LESSON 15

Disappearing M&Ms™—Looking at Exponential Decay

LEARNING OBJECTIVES

- Today I am: conducting an experiment with M&Ms™.
- So that I can: understand exponential decay.
- I'll know I have it when I can: determine a way to change the experiment to show exponential growth.

Opening Activity

Source: adapted from the Virginia Department of Education

© Amy_Michelle/Shutterstock.com

Your group will need: 1 bag of M&Ms™, paper plate, cup

1. **Collecting the Data**

   A. Empty your bag of M&Ms™ onto the paper plate and count the M&Ms™. Record the number of candies on the paper plate.

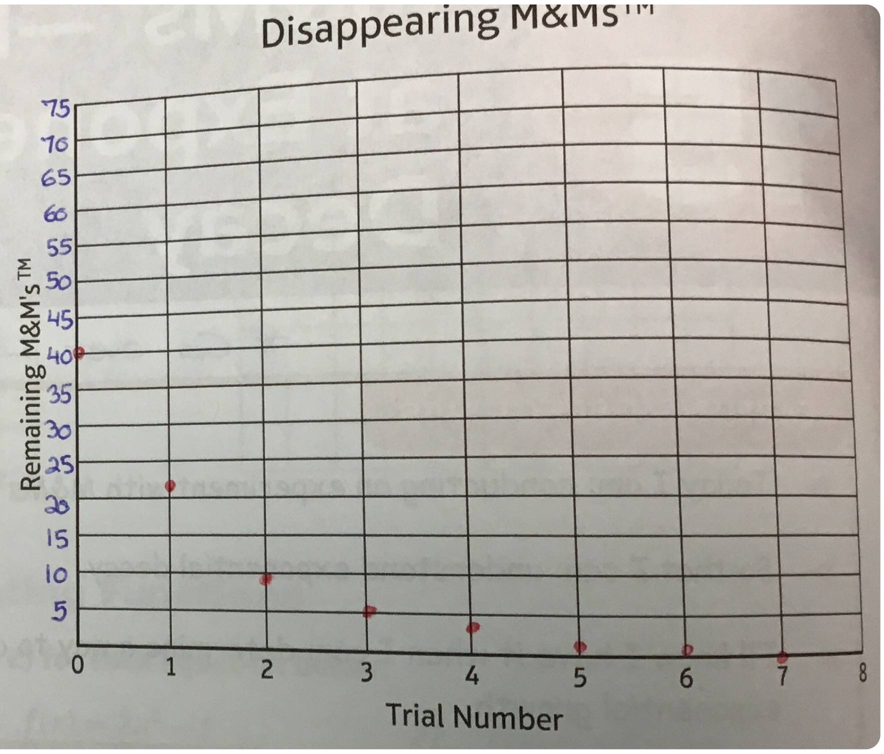
   B. Then place the M&Ms™ in a cup and mix them well.

   C. Pour them out on the paper plate, count the number that show an "m," and place them back in the cup. The others are removed. Record the number of M&Ms™ that show an "m" in your data table.

   D. Repeat the procedure. Continue until the number of M&Ms™ remaining is less than 5, but greater than 0.

E. Graph the data in the grid below. You'll need to create the vertical scale for your graph.

Disappearing M&Ms™	
Trial Number	**Number of M&Ms™ Remaining**
0 (initial amount)	40
1	22
2	9
3	5
4	3
5	1
6	1
7	0
8	



Interpreting the Data

2. Alex and Kevin started with 46 M&Ms™ and wrote the equation $y = 46\left(\frac{1}{2}\right)^x$, where y = number of M&Ms™ remaining and x = trial number. Where does the $\frac{1}{2}$ come from? Does this make sense for your trials?

common ratio $= \frac{1}{2}$ (decay)

3. From your experiment and using the exponential equation, $\boxed{y = a(b)^x}$, what value do you have for ***a***? What does ***a*** represent?

a = starting (initial) amount

$y = a \cdot b^x$

4. A. Write an equation that would approximate your data.

$$y = 40\left(\frac{1}{2}\right)^x \qquad x \geq 0$$

diff data

B. When $x = 0$, what is your function value? Compare this to the value in your data table.

$$y = 40\left(\frac{1}{2}\right)^0$$
$$= 40(1) = 40$$

trial 0 → 40 m&ms

5. If you started with 40 M&Ms™, how many trials do you think it would take before the number of M&Ms™ was between five and zero? What equation would model this new, initial value?

Beyond M&M™ Decay

6. What other objects could be used that would follow the same exponential model as in the previous experiment? What objects could you use to change the value of ***b***?

Coin tossing (50%)

7. How could you use M&Ms™ to model exponential growth instead of exponential decay?

You can add more m&ms instead

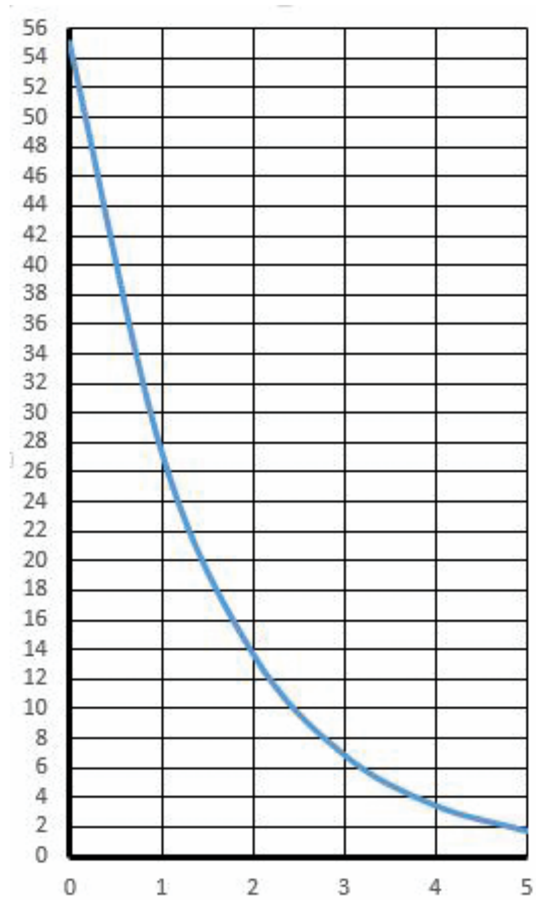
Analyzing Sample Data

8. Group A collected the following data.

A. Prove that the equation $y = 55\left(\frac{1}{2}\right)^x$ approximates Group A's data. The equation is graphed below.

$y = a \cdot b^x$

x	y
0	55
1	28
2	14
3	6
4	3
5	2



B. Describe how the 55 used in the formula is related to the data in the table.

55 → initial value.

C. Describe how the $\frac{1}{2}$ used in the formula is related to the data in the table.

$\frac{1}{2}$ → decay value.

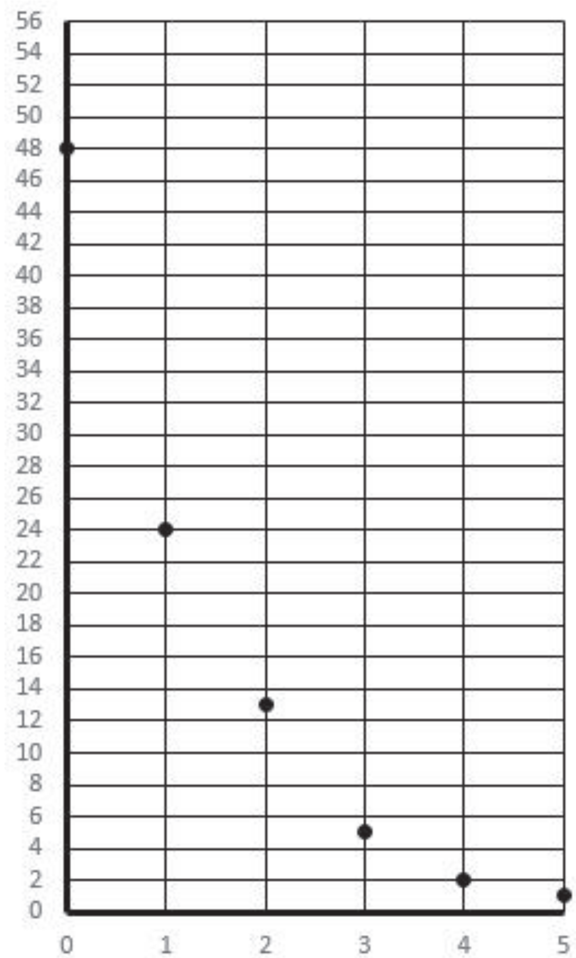
9. Group B collected the following data.

A. Write an equation to approximate Group B's data. The data is graphed below.

x	y
0	48
1	24
2	13
3	5
4	2
5	1

$b = \frac{24}{48}$

$= \frac{1}{2}$



$y = a \cdot b^x$

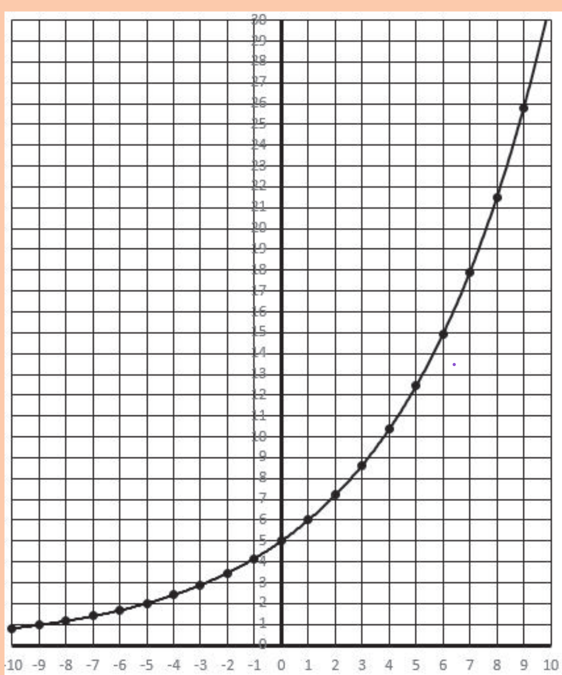
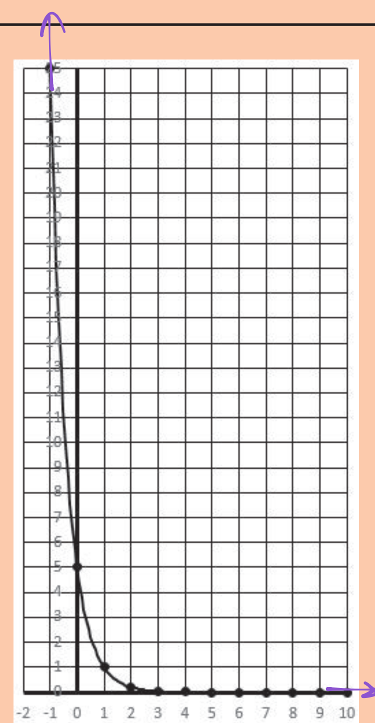
$y = 48 \cdot \left(\frac{1}{2}\right)^x$

B. Use your equation to create a table of values. Then use those points to draw the curve.

C. Were the data points you calculated the same as the ones from Group B? Explain any differences.

No, real world data can vary.

Lesson Summary

Exponential Growth	Exponential Decay
The explicit formula $f(t) = ab^t$ models exponential growth, where a represents the initial value of the sequence, b > 1 represents the growth factor per unit of time, and t represents units of time.	The explicit formula $f(t) = ab^t$ models exponential decay, where a represents the initial value of the sequence, b < 1 represents the decay factor per unit of time, and t represents units of time.
Example: $f(t) = 5 \cdot (1.2)^t$ $a =$ 5 $b =$ 1.2 growth $f(0) =$ 5 Domain: $(-\infty, \infty)$ Range: $(0, \infty)$	Example: $f(t) = 5 \cdot (0.2)^t$ $a =$ 5 $b =$ 0.2 decay $f(0) =$ 5 Domain: $(-\infty, \infty)$ Range: $(0, \infty)$
	

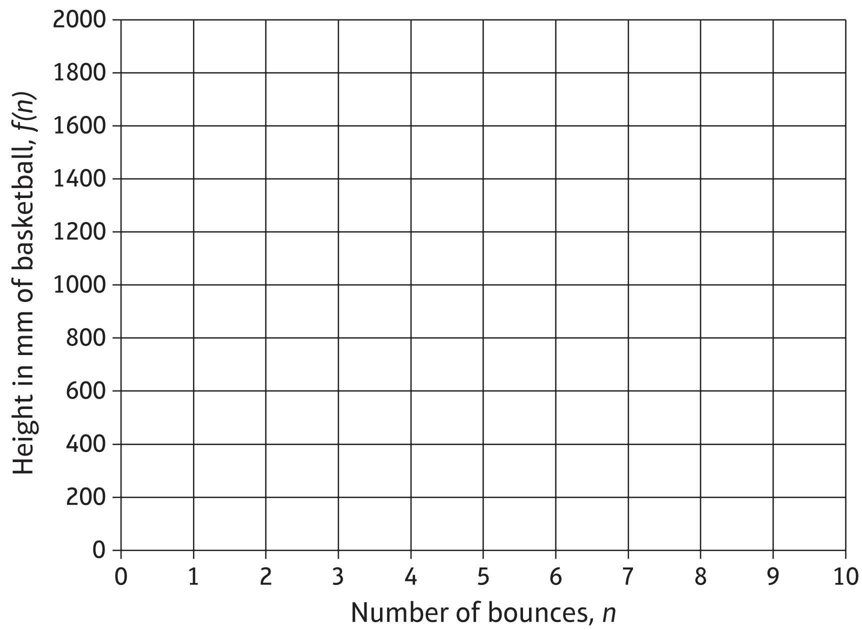
NAME: ______________________ PERIOD: __________ DATE: __________

Homework Problem Set

1. According to the International Basketball Association (FIBA), a basketball must be inflated to a pressure such that when it is dropped from a height of 1,800 mm, it rebounds to a height of 1,300 mm. Maddie decides to test the reboundability of her new basketball. She assumes that the ratio of each rebound height to the previous rebound height remains the same at $\frac{1300}{1800}$. Let $f(n)$ be the height of the basketball after n bounces.

A. Complete the chart below to reflect the heights Maddie expects to measure.

n	$f(n)$
0	1,800
1	
2	
3	
4	



B. Write the explicit formula for the sequence that models the height of Maddie's basketball after any number of bounces.

C. Plot the points from the table. Connect the points with a smooth curve, and then use the curve to estimate the bounce number at which the rebound height drops below 200 mm.

Evaluate each function at the given values.

2. $f(x) = 3 \cdot 4^x$ at $x = -1$ and $x = 1$

$f(-1) = 3 \cdot 4^{-1}$

$= 3 \cdot \frac{1}{4} = \frac{3}{4}$

$f(1) =$

3. $f(x) = -1 \cdot 2^x$ at $x = -2$ and $x = 0$

$f(-2) = -1 \cdot 2^{-2}$

$= -1 \cdot \frac{1}{2^2} = -1 \cdot \frac{1}{4} = -\frac{1}{4}$

4. $f(x) = \frac{1}{2} \cdot 4^x$ at $x = 1$ and $x = 3$

For each equation, table or graph below, determine if the function is showing exponential growth or exponential decay.

5. $y = 3^{x-1}$

$y = 1 \cdot 3^{x-1}$

$b > 3$

growth

6. $y = 0.8^x$

$b = 0.8$

decay

7. $f(x) = \left(\frac{1}{5}\right)^{x+1}$

8. $f(x) = 1.5^x$

9.

x	y
1	0.5
2	0.25
3	0.125
4	0.0625

10.

x	y
1	6
2	18
3	54
4	162

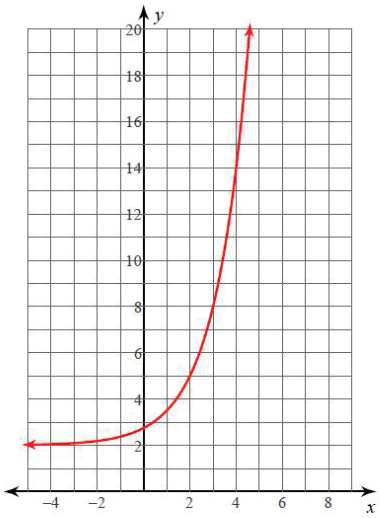
11.

x	y
1	3
2	1
3	$\frac{1}{3}$
4	$\frac{1}{9}$

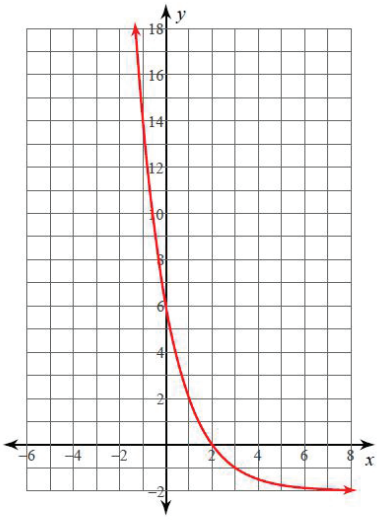
12.

x	y
1	-6
2	-12
3	-24
4	-48

13.



14.



Match the equations to their graph.

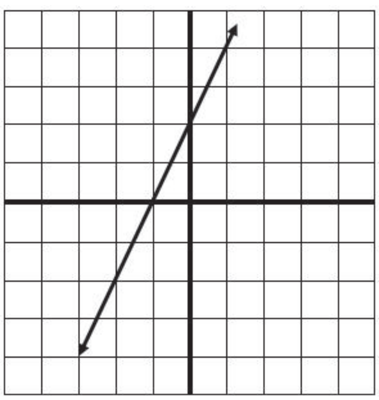
15. $f(x) = 4 \cdot 2^x$

16. $f(x) = 2x + 2$

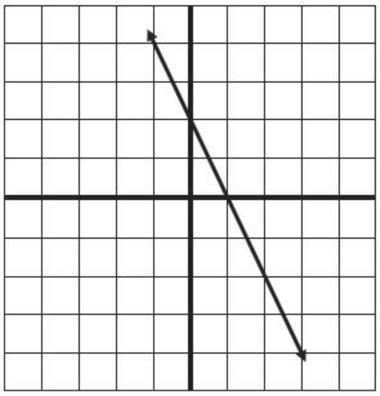
17. $f(x) = -2x + 2$

Graphs

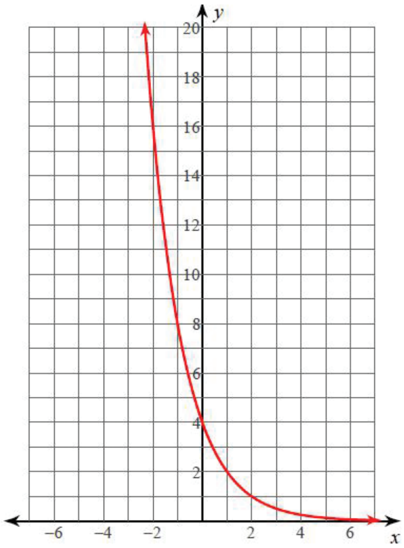
A.



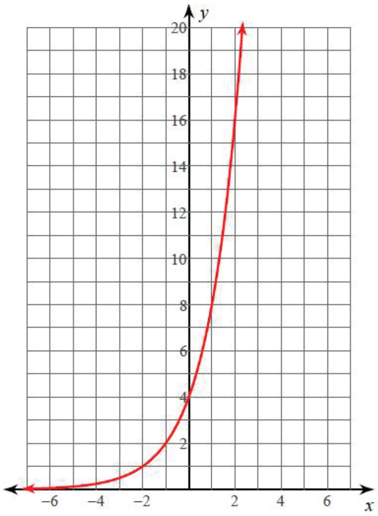
B.



C.



D.



18. Create a table of values and then graph the function $f(x) = -5 \cdot 3^x$.

x	$f(x)$
-3	$-0.\overline{185}$
-1	$-1.\overline{6}$
0	-5
1	-15
2	-45
3	
4	
5	

$-5 \cdot 3^{-3}$

$-5 \cdot \frac{1}{27} = \frac{-5}{27}$

$-5 \cdot 3^{-1}$

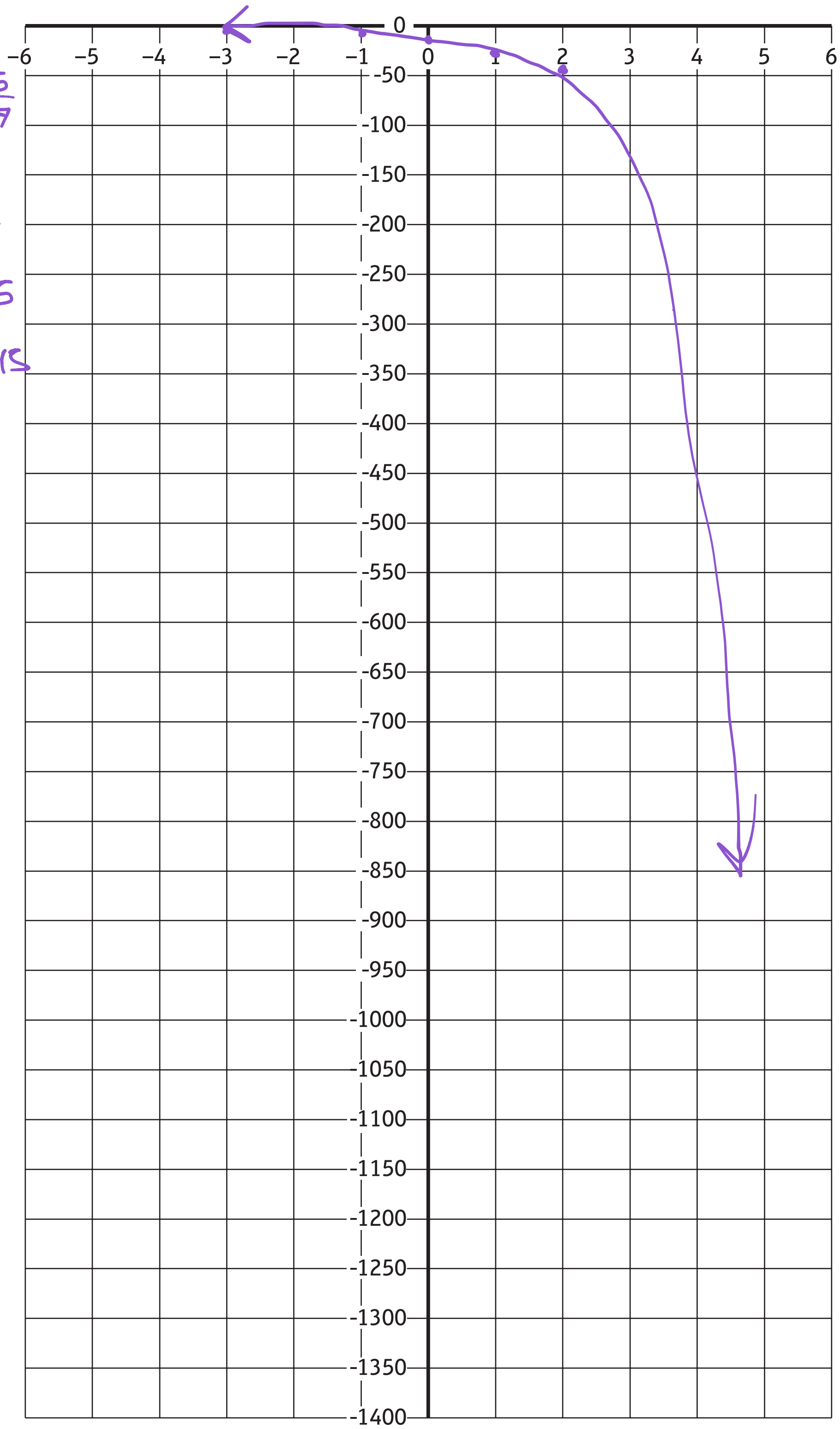
$-5 \cdot \frac{1}{3} = \frac{-5}{3}$

$-5 \cdot 3^0 = -5$

$-5 \cdot 3^1 = -15$

$-5 \cdot 3^2 =$

$-5 \cdot 9 =$



Spiral REVIEW—Percent Calculations

Determine each of the following without a calculator.

19. 50% of 100

20. 50% of 200

21. 50% of 10

22. 50% of 1

23. 100% of 100

24. 100% of 200

25. 100% of 10

26. 100% of 1

27. 10% of 100

28. 10% of 200

29. 10% of 10

30. 10% of 1

31. 60% of 100

32. 60% of 200

33. 60% of 10

34. 60% of 1

35. 200% of 100

36. 200% of 200

37. 200% of 10

38. 200% of 1