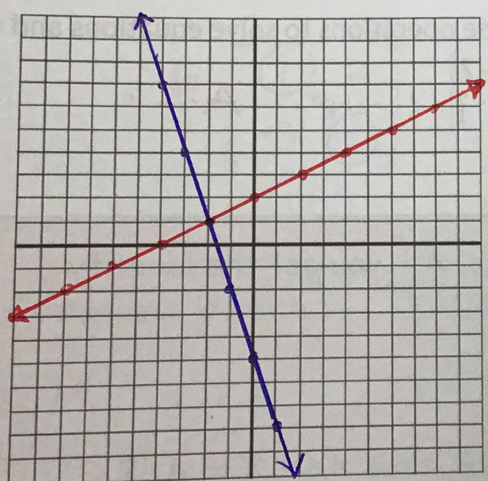
# LESSON 6 Using Matrices to Solve Systems of Equations

## Opening Exercise

In the last unit, you learned to solve systems of equations by graphing, and by the substitution and elimination methods. Systems of equations can also be solved using matrices. Before we use matrices, let's use a simple example to review solving systems of equations.

1. Solve the system $\begin{matrix} x - 2y = -4 \\ 3x + y = -5 \end{matrix}$ by graphing and then use either the substitution method or the elimination method to verify your results.

| Graphing Method | Substitution or Elimination Method |
|---|---|
| $x - 2y = -4 \rightarrow -2y = -x - 4$<br>$3x + y = -5$ $\quad y = \frac{x}{2} + 2$<br>$y = -3x - 5$<br> | $x - 2y = -4$ $\quad x - 2y = -4$<br>$2(3x + y = -5)$ $\quad \underline{6x + 2y = -10}$<br>$7x = -14$<br>$x = -2$<br>Back sub<br>$x - 2y = -4$<br>$-2 - 2y = -4$<br>$-2y = -2$<br>$y = 1$<br>$(-2, 1)$ |

The solution to $\begin{matrix} x - 2y = -4 \\ 3x + y = -5 \end{matrix}$ is $x =$ <u>$-2$</u> and $y =$ <u>$1$</u>.

To solve the system, $\begin{aligned} x - 2y &= -4 \\ 3x + y &= -5 \end{aligned}$, with matrices, we have to see this system as three matrices as shown below.

$$\underbrace{\begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}}_{\substack{\text{coefficients} \\ A}} \cdot \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\substack{\text{variable} \\ x}} = \underbrace{\begin{bmatrix} -4 \\ -5 \end{bmatrix}}_{\substack{\text{constants} \\ b}}$$

2. What does each matrix represent?

$$[\text{coeff}]\,[\text{variables}] = [\text{constants}]$$

$$A \cdot x = b$$

3. Determine $\begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$. How does it relate to the original equation?

$$\begin{bmatrix} x - 2y \\ 3x + y \end{bmatrix}$$

$$A \cdot x = b$$

$$\underbrace{A^{-1} A}\cdot x = A^{-1} b$$

$$I \cdot x = A^{-1} b$$

$$\boxed{x = A^{-1} b}$$

To solve for the matrix $\begin{bmatrix} x \\ y \end{bmatrix}$, we need to use some type of operation to "move" the matrix $\begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$ to the other side of the equation. In algebra we use inverse operations to solve equations and we'll do the same here. We need the inverse of the matrix $\begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$.

$A^{-1} \rightarrow$ inverse

When you multiply a matrix, $A$, by its inverse, $A^{-1}$, you get the identify matrix, $I$.

$$AA^{-1} = A^{-1}A = I$$

Suppose A, X and B are matrices and AX = B, then $AA^{-1}X = A^{-1}B$.

This gives $IX = A^{-1}B$ or $X = A^{-1}B$

To determine the inverse, $A^{-1}$, you'll need a special number called the determinant, and it is written as $|A|$ or det $A$.

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then the determinant of $A$ or det $A$ is given by the formula

$$|A| = ad - bc.$$

4. Find the determinant of the matrix $\begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$.

determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$a \cdot d - c \cdot b$

$$|A| = (1)(1) - (-2)(3)$$
$$= 1 + 6$$
$$= 7$$

To find the inverse you'll use the reciprocal of the inverse and then manipulate the matrix. The steps are given below.

negate

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $|A| = ad - bc$, then $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

negate

5. If the determinant of the matrix is 0, then no inverse exists. Why is that so?

$|A| = 0 \rightarrow A^{-1}$ does not exist.

6. What is the inverse of matrix $\begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$? (We'll call this Matrix A from here on.)

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 1 & 2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 1/7 & 2/7 \\ -3/7 & 1/7 \end{bmatrix}$$

$A^{-1} \cdot A = I$?

$$\begin{bmatrix} 1/7 & 2/7 \\ -3/7 & 1/7 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1/7 & 2/7 \end{bmatrix}\begin{bmatrix} \frac{1}{7}+\frac{6}{7} & -\frac{2}{7}+\frac{2}{7} \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -3/7 & 1/7 \end{bmatrix} \begin{bmatrix} -\frac{3}{7}+\frac{3}{7} & \frac{6}{7}+\frac{1}{7} \end{bmatrix} = \begin{bmatrix} 0 & 1 \end{bmatrix}$$



7. Solve the system using $A^{-1}$.

$$\begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ -5 \end{bmatrix}$$

Write $A^{-1}$ here.

And here.

2x2 2x1 → 2x1

$$\underbrace{\begin{bmatrix} 1/7 & 2/7 \\ -3/7 & 1/7 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}}_{} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1/7 & 2/7 \\ -3/7 & 1/7 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$x = -2$
$y = 1$

$$\begin{bmatrix} -4 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} 1/7 & 2/7 \\ -3/7 & 1/7 \end{bmatrix} \begin{bmatrix} -\frac{4}{7} - \frac{10}{7} \\ \frac{12}{7} - \frac{5}{7} \end{bmatrix}$$

$$\begin{bmatrix} -14/7 \\ 7/7 \end{bmatrix}$$

8. Does your answer agree with your work in Exercise 1?

Yes.

9. Which method, graphing, substitution, elimination or matrices do you like best? Why?

We've been looking at transportation problems throughout this unit, but most transportation problems are so complex that you need computers or graphing calculators to solve them. Most require at least 4 × 4 matrices. We'll look at some typical 2-variable system problems which will give us 2 × 2 matrices.

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10. Admission to the county fair is $2 for children and $5 for adults. Last Sunday, 1900 people came to the fair and they collected $5900 at the entrance booth.

A. If C represents the number of children who attended and A represents the number of adults, what are the two equations to represent this situation?

$$C + A = 1900$$

$$2C + 5A = 5900$$

B. Write the matrix equation that represents this system.

$$\begin{bmatrix} 1 & 1 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} C \\ A \end{bmatrix} = \begin{bmatrix} 1900 \\ 5900 \end{bmatrix}$$

C. Find the determinant and the inverse, and then solve the system with matrices.

$$|A| = 5 - 2 = 3$$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 5 & -1 \\ -2 & 1 \end{bmatrix}$$

=

$$\begin{bmatrix} C \\ A \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 5 & -1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1900 \\ 5900 \end{bmatrix}$$

$$\begin{bmatrix} C \\ A \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9500 + (-5900) \\ -3800 + 5900 \end{bmatrix}$$

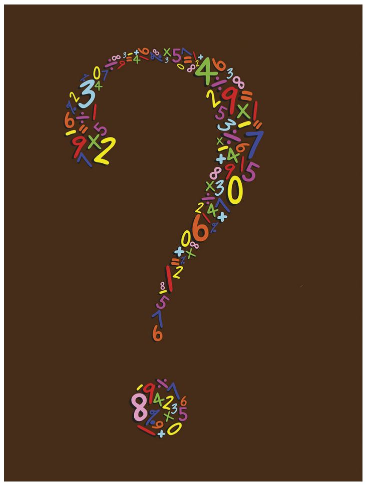
$$\begin{bmatrix} C \\ A \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 3600 \\ 2100 \end{bmatrix}$$

D. Number of children: 1200 Number of adults: 700

$$\begin{bmatrix} C \\ A \end{bmatrix} = \begin{bmatrix} 1200 \\ 700 \end{bmatrix}$$

11. The sum of two numbers is 1 and their difference is 15.

    A. Write a system of equations for the two numbers. Be sure to define the variables you are using.





    B. Solve the system using matrices.

# Lesson Summary

You can use matrices to solve systems of equations. Complete the example below by following the steps given.

| Steps Explained | Example: Solve $\begin{aligned} 2x - \phantom{2}y &= -8 \\ -x + 2y &= \phantom{-}7 \end{aligned}$ using matrices |
|---|---|
| 1. Write the matrix equation. | |
| 2. Find the determinant. | |
| 3. Determine the inverse matrix. | |
| 4. Use the inverse to isolate the variable matrix. | |
| 5. Multiply to solve for the variables. | |
| 6. Check your answer. | |

NAME: ____________ PERIOD: ____________ DATE: ____________

# Homework Problem Set

1. Solve the system, $\begin{aligned} x - 2y &= 1 \\ x + 4y &= 8 \end{aligned}$ using matrices.

**In Exercise 5, you explained why matrices with a determinant of 0 has no inverse. Find the determinant of each matrix and then decide which have no inverse.**

2. $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

3. $\begin{bmatrix} 2 & -1 \\ 4 & 0 \end{bmatrix}$

4. $\begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix}$

5. $\begin{bmatrix} 5 & 2 \\ -10 & 4 \end{bmatrix}$

6. Julie went to the Taco Truck and bought 5 tacos and 2 burritos for $12.50. Kent bought 3 tacos and 4 burritos for $14.50. Use matrices to determine how much each taco costs.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