NAME: ______________ PERIOD: ______________ DATE: ______________

Homework Problem Set

1. Consider the matrices.

$$A = \begin{bmatrix} 3 & 1 & -\frac{1}{2} \\ 2 & \frac{2}{3} & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

A. Would you consider B to be an identity matrix for A? Why or why not?

No, Identity matrices must be square and the main diagonal is all 1, and all other elements are 0. Also, if you multiplied A and B, you would not get A.

B. Would you consider $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ or $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ an identity matrix for A? Why or why not?

$I_2 A = A$ and $A I_3 = A$ $\longrightarrow$ but neither can commute based on dimensions.

Identity for A on left

Identity for A on right

2. Recall the bus and trolley matrices from the lesson:

$$B = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 2 & 0 & 2 & 2 \\ 2 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A. Explain why it makes sense that $BI = IB$ in the context of the problem.

Since there is only one way to take a trolley in each city, the result is the same no matter if you take it before or after the bus.

B. Multiply out BI and show $BI = IB$.

$$BI = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 2 & 0 & 2 & 2 \\ 2 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix} \qquad IB = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 2 & 0 & 2 & 2 \\ 2 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$$

C. Consider the multiplication that you did in Part B. What about the arrangement of the entries in the identity matrix causes $BI = B$?

When you multiply BI, you are multiplying the rows of B by the columns of I. Since every entry in the columns of I but the row you are currently multiplying is by zero, you only get a single value of B to carryover, and it is carried over in the same position.

3. Let $A = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 2 & 1 \\ 2 & 1 & 0 & 2 \\ 1 & 2 & 1 & 0 \end{bmatrix}$ represent airline flights of one airline between 4 cities.

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A. We use the notation A^2 to represent the product $A \cdot A$. Calculate A^2. What do the entries in matrix A^2 represent?

$$\begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 2 & 1 \\ 2 & 1 & 0 & 2 \\ 1 & 2 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 2 & 1 \\ 2 & 1 & 0 & 2 \\ 1 & 2 & 1 & 0 \end{bmatrix} = \boxed{A^2 = \begin{bmatrix} 5 & 5 & 4 & 3 \\ 5 & 5 & 2 & 6 \\ 3 & 6 & 6 & 5 \\ 4 & 2 & 5 & 6 \end{bmatrix}}$$

rows $= i$ columns $= j$

Each entry of A^2 is the # of ways to get from city i to city j w/ one stop in between.

B. Jade wants to fly from City 1 to City 4 with exactly one stop. How many different ways are there for her to travel?

$(A)^2_{1,4} =$ $\boxed{\text{3 different ways to travel}}$

C. Now Jade wants to fly from City 1 to City 4 with exactly two stops. How many different ways are there for her to choose?

$A^2 \cdot A = A^3$

$$\begin{bmatrix} 5 & 5 & 4 & 3 \\ 5 & 5 & 2 & 6 \\ 3 & 6 & 6 & 5 \\ 4 & 2 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 2 & 1 \\ 2 & 1 & 0 & 2 \\ 1 & 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 16 & 15 & 18 & 23 \\ 15 & 19 & 21 & 19 \\ 23 & 19 & 20 & 24 \\ 18 & 21 & 14 & 20 \end{bmatrix}$$

$(A^3)_{1,4} =$ $\boxed{\text{23 different ways}}$