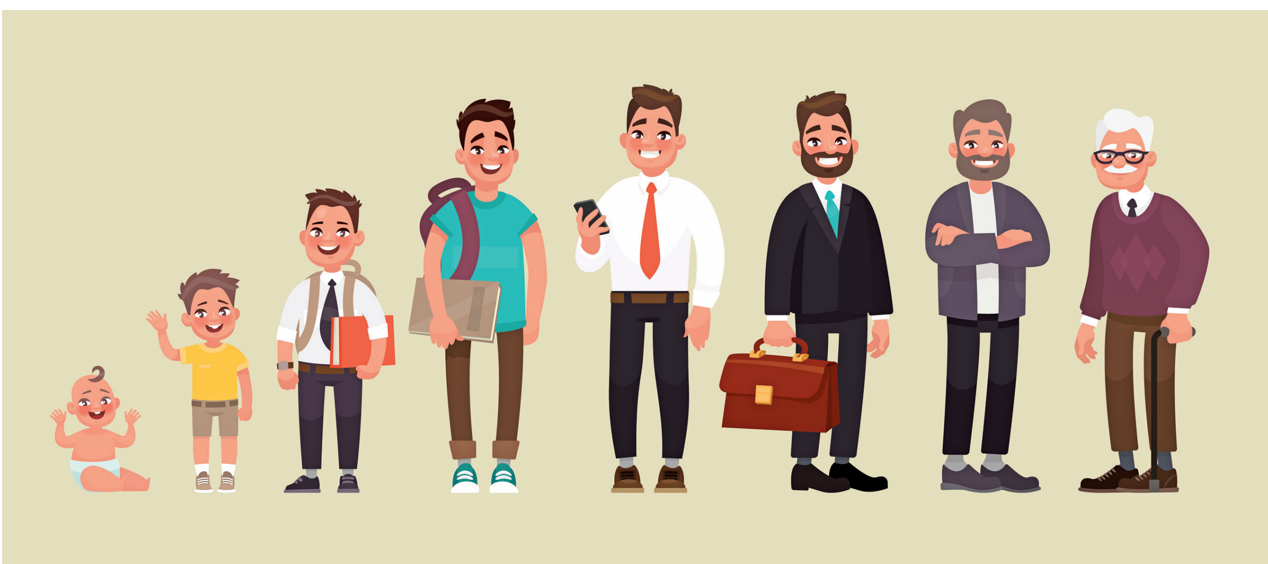
LESSON 11

Arithmetic versus Geometric Sequences

LEARNING OBJECTIVES

- Today I am: playing a matching game.
- So that I can: identify when a sequence is arithmatic or geometric.
- I'll know I have it when I can: tell if a word problem is best modeled with an arithmetic or a geometric sequence.



Do you think the age of this person is shown as a arithmetic sequence or a geometric sequence?

$\sqrt{500} = 10\sqrt{5}$

$10 \cdot 10 \cdot 5$

$\sqrt{128} = 8\sqrt{2}$

$64 \cdot 2$

$8 \cdot 8 \cdot 2$

$\sqrt{50x^2y^4} = \boxed{5|x|y^2\sqrt{2}}$

$5 \cdot 5\,\boxed{2} \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$

$\sqrt{162m^3n^5}$

$9 \cdot 9\,\boxed{2}\ m \cdot m \cdot m\ n\,n\,n\,n\,n$

$9mn^2\sqrt{2mn}$

$\sqrt[3]{24x^2y^7z^5}$

$2 \cdot 2 \cdot 2\,\boxed{3}$

$2y^2z\sqrt[3]{3x^2yz^2}$

Arith: $A(n) = A(1) + d(n-1)$

Geo: $A(n) = A(1) \cdot (r)^{n-1}$

Opening Activity

You will need: *Sequences Mix Up* cards, scissors, glue or tape

1. With your partner, sort the sequence, rule, recursive formula and explicit formula cards. Each sequence will be matched with one of each of the other cards. Then glue or tape the cards into the chart below.

(+ or −) $n = 1, 2, 3, 4$

Sequence	Arithmetic or Geometric (×)	Rule (d, r)	Recursive Formula	Explicit Formula (General)
n = 1 2 3 4 $-2, 2, 6, 10, \ldots$ $A(1) = -2$ $A(3) = 6$ $A(2) = 2$ $A(4) = 10$	Arithmetic	$+4$	$A(n+1) = A(n) + 4$ $A(1) = -2$, $n \geq 1$	$A(n) = -2 + 4(n-$ $= 4n - 6$ $n \geq 1$
$2, 4, 8, 16, \ldots$	Geometric	$\times 2$	$A(n+1) = 2 \cdot A(n)$ $A(1) = 2$, $n \geq 1$	$A(n) = 2 \cdot (2)^{n-1}$ $n \geq 1$
$\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$	Arithmetic	$+\frac{1}{2}$	$A(n+1) = A(n) + \frac{1}{2}$ $A(1) = \frac{1}{2}$, $n \geq 1$	$A(n) = \frac{1}{2} + \frac{1}{2}(n-1)$ $= \frac{1}{2}n$ $n \geq 1$
$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$	Geometric	$\times \frac{1}{3}$	$A(n+1) = \frac{1}{3}A(n)$ $A(1) = 1$, $n \geq 1$	$A(n) = 1 \cdot (\frac{1}{3})^{n-1}$ $n \geq 1$
$10, 1, 0.1, 0.01, 0.001, \ldots$	Geometric	$\times \frac{1}{10}$	$A(n+1) = \frac{1}{10} A(n)$ $A(1) = 10$, $n \geq 1$	$A(n) = 10(\frac{1}{10})^{n-1}$ $n \geq 1$
$4, -1, -6, -11, \ldots$	Arithmetic	-5	$A(n+1) = A(n) - 5$ $A(1) = 4$, $n \geq 1$	$A(n) = 4 - 5(n-1)$ $= 4 - 5n + 5$ $= -5n + 9$ $n \geq 1$

Discussion

2. How can you tell a sequence is geometric? What do you look for in the sequence?

3. What is the best way to identify a recursive rule from an arithmetic one?

4. Was there anything you needed to do with the sequences to be able to easily match the formulas to the sequences?

Practice

For Exercises 5–7, are the sequences arithmetic, geometric, or neither?

5. $4, \frac{5}{2}, 1, -\frac{1}{2}, \ldots\ldots$

$d = \frac{5}{2} - 4 = -\frac{3}{2}$

Explicit

$F(n) = F(1) + d(n-1)$

$F(n) = 4 - \frac{3}{2}(n-1)$

$= 4 - \frac{3}{2}n + \frac{3}{2}$

$\boxed{F(n) = -\frac{3}{2}n + \frac{11}{2}}$, $n \geq 1$

Recursive

$F(n+1) = F(n) - \frac{3}{2}$

$F(1) = 4$

$n \geq 1$

6. 8, 12, 18, 27, . . .

$r = \frac{12}{8} = \boxed{\frac{3}{2}}$

Explicit

$F(n) = F(1) \cdot r^{n-1}$

$F(n) = 8\left(\frac{3}{2}\right)^{n-1}$

$n \geq 1$

Recursive: $\boxed{F(n+1) = \frac{3}{2} \cdot F(n) \quad F(1) = 8, \ n \geq 1}$

7. 2, 5, 10, 17, 26, . . .

+3 +5 +7 +9

neither

8. Starting May 1, a new store will begin giving away 500 posters as a promotion. Each day, 4 posters will be given away. If the store is open 7 days a week, how many posters will the store have left when it opens for business on May 14?

$n = 14$

500, 496, 492, ...

$d = -4$

$F(1) = 500$

$F(n) = F(1) + d(n-1)$

$F(n) = 500 - 4(n-1)$

$= 500 - 4n + 4$

$F(n) = -4n + 504$

$F(14) = -4(14) + 504$

$= \boxed{448}$

9. A radioactive substance decreases in the amount of grams by one third each year. If the starting amount of the substance in a rock is 1,701 grams, how many grams will be left after 4 years?

$n = 4$

decrease by $\frac{1}{3}$

$r = \boxed{\frac{2}{3}}$ left

$F(1) = 1701$

$F(n) = 1701\left(\frac{2}{3}\right)^{n-1}$

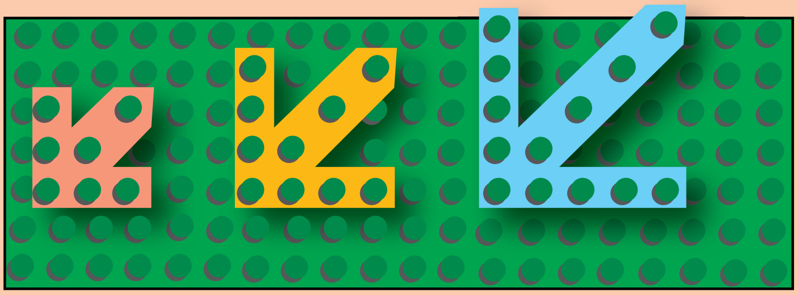
$F(4) = 1701\left(\frac{2}{3}\right)^{4-1}$

$= 1701\left(\frac{2}{3}\right)^{3} \rightarrow 1701 \cdot (2/3)^\wedge 3$

Lesson Summary

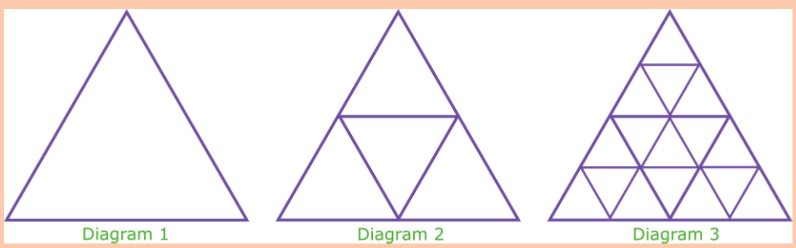
Sequences can be formed from many different patterns. We have looked at arithmetic and geometric sequences, sequences made from geometric shapes, and sequence patterns in nature.

Arithmetic sequences are formed by adding or subtracting a constant amount.



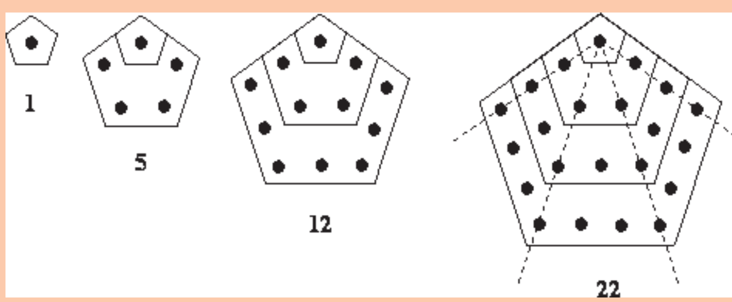
Source: https://mathybeagle.com/tag/arithmetic-progression/

Geometric sequences are formed by multiplying or dividing by a constant amount.



Source: http://www.transum.org/Maths/Exam/Online_Exercise.asp?Topic=Sequences

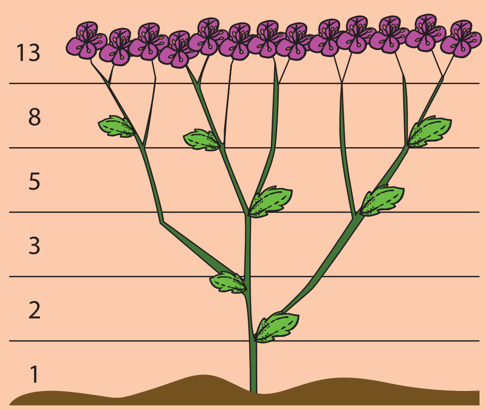
Geometric shape sequences are formed by looking at the number of dots used to create different polygons.



Pentagonal Numbers

Source: http://www.mathpages.com/home/kmath623/kmath623.htm

Patterns in nature are found by counting the number of petals, spirals, branches or leaves in plants.



Source: http://jwilson.coe.uga.edu/emat6680/parveen/fib_nature.htm

NAME: ______________________ PERIOD: __________ DATE: __________

Homework Problem Set

For Problems 1–4, list the first five terms of each sequence, and identify them as arithmetic or geometric.

1. $A(n) = A(n-1) + 4$ for $n \geq 1$ and $A(1) = -2$

2. $A(n) = \frac{1}{4} \cdot A(n-1)$ for $n \geq 1$ and $A(1) = 8$

3. $A(n) = A(n-1) - 19$ for $n \geq 1$ and $A(1) = -6$

4. $A(n) = \frac{2}{3} A(n-1)$ for $n \geq 1$ and $A(1) = 6$

For Problems 5–8, identify the sequence as arithmetic or geometric, and write a recursive formula for the sequence. Be sure to identify your starting value.

5. 14, 21, 28, 35, . . .

6. 4, 40, 400, 4000, . . .

7. 49, 7, 1, $\frac{1}{7}$, $\frac{1}{49}$, . . .

8. -101, -91, -81, -71, . . .

9. The local football team won the championship several years ago, and since then, ticket prices have been increasing \$20 per year. The year they won the championship, tickets were \$50. Write a recursive formula for a sequence that models ticket prices. Is the sequence arithmetic or geometric?

10. A radioactive substance decreases in the amount of grams by one-third each year. If the starting amount of the substance in a rock is 1,452 g, write a recursive formula for a sequence that models the amount of the substance left after the end of each year. Is the sequence arithmetic or geometric?

11. After knee surgery, your trainer tells you to return to your jogging program slowly. He suggests jogging for 12 minutes each day for the first week. Each week thereafter, he suggests that you increase that time by 6 minutes per day. How many weeks will it be before you are up to jogging 60 minutes per day?

12. Brian gets a starting wage of $15 and an annual raise of $1.50 per hour. What will Brian's hourly wage be during his tenth year?

to diminish in value each year

13. A car depreciates 30% every year. Find the value of a 5 year old car if the original price was \$30,000.

14. Scott is saving to buy a guitar. In the first week, he put aside \$42 that he received for his birthday, and in each of the following weeks, he added \$8 to his savings. He needs \$400 for the guitar that he wants. In which week will he have enough money for the guitar?

15. A virus reproduces by dividing into two, and after a certain growth period, it divides in two again. How many viruses will be in a system starting with a single virus after 10 divisions?