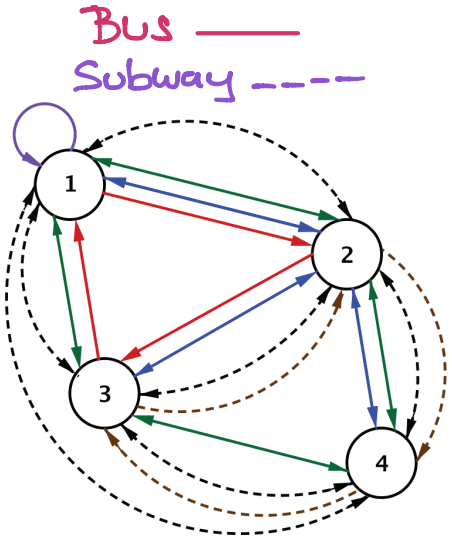
LESSON 3

Networks and Matrix Arithmetic

Opening Exercise

Suppose a subway line also connects the four cities. Here is the subway and bus line network. The bus routes connecting the cities are represented by solid lines, and the subway routes are represented by dashed arcs.



1. Write a matrix B to represent the bus routes and a matrix S to represent the subway lines connecting the four cities.

$$B = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \text{ To} \\ \begin{bmatrix} 1 & 3 & 1 & 0 \\ 2 & 0 & 2 & 2 \\ 2 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix} \end{array} \quad \text{From}$$

$$S = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 \\ 1 & 2 & 0 & 1 \\ 1 & 1 & 2 & 0 \end{bmatrix}$$

Exploratory Challenge

Use the network diagram from the Opening Exercise and your two matrices to help you complete this challenge with your group.

2. Suppose the number of bus routes between each city were doubled.

   A. What would the new bus route matrix be?

$$2B = 2\begin{bmatrix} 1 & 3 & 1 & 0 \\ 2 & 0 & 2 & 2 \\ 2 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 2 & 0 \\ 4 & 0 & 4 & 4 \\ 4 & 2 & 0 & 2 \\ 0 & 4 & 2 & 0 \end{bmatrix}$$

   B. Mathematicians call this matrix $2B$. Why do you think they call it that?

Mult. every element by 2

3. A. What would be the meaning of *10B* in this situation?

   B. Write the matrix *10B*.

$$10\begin{bmatrix}1&3&1&0\\2&0&2&2\\2&1&0&1\\0&2&1&0\end{bmatrix}=\begin{bmatrix}10&30&10&0\\20&0&20&20\\20&10&0&10\\0&20&10&0\end{bmatrix}$$

   C. How would you describe the process you used to create the matrix *10B*?

4. Suppose we ignore whether or not a line connecting cities represents a bus or subway route.

   A. Create one matrix that represents all the routes between the cities in this transportation network.

$$B+S=\begin{bmatrix}1&3&1&0\\2&0&2&2\\2&1&0&1\\0&2&1&0\end{bmatrix}+\begin{bmatrix}0&1&1&1\\1&0&1&2\\1&2&0&1\\1&1&2&0\end{bmatrix}=\begin{bmatrix}1&4&2&1\\3&0&3&4\\3&3&0&2\\1&3&3&0\end{bmatrix}$$

$B_{1,2} + S_{1,2}$

   B. Why would it be appropriate to call this matrix $B + S$? Explain your reasoning.

   C. Why would *B* and *S* have to be the same dimensions in order to find their sum?

$$\begin{bmatrix}a&b\\c&d\end{bmatrix}+\begin{bmatrix}\quad&\quad\\\quad&\quad\end{bmatrix}=?$$ not possible

add/sub must have the same dimensions.

5. Suppose that April's Pet Supply has three stores in Cities 1, 2, and 3. Ben's Pet Mart has two stores in Cities 1 and 2. Each shop sells the same type of dog crates in size 1 (small), 2 (medium), 3 (large), and 4 (extra large). April's and Ben's inventory in each city are shown in the tables below.

	April's Pet Supply		
	City 1	City 2	City 3
Size 1	3	5	1
Size 2	4	2	9
Size 3	1	4	2
Size 4	0	0	1

	Ben's Pet Mart		
	City 1	City 2	City 3
Size 1	2	3	0
Size 2	0	2	0
Size 3	4	1	0
Size 4	0	0	0

A. Create a matrix A so that $a_{i,j}$ represents the number of crates of size i available in April's store j.

$$\begin{bmatrix} 3 & 5 & 1 \\ 4 & 2 & 9 \\ 1 & 4 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

4x3

rows columns

B. Explain how the matrix $B = \begin{bmatrix} 2 & 3 & 0 \\ 0 & 2 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ can represent the dog crate inventory at Ben's Pet Mart.

4x3

C. Suppose that April and Ben merge their inventories. Write a matrix that represents their combined inventory in each of the three cities.

$$\begin{bmatrix} 3 & 5 & 1 \\ 4 & 2 & 9 \\ 1 & 4 & 2 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 0 \\ 0 & 2 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 8 & 1 \\ 4 & 4 & 9 \\ 5 & 5 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Lesson Summary

Complete the example for each operation.

Matrix Operations

Operation	Symbols	How to Calculate	Examples
Scalar Multiplication	kA	Multiply each element of matrix A by the real number k.	$2\begin{bmatrix}1&1&2\\2&2&4\\3&0&0\end{bmatrix}=$
The Sum of Two Matrices	$A+B$	Add corresponding elements in each row and column of A and B. Matrices A and B MUST have the same dimensions.	$\begin{bmatrix}1&1&2\\2&2&4\\3&0&0\end{bmatrix}+\begin{bmatrix}2&2&4\\1&3&5\\1&-2&1\end{bmatrix}=$
The Difference of Two Matrices	$A-B$ $=A+(-1)B$	Subtract corresponding elements in each row and column of A and B. The matrices MUST have the same dimensions.	$\begin{bmatrix}1&1&2\\2&2&4\\3&0&0\end{bmatrix}+\begin{bmatrix}-2&-2&-4\\-1&-3&-5\\-1&+2&-1\end{bmatrix}=$ $\begin{bmatrix}-1&-1&-2\\1&-1&-1\\2&2&-1\end{bmatrix}$

$$\begin{bmatrix}3&4\\-5&7\end{bmatrix}-2\begin{bmatrix}0&4\\-8&-7\end{bmatrix}$$

$$\begin{bmatrix}3&4\\-5&7\end{bmatrix}+\begin{bmatrix}0&-8\\16&14\end{bmatrix}=\begin{bmatrix}3&-4\\11&21\end{bmatrix}$$

NAME: _____ PERIOD: _____ DATE: _____

Homework Problem Set

For the matrices given below, perform each of the following calculations or explain why the calculation is not possible.

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 5 & 2 & 9 \\ 6 & 1 & 3 \\ -1 & 1 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 6 & 0 \\ 3 & 0 & 2 \\ 1 & 3 & -2 \end{bmatrix}$$

1. $A + B$	2. $2A - B$
3. $A + C$	4. $-2C$
5. $4D - 2C$	6. $3B - 3B$
7. $5B - C$	8. $B - 3A$

9. Let $A = \begin{bmatrix} 3 & \frac{2}{3} \\ -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} \frac{1}{2} & \frac{3}{2} \\ 4 & 1 \end{bmatrix}$

   A. If $C = 6A + 6B$, determine matrix C.

   B. If $D = 6(A + B)$, determine matrix D.

   C. What is the relationship between matrices C and D? Why do you think that is?

10. Let $A = \begin{bmatrix} 3 & 2 \\ -1 & 5 \\ 3 & -4 \end{bmatrix}$ and X be a 3×2 matrix. If $A + X = \begin{bmatrix} -2 & 3 \\ 4 & 1 \\ 1 & -5 \end{bmatrix}$, then determine X.

11. Let $A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 1 & 2 \\ 4 & 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix}$ represent the bus routes of two companies between three cities.

   A. Let $C = A + B$. Find matrix C. Explain what the resulting matrix and entry $c_{1,3}$ mean in this context.

   B. Let $D = B + A$. Find matrix D. Explain what the resulting matrix and entry $d_{1,3}$ mean in this context.

   C. What is the relationship between matrices C and D? Why do you think that is?