LESSON 13

Linear and Exponential Investigations

LEARNING OBJECTIVES

- Today I am: doing a variety of investigations.
- So that I can: explore linear and exponential growth and decay.
- I'll know I have it when I can: summarize my findings.

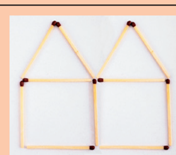
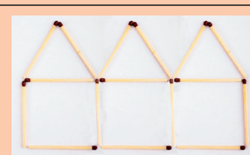
Opening Exercise

In this lesson, you'll be exploring linear and exponential function in five different investigations.

Investigation I: Matchstick Houses

Source: adapted from http://www.transum.org/Maths/Activity/Matchstick_Patterns/

A. Determine the number of matchsticks in each "house" and record your information in the table below. Then sketch a picture of the fourth term "house".

Picture of Matchstick "House"				
Term Number	1	2	3	4
Number of Matchsticks	6			

B. Is this linear or exponential? Decay or growth?

C. Write a formula to determine the number of matchsticks needed for term *t*.

Investigation 2: Eliminating Sixes

You will need: access to a computer

Directions

- Go to the website https://www.random.org/dice/.
- Use the pull down menu to select 60 dice to roll and then select "Roll Dice."
- Count the number of dice that show six dots and record it in the table on the next page. Then subtract the number of sixes to get the new number of dice to roll.
- Use the "Go Back" button to select a new number of dice to roll.
- Continue to count the number of sixes, subtract those from the number of dice and reroll with the new number of dice.
- Continue rolling the dice until you have 2 dice left or you have completed 25 trials.

Graph the number of dice for each roll in the grid provided on the next page, then answer the questions below.

Reflection

A. About what fraction of the original amount of dice are left after the first roll?

B. Is this linear or exponential? Decay or growth?

C. Write a formula to model the number of dice left after r rolls.

D. What is the domain for this situation? What is the range?

Dice Roll (trial number)	Number of Dice to Roll	Number of Sixes (subtract from Number of Dice to Roll for next trial)
0	60	
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
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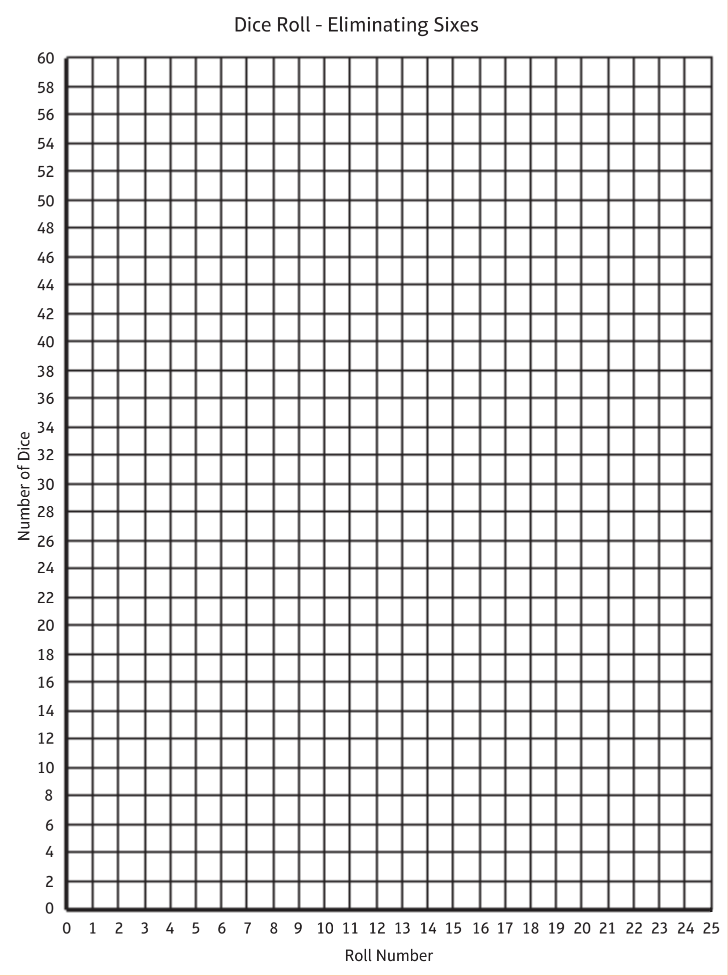
22		
23		
24		
25		



Investigation 3: A Sticky Situation

You will need: one length of masking tape, scissors

Directions

- Cut one piece of masking tape to match the outline shown on the next page. Place it over the outlined tape (Tape 0).
- Cut another piece of masking tape that is three times as long as the first piece. Place it next to the first piece of tape.
- Cut another piece of tape that is three times as long as the second piece of tape. Place it next to the second piece of tape.
- Continue doing this until you run out of tape or the tape strip will no longer fit on your paper.

Organizing Your Work

A. Write the length you expected to have at each step.

Tape Number	0	1	2	3	4	5	6
Expected Length of Each Piece of Tape (centimeters)	0.5						
Actual Length of Each Piece of Tape (centimeters)							

You will need: a ruler

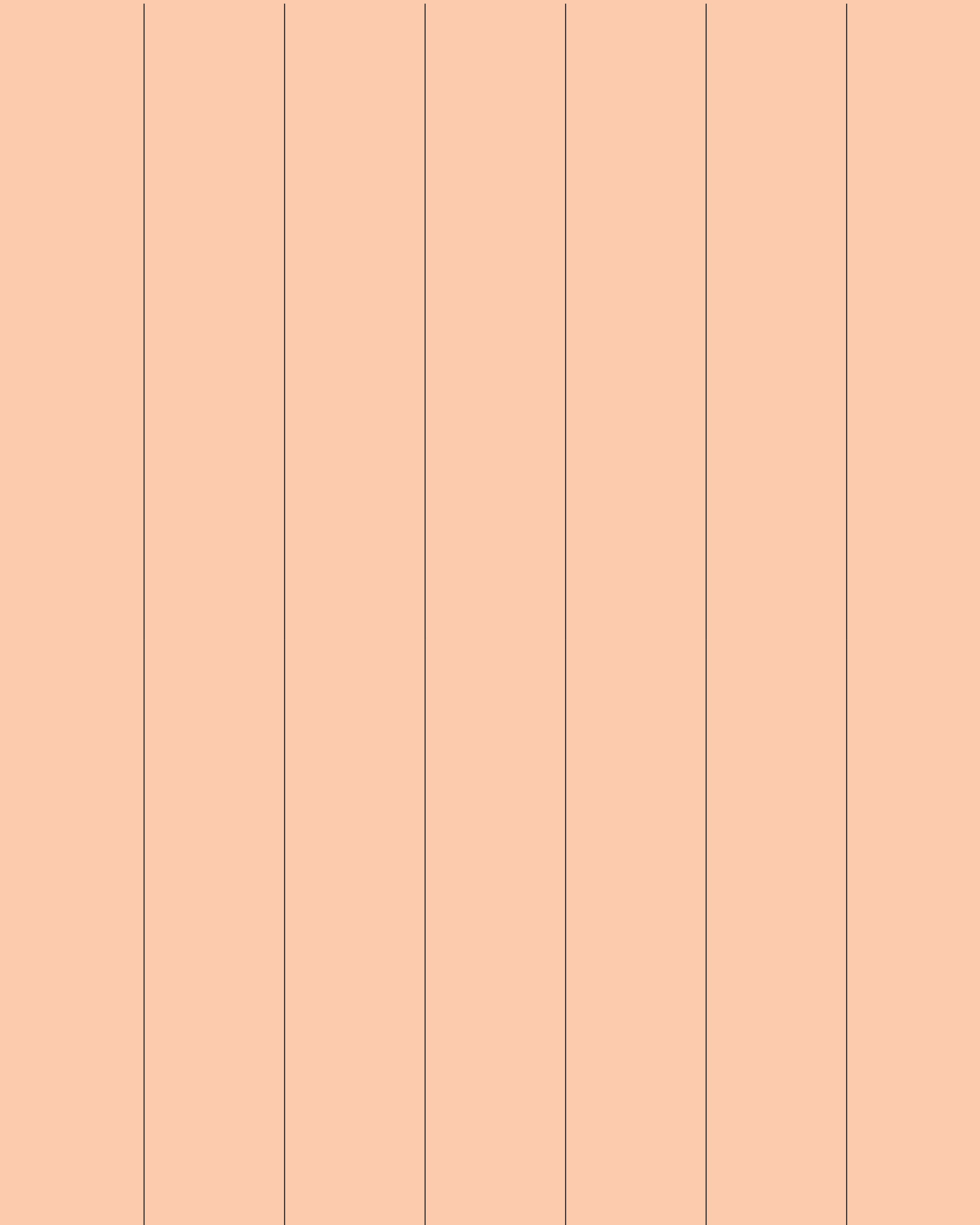
Measuring

B. Measure each piece of tape to the nearest tenth of a centimeter and record those lengths in the table. Why might there be differences between the expected and actual measurements?

C. Is this linear or exponential? Decay or growth?

D. Write a formula to determine the expected length of tape at each tape number.

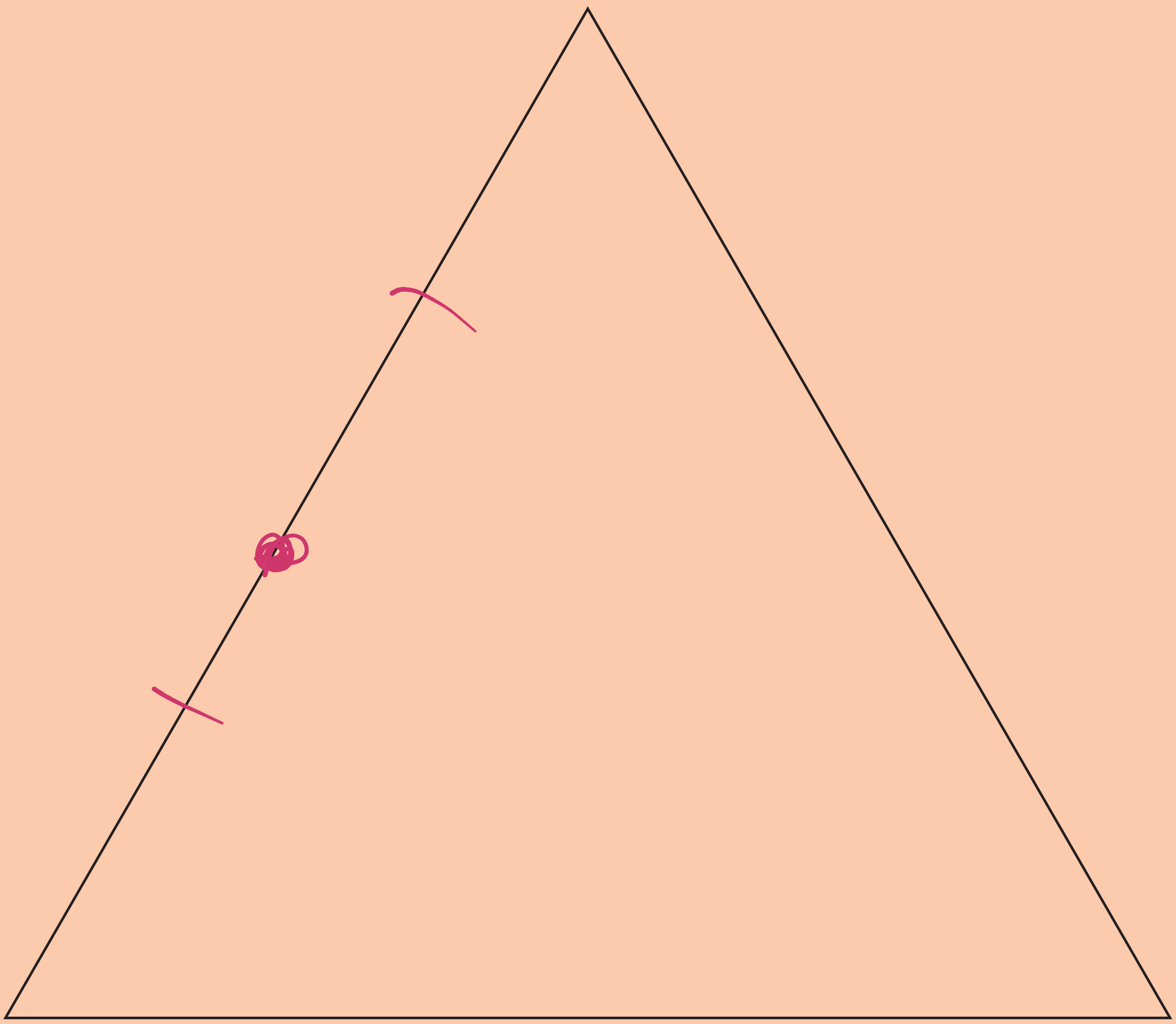
E. What is the domain for this situation? What is the range?


Tape 0
Tape 1
Tape 2
Tape 3
Tape 4
Tape 5
Tape 6

Investigation 4: Bisecting a Triangle

You will need: a ruler

A. Measure the length of each side of the triangle below in centimeters. Round to the nearest centimeter. Record all data in the table.



B. Mark on each side of the triangle the exact middle of the side. This point is the *midpoint* and you are *bisecting* each side.

C. Connect two of the midpoints. Now connect another two midpoints and then the last set of midpoints. Every midpoint should be connected to the other two midpoints.

D. Repeat Steps A, B and C.

E. Repeat Steps A, B and C.

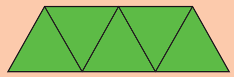
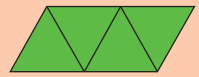
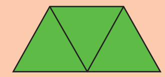
Term Number	1	2	3	4
Length of each side of the triangle	________ cm.			

F. Write a formula to determine the length of the triangles' sides for term *t*.

G. This was an example of exponential decay. How could this activity be changed to show exponential growth?

Investigation 5: Triangle Patterns

A. Determine the number of triangles in each term and record your information in the table below. Then sketch a picture of the fourth term triangle drawing.

Triangle Drawing				
Term Number	1	2	3	4
Number of Triangles	5			

B. Is this linear or exponential situation? Decay or growth?

C. Write a formula to determine the number of triangles needed for term *t*.

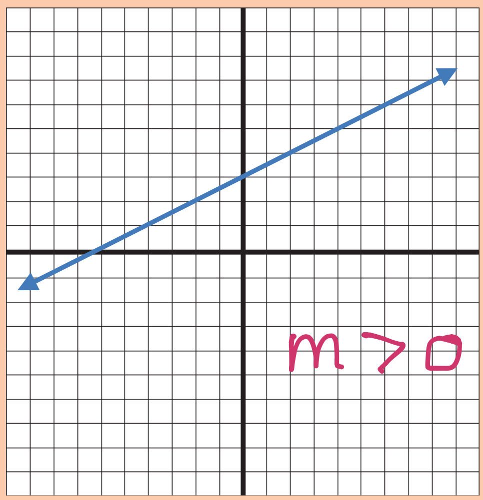
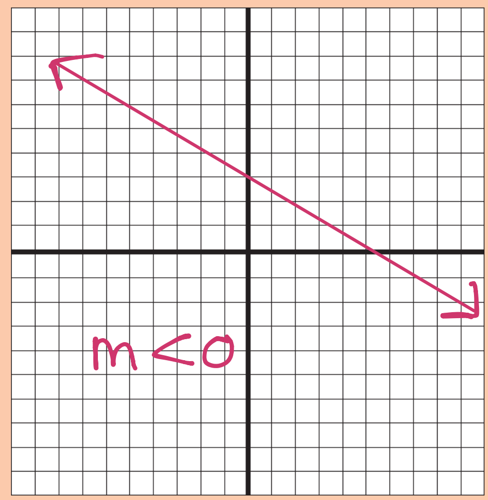
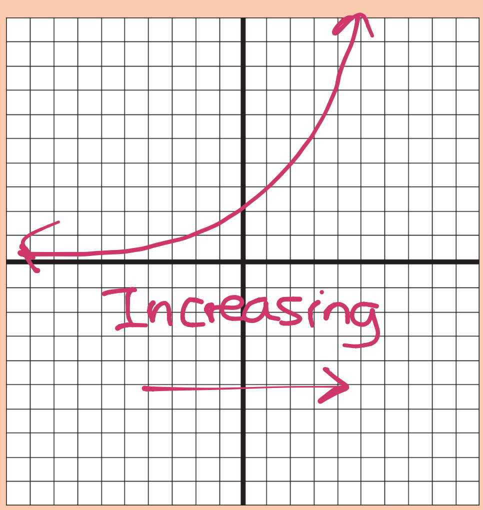
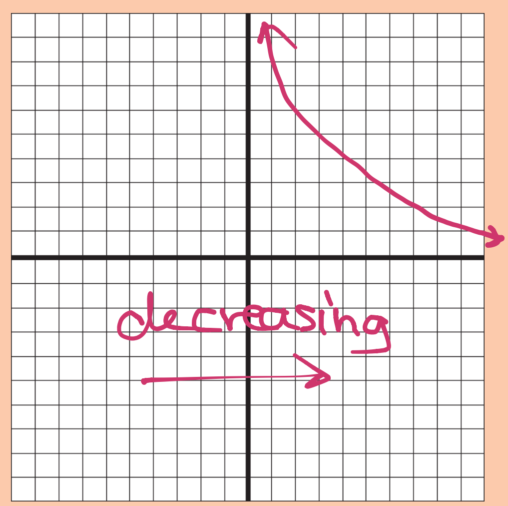
D. What is the domain of this sequence? What is the range?

E. Draw a new sequence of shapes that fits the sequence shown in the last row of the table below.

Drawing				
Term Number	1	2	3	4
Number of __________	3	5	7	9

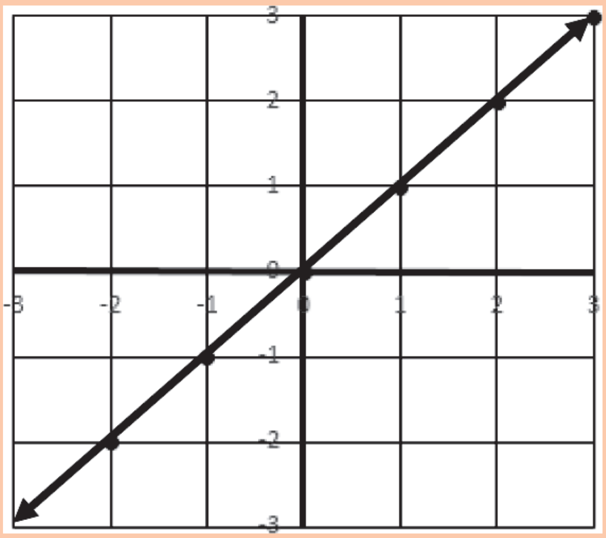
Lesson Summary

For each grid below, sketch a graph that shows the two characteristics. The first one for Linear Growth has been done for you.

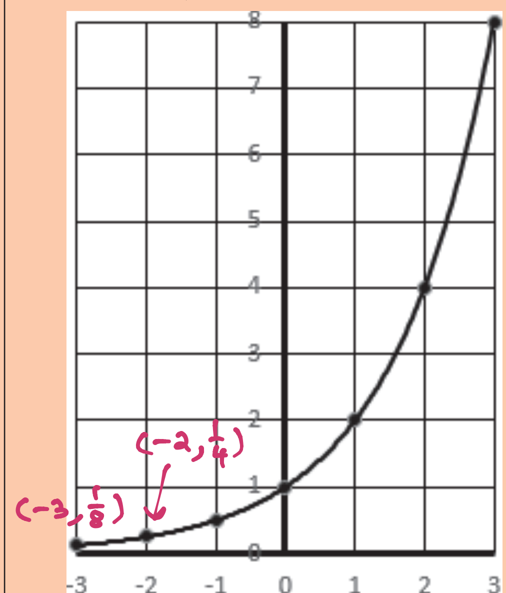
	Growth	Decay
Linear		
Exponential		

Both the linear and exponential functions have a parent graph. A parent graph is the most basic form of the function. The parent graphs and their equation are given below. You'll learn more about parent graphs in Unit 7. Fill in the table of values for each function.

Linear
$y = x$ or $f(x) = x$



Exponential
$y = 2^x$ or $f(x) = 2^x$



x	$y = f(x)$
-3	-3
-2	-2
-1	-1
0	0
1	1
2	2
3	3

x	$y = f(x)$	
-3	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$	$(-3, \frac{1}{8})$
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$	$(-2, \frac{1}{4})$
-1	$2^{-1} = \frac{1}{2}$	
0	$2^0 = 1$	
1	$2^1 = 2$	
2	$2^2 = 4$	
3	$2^3 = 8$	
4	$2^4 = 16$	

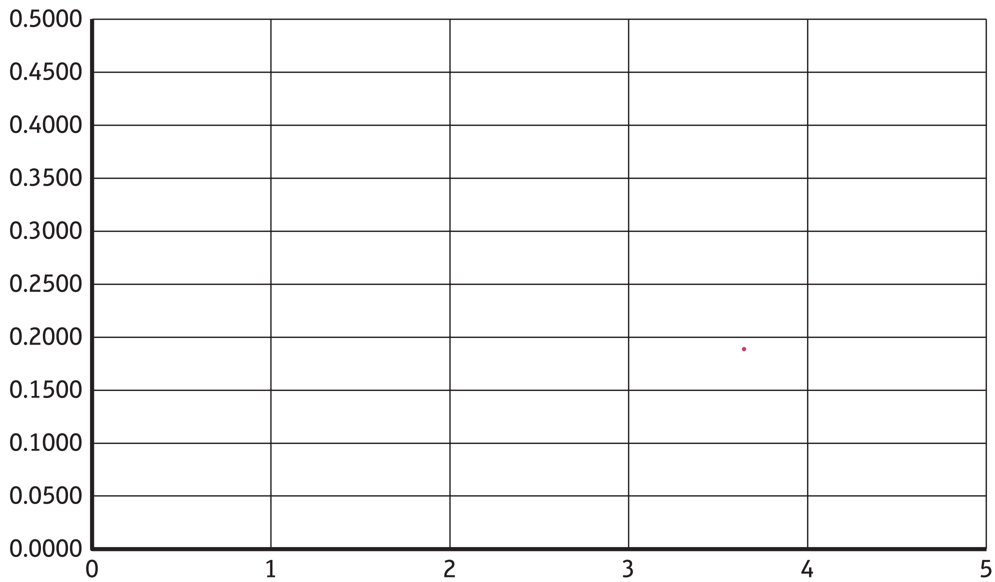
NAME: ______________________ PERIOD: __________ DATE: __________

Homework Problem Set

For each table in Problems 1–6, graph the data then classify the data as describing a linear relationship, an exponential growth relationship, an exponential decay relationship, or neither. If the relationship is linear or exponential, write a formula that models the data.

1.

x	$f(x)$
1	$\frac{1}{2}$
2	$\frac{1}{4}$
3	$\frac{1}{8}$
4	$\frac{1}{16}$
5	$\frac{1}{32}$

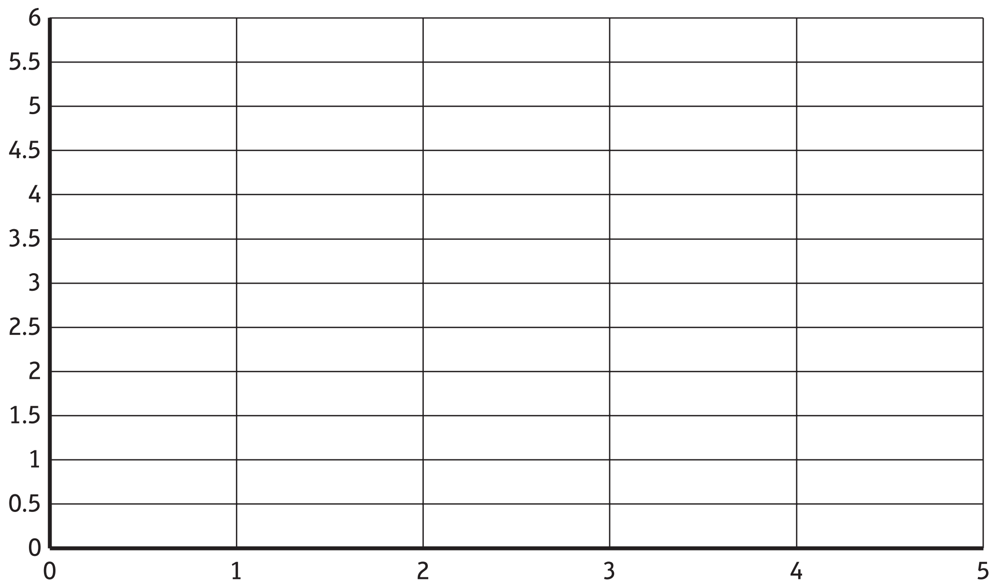


Linear or Exponential or Neither? Growth or Decay?

Equation if linear or exponential: ______________________

2.

x	*f*(*x*)
1	1.4
2	2.5
3	3.6
4	4.7
5	5.8

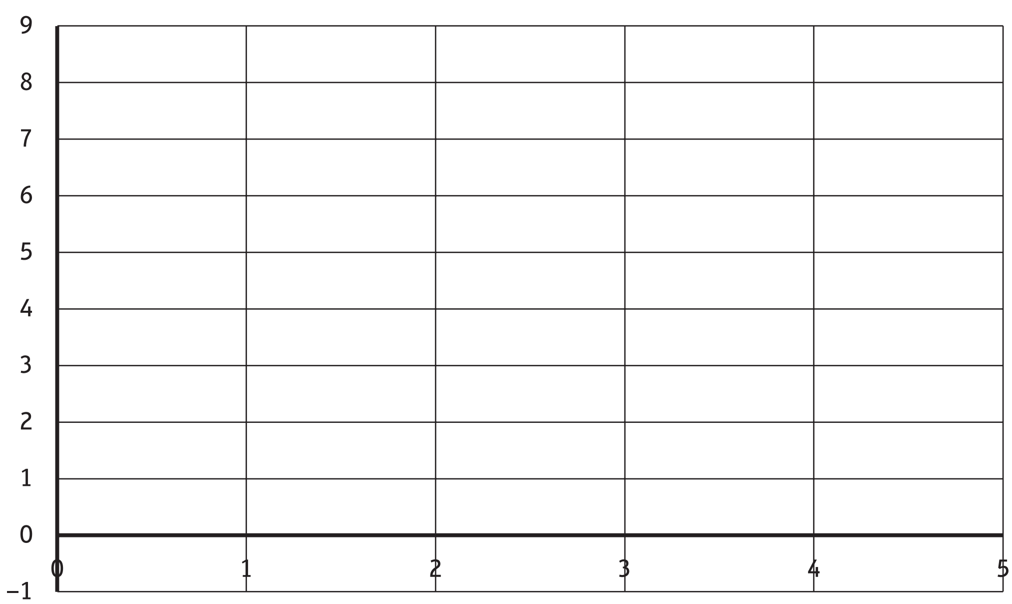


Linear or Exponential or Neither? Growth or Decay?

Equation if linear or exponential: ____________________

3.

x	*f*(*x*)
1	−1
2	0
3	2
4	5
5	9

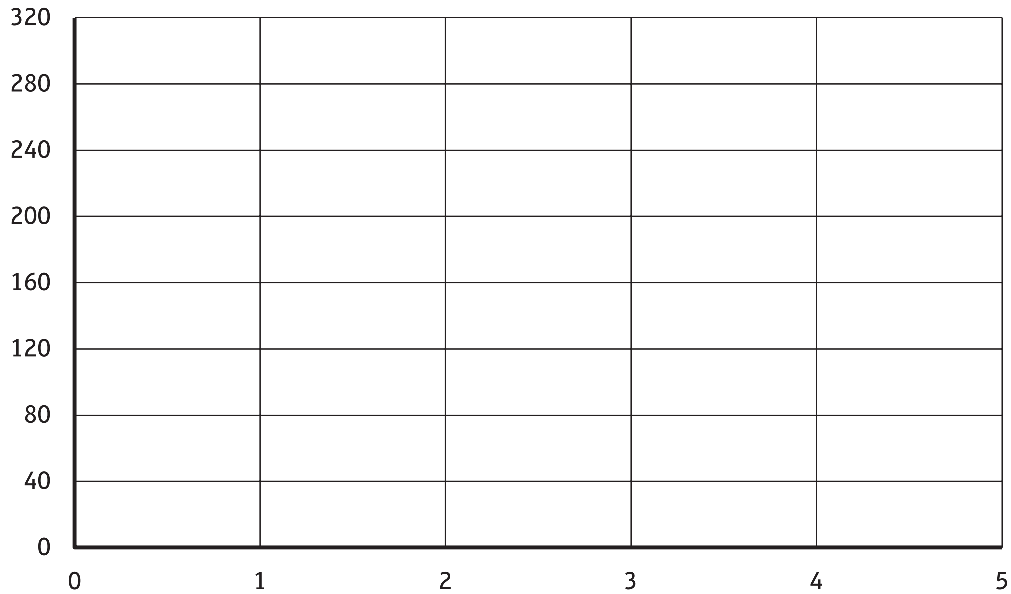


Linear or Exponential or Neither? Growth or Decay?

Equation if linear or exponential: ____________________

4.

x	$f(x)$
1	20
2	40
3	80
4	160
5	320



Linear or Exponential or Neither? Growth or Decay?

Equation if linear or exponential: ____________________

5.

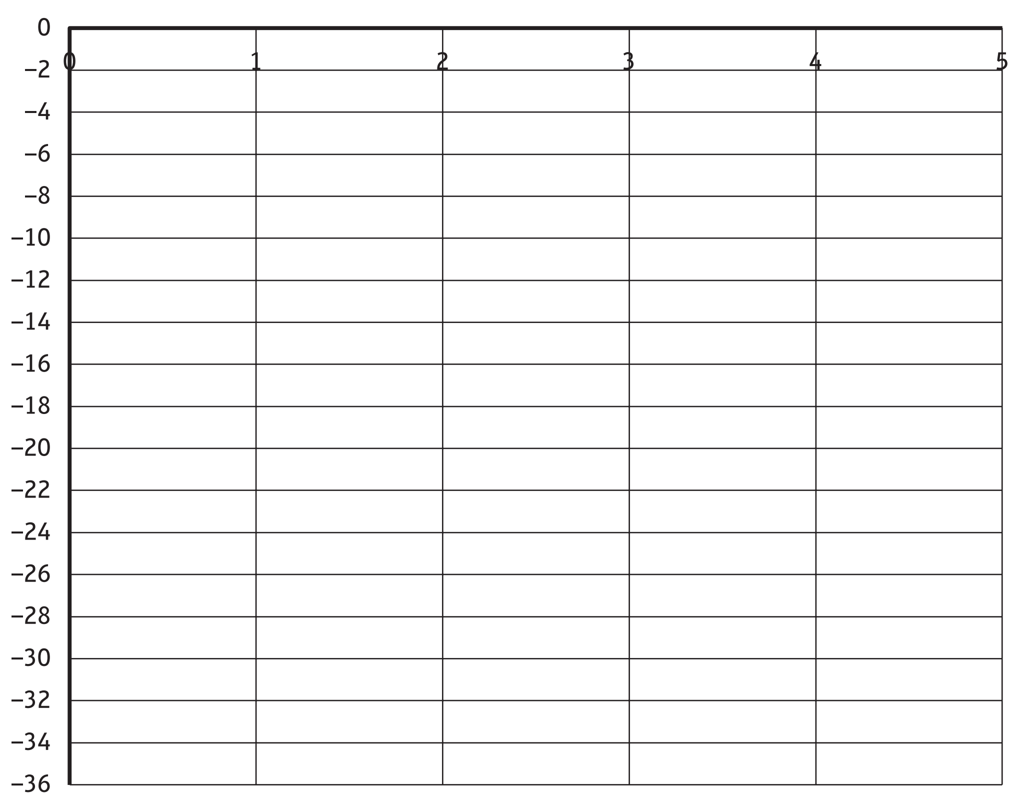
x	$f(x)$
1	−5
2	−12
3	−19
4	−26
5	−33

Linear or Exponential or Neither?

Growth or Decay?

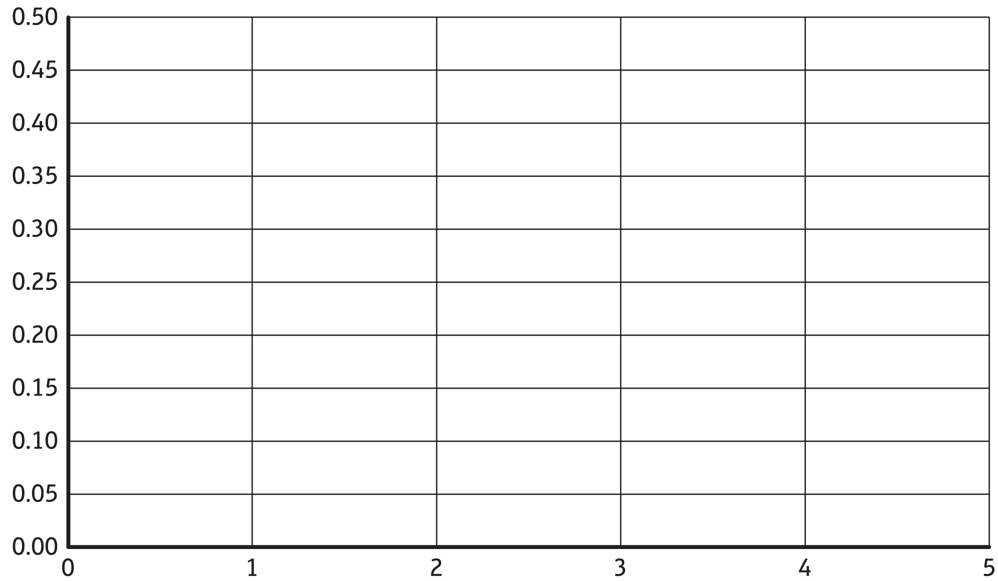
Equation if linear or exponential:

____



6.

x	$f(x)$
1	$\frac{1}{2}$
2	$\frac{1}{3}$
3	$\frac{1}{4}$
4	$\frac{1}{5}$
5	$\frac{1}{6}$



Linear or Exponential or Neither? Growth or Decay?

Equation if linear or exponential: ____________________

Spiral REVIEW—Function Notation and Evaluating Functions

Determine the value of each of the following given, $f(x) = -x + 4$.

7. $f(0)$

8. $f(4)$

9. $f(-4)$

10. $f(2)$

11. $f(-2)$

12. $f\left(\frac{1}{2}\right)$

Spiral REVIEW—Domain and Range

For each of the following, state the domain and range in interval notation if possible.

13. $f(x) = -2x + 1$

Domain: ____________

Range: ____________

14. $f(x) = 2^x$

Domain: ____________

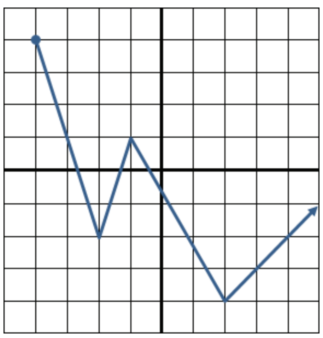
Range: ____________

15. $f(x) = |x|$

Domain: ____________

Range: ____________

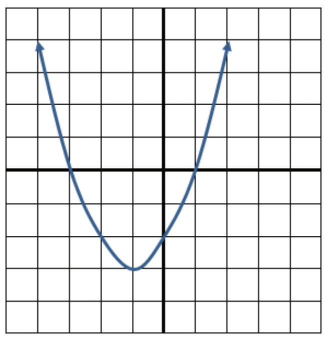
16.



Domain: ____________

Range: ____________

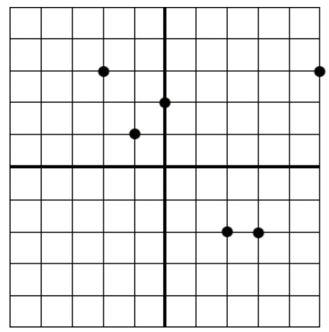
17.



Domain: ____________

Range: ____________

18.



Domain: ____________

Range: ____________

19. $\{(1, 3), (-4, 9), (-2, -7)\}$

Domain: ____________

Range: ____________

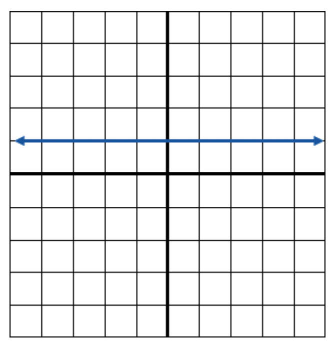
20.

x	y
3	3
2	2
−1	−1
0	0

Domain: ____________

Range: ____________

21.



Domain: ____________

Range: ____________