NAME: ____________________ PERIOD: ____________________ DATE: ______________

# Homework Problem Set

1. Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ represent the bus routes of two companies between two cities. Find the product $A \cdot B$, and explain the meaning of the entry in row 1, column 2 of $A \cdot B$ in the context of this scenario.



$$A \cdot B = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 1+12 & 2+9 \\ 2+0 & 4+0 \end{bmatrix} = \boxed{\begin{bmatrix} 13 & 11 \\ 2 & 4 \end{bmatrix}}$$

$\boxed{\text{Row}_1\ \text{Column}_2 = 11}$ ⟹ 11 possible routes from city 1→city 2 starting with a bus from company A and then a bus from company B.

2. Let $A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 1 & 2 \\ 4 & 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix}$ represent the bus routes of two companies between three cities.

   A. Let $C = A \cdot B$. Find matrix $C$, and explain the meaning of entry $c_{1,3}$.

$$C = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 1 & 2 \\ 4 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 3 \\ 2 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 2+6+2 & 1+6+6 & 3+3+2 \\ 6+2+2 & 3+2+6 & 9+1+2 \\ 8+6+2 & 4+6+6 & 12+3+2 \end{bmatrix} = \boxed{\begin{bmatrix} 10 & 13 & 8 \\ 10 & 11 & 12 \\ 16 & 16 & 17 \end{bmatrix}}$$

$\boxed{C_{1,3} = 8}$ → 8 routes from city 1→3 by taking a Bus from company A and then company C

   B. Nina wants to travel from City 3 to City 1 and back home to City 3 by taking a direct bus from Company A on the way to City 1 and a bus from Company B on the way back home to City 3. How many different ways are there for her to make this trip?

$\boxed{\begin{bmatrix} 10 & 13 & 8 \\ 10 & 11 & 12 \\ 16 & 16 & 17 \end{bmatrix}}$ City 3 → 3 $\boxed{C_{3,3} = 17}$ → 17 ways to go from City 3 and back to City 3

   C. Oliver wants to travel from City 2 to City 3 by taking first a bus from Company A and then taking a bus from Company B. How many different ways can he do this?

$C_{2,3} = 12$ $\boxed{\text{12 ways}}$ to make trip.

   D. How many routes can Oliver choose from if travels from City 2 to City 3 by first taking a bus from Company B and then taking a bus from Company A?

$$B \cdot A = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & 2 \\ 3 & 1 & 2 \\ 4 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 2+3+12 & 6+1+9 & 4+2+6 \\ 2+6+4 & 6+2+3 & 4+4+2 \\ 1+9+4 & 3+3+3 & 2+6+2 \end{bmatrix} = \boxed{\begin{bmatrix} 17 & 16 & 12 \\ 12 & 11 & 10 \\ 14 & 9 & 10 \end{bmatrix}}$$

$\boxed{C_{2,3} = \text{10 ways}}$

3. Consider the matrices

$$A = \begin{bmatrix} 3 & 1 & -\frac{1}{2} \\ 2 & \frac{2}{3} & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Multiply $AB$ and $BA$ or explain why you cannot.

$$AB = \begin{bmatrix} 3 & 1 \\ 2 & \frac{2}{3} \end{bmatrix} \qquad BA = \begin{bmatrix} 3 & 1 & -\frac{1}{2} \\ 2 & \frac{2}{3} & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

**For the matrices given below, perform each of the following calculations or explain why the calculation is not possible.**

$$A = \begin{bmatrix} \frac{1}{2} & 3 \\ 2 & \frac{2}{3} \end{bmatrix} \qquad B = \begin{bmatrix} 9 & -1 & 2 \\ -3 & 4 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 1 & 3 \\ 1 & 0 & 1 \\ 3 & 1 & 3 \end{bmatrix} \qquad D = \begin{bmatrix} 2 & 0 & -2 & \frac{1}{2} \\ 3 & 2 & 1 & 0 \end{bmatrix}$$

4. $AB$ $(2\times 2)\cdot(2\times 3) \rightarrow (2\times 3)$

$$\begin{bmatrix} \frac{1}{2} & 3 \\ 2 & \frac{2}{3} \end{bmatrix} \times \begin{bmatrix} 9 & -1 & 2 \\ -3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \frac{9}{2}+-9 & -\frac{1}{2}+12 & 1+3 \\ 18-2 & -2+\frac{8}{3} & 4+\frac{2}{3} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{9}{2} & \frac{23}{2} & 4 \\ 16 & \frac{2}{3} & \frac{14}{3} \end{bmatrix}$$

5. $BC$ $(2\times 3)\cdot(3\times 3) = (2\times 3)$

$$\begin{bmatrix} 9 & -1 & 2 \\ -3 & 4 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 1 & 3 \\ 1 & 0 & 1 \\ 3 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 27+-1+6 & 9+0+2 & 27+-1+6 \\ -9+4+3 & -3+0+1 & -9+4+3 \end{bmatrix}$$

$$= \begin{bmatrix} 32 & 11 & 32 \\ -2 & -2 & -2 \end{bmatrix}$$

6. $AC$ $(2\times 2)\cdot(3\times 3)$

different #s

We cannot multiply them. A has 2 columns and C has 3 rows.

7. $AD$ $(2\times 2)\cdot(2\times 4) \rightarrow (2\times 4)$

$$\begin{bmatrix} \frac{1}{2} & 3 \\ 2 & \frac{2}{3} \end{bmatrix} \times \begin{bmatrix} 2 & 0 & -2 & \frac{1}{2} \\ 3 & 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1+9 & 0+6 & -1+3 & \frac{1}{4}+0 \\ 4+2 & 0+\frac{4}{3} & -4+\frac{2}{3} & 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 6 & 2 & \frac{1}{4} \\ 6 & \frac{4}{3} & -\frac{10}{3} & 1 \end{bmatrix}$$

8. $A^2$ $(2\times 2)\cdot(2\times 2) \rightarrow (2\times 2)$

$$\begin{bmatrix} \frac{1}{2} & 3 \\ 2 & \frac{2}{3} \end{bmatrix} \times \begin{bmatrix} \frac{1}{2} & 3 \\ 2 & \frac{2}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{4}+6 & \frac{3}{2}+2 \\ 1+\frac{4}{3} & 6+\frac{4}{9} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{25}{4} & \frac{7}{2} \\ \frac{7}{3} & \frac{58}{9} \end{bmatrix}$$

9. $C^2$ $(3\times 3)\cdot(3\times 3) \rightarrow (3\times 3)$

$$\begin{bmatrix} 3 & 1 & 3 \\ 1 & 0 & 1 \\ 3 & 1 & 3 \end{bmatrix} \times \begin{bmatrix} 3 & 1 & 3 \\ 1 & 0 & 1 \\ 3 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 9+1+9 & 3+0+3 & 9+1+9 \\ 3+0+3 & 1+0+1 & 3+0+3 \\ 9+1+9 & 3+0+3 & 9+1+9 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 6 & 19 \\ 6 & 2 & 6 \\ 19 & 6 & 19 \end{bmatrix}$$

10. $2A + B$

2A and B have different dimensions, so they cannot be added.

11. $B + BC$ $BC$ $(2\times 3)\cdot(3\times 3)$ $(2\times 3)$

$$BC = \begin{bmatrix} 9 & -1 & 2 \\ -3 & 4 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 1 & 3 \\ 1 & 0 & 1 \\ 3 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 32 & 11 & 32 \\ -2 & -2 & -2 \end{bmatrix}$$

$$B+BC \begin{bmatrix} 9 & -1 & 2 \\ -3 & 4 & 1 \end{bmatrix} + \begin{bmatrix} 32 & 11 & 32 \\ -2 & -2 & -2 \end{bmatrix} =$$

$$\begin{bmatrix} 41 & 10 & 34 \\ -5 & 2 & -1 \end{bmatrix}$$

12. Let $F$ be an $m \times n$ matrix. Then what do you know about the dimensions of matrix $G$ in the problems below if each expression has a value?

A. $F + G$

G must have the same dimension $(m \times n)$

B. $FG$

G must have n rows but any columns

C. $GF$

G must have m columns but any rows

13. Consider an $m \times n$ matrix $A$ such that $m \neq n$. Explain why you cannot evaluate $A^2$.

Because the column of A does not match the row of A

$(m \times n) \cdot (m \times n)$

Not possible

14. Let $A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 1 \\ 2 & 2 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$ represent the routes of three airlines $A$, $B$, and $C$ between three cities.

A. Zane wants to fly from City 1 to City 3 by taking Airline $A$ first and then Airline $B$ second. How many different ways are there for him to travel?

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$$A \cdot B = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 1 \\ 2 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 1 \\ 2 & 6 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

row 1, column 3

There is only one way.

B. Zane did not like Airline $A$ after the trip to City 3, so on the way home, Zane decides to fly Airline $C$ first and then Airline $B$ second. How many different ways are there for him to travel?

$$C \cdot B = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 2 & 3 \end{bmatrix} \rightarrow C_{1,3} = 2 \text{ ways}$$