LESSON 16 A Closer Look at Linear and Exponential Functions

LEARNING OBJECTIVES

- Today I am: examining the difference between linear and exponential functions.
- So that I can: see how quickly an exponential function can overtake a linear function.
- I'll know I have it when I can: determine if $y = 3x$ is always greater than $y = 1.02^x$.

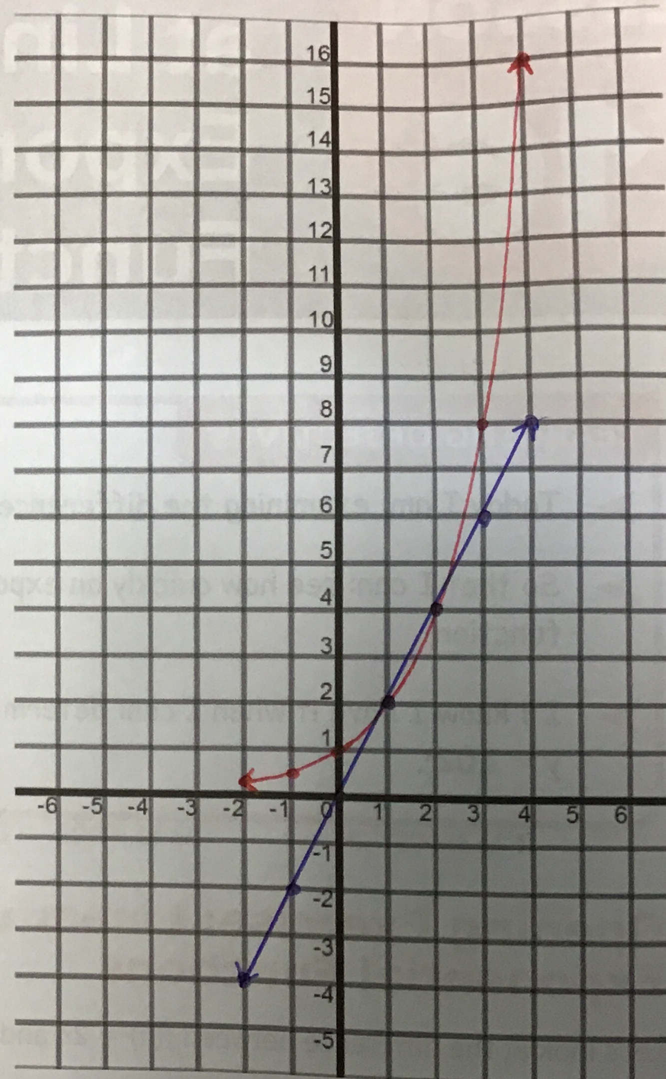
Opening Exercise: Linear versus Exponential Functions

Let's look at the difference between $f(n) = 2n$ and $f(n) = 2^n$.

1. Complete the tables below, and then graph the points $(n, f(n))$ on a coordinate plane for each of the formulas.

n	$f(n) = 2n$
−2	−4
−1	−2
0	0
1	2
2	4
3	6
4	8

n	$f(n) = 2^n$
−2	1/4
−1	1/2
0	1
1	2
2	4
3	8
4	16



2. Describe the change in each sequence when n increases by 1 unit for each sequence.

$f(n) = 2n$ increases constantly by $\frac{2}{1}$

$f(n) = 2^n$ increases exponentially by a factor of 2

Let's summarize what we've learned so far in this unit about functions.

3. Suppose that the input-output pairs of a bivariate data set have the following property:

   A. For every two inputs that are a given difference apart, the ***difference*** in their corresponding outputs is ***constant***. Then, an appropriate model for that data set could be a linear function. (common diff)

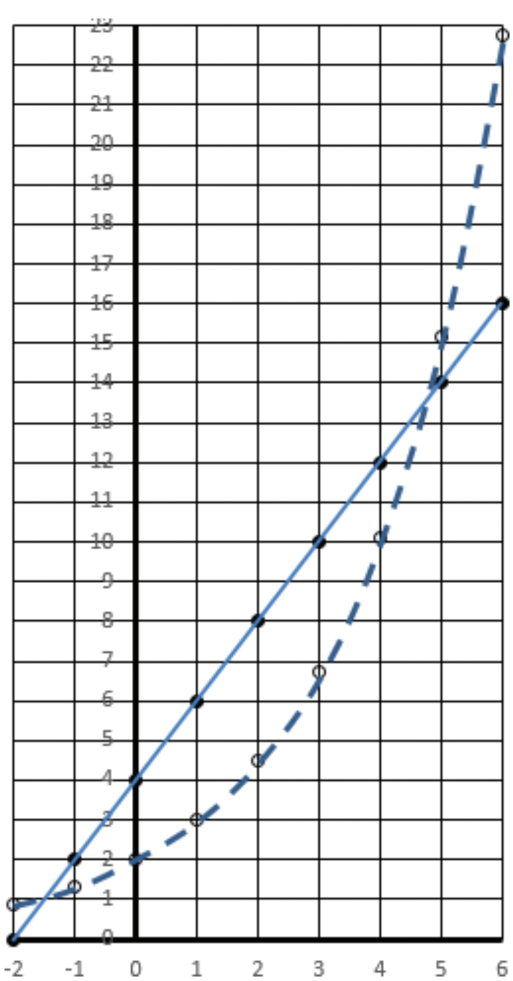
   B. For every two inputs that are a given difference apart, the ***quotient*** of their corresponding outputs is ***constant***. Then, an appropriate model for that data set could be an exponential function. ((common ratio)

   * C. An ***increasing*** exponential function will eventually surpass/exceed any linear function.

Sometimes this is not apparent in a graph displayed on a graphing calculator; that is because the graphing window does not show enough of the graph to show the sharp rise of the exponential function in contrast with the linear function.

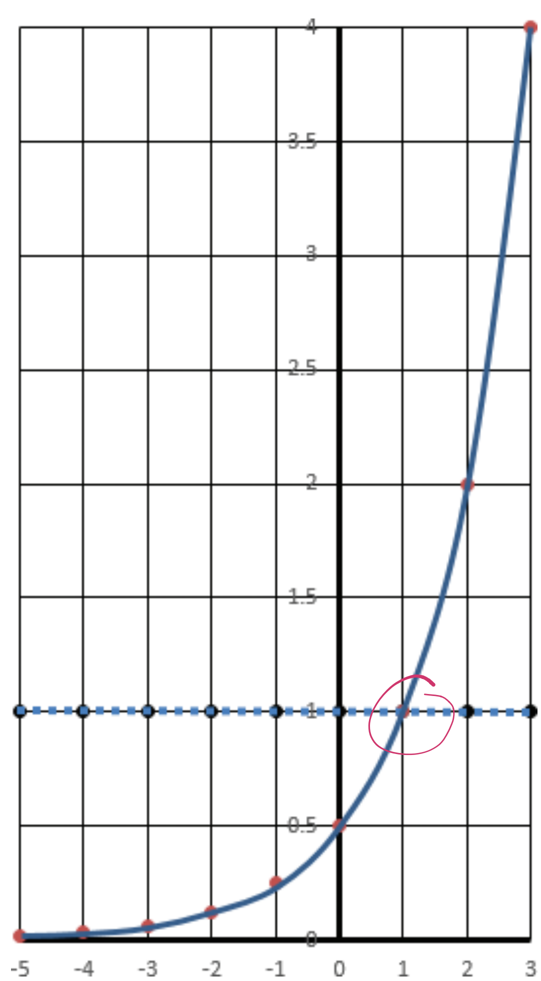
4. For each graph below, identify when the exponential function will be greater than the linear function.

A.



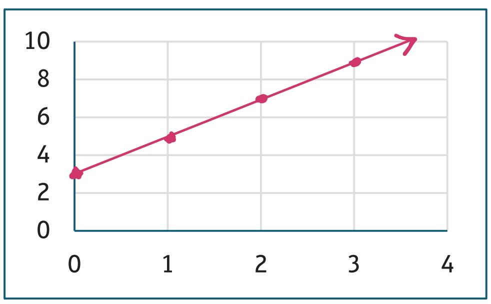
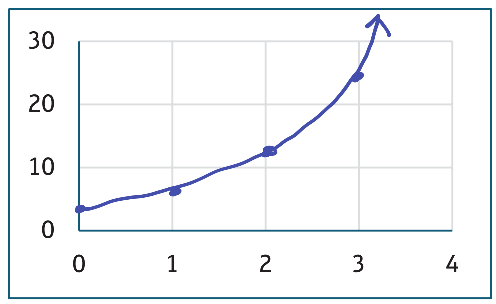
when x > 5

B.



when x > 1

5.

	Linear Model	Exponential Model
General Form	$f(x) = ax + b$	$f(x) = a(b)^x$
Meaning of Parameters *a* and *b*	a = slope b = y-intercept	a = initial value (y-inter) b = growth/decay factor
Example	$f(x) = 2x + 3$	$f(x) = 3(2)^x$ 3 2^2 3·4
Rule for Finding $f(x + 1)$ from $f(x)$	start at (0,3) then use the slope $\frac{2\uparrow}{1\rightarrow}$ (1,5)	start (0,3) mult. by 2 to y-value
Table	x | f(x) 0 | 3 1 | 5 2 | 7 3 | 9	x | f(x) 0 | 3 1 | 6 2 | 12 3 | 24
Graph		
Story Problem Example		

6. Using a calculator, Joanna made the following table and then made the following conjecture: $3x$ is always greater than $(1.02)^x$. Is Joanna correct? Explain.

x	$(1.02)^x$	$3x$
1	1.02	3
2	1.0404	6
3	1.0612	9
4	1.0824	12
5	1.1041	15

No, exponential will eventually surpass linear function.

Lesson Summary

For each space in the grid below, write the generic equation that describes the two characteristics. The first one for Linear Growth has been done for you.

	Growth	Decay
Linear	$y = mx + b$ m is the slope $m > 0$ b is the y-intercept b can be any value	$y = mx + b$ $m < 0$ $b = y$-intercept
Exponential	$y = a \cdot b^x$ $b > 1$ $x \geq 0$ a = initial amount	$y = a \cdot b^x$ $0 < b < 1$ a = initial amount

NAME: ______________________ PERIOD: __________ DATE: __________

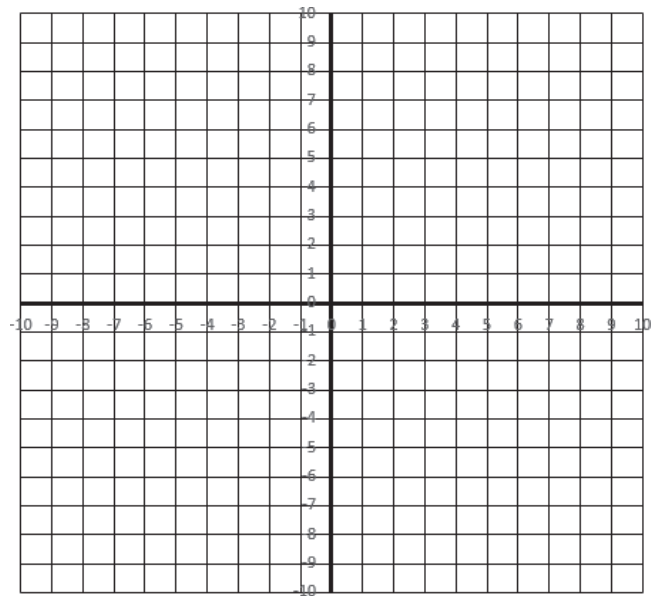
Homework Problem Set

Fill in the chart by stating the next terms in each sequence, describing the sequence as arithmetic or geometric, determining the common difference or ratio, and finally writing the formula.

	Sequence	Arithmetic or Geometric?	Common Difference or Common Ratio	Formula
1.	2, 5, 8, ______, ______			
2.	2, 6, 18, ______, ______			
3.	−2, −4, −8, ______, ______			
4.	−2, −4, −6, ______, ______			
5.	1, 2, 3, ______, ______			
6.	1, 3, 9, ______, ______			
7.	−1, −4, −7, ______, ______			
8.	−1, −4, −16, ______, ______			

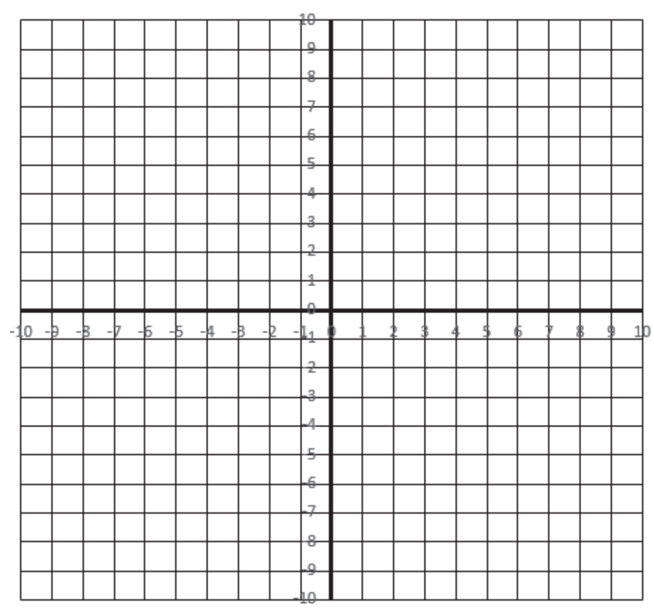
Graph each set of functions on the same grid. Then state which is linear and which is exponential and whether they are showing growth or decay. Create a table of values if necessary to graph the equations.

9. A. $f(x) = \frac{1}{2}x - 4$ B. $f(x) = \left(\frac{1}{2}\right)^x - 4$



C. Where do the two graphs meet? When is the exponential function greater than the linear function?

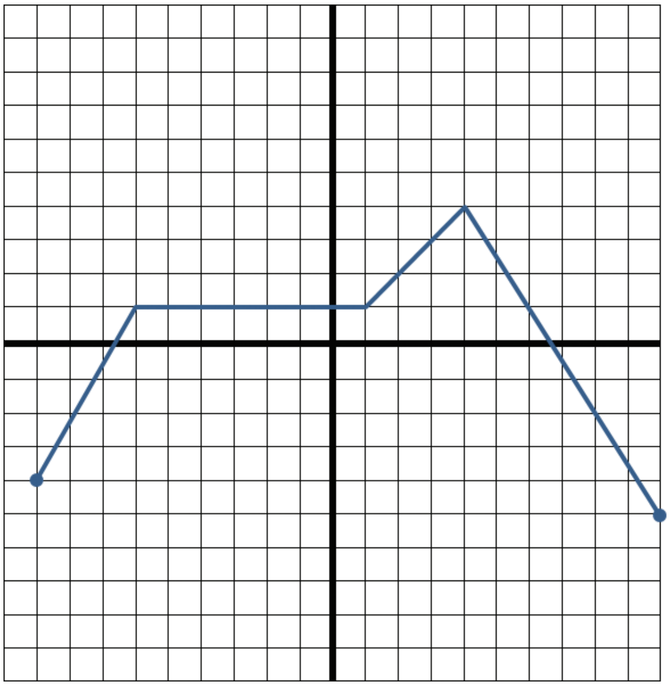
10. A. $f(x) = 3x + 1$ B. $f(x) = 3^x + 1$



C. Where do the two graphs meet? When is the exponential function greater than the linear function?

Spiral REVIEW—Evaluating Functions with a Graph

Use the graph of *f*(*x*) to determine each of the following values.

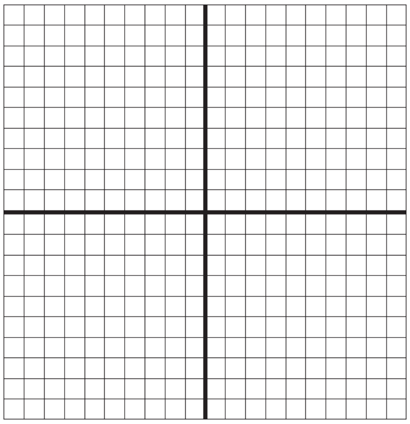


11. $f(-9)$
12. $f(-6)$
13. $f(-3)$
14. $f(-1)$
15. $f(0)$
16. $f(2)$
17. $f(4)$
18. $f(6)$
19. $f(8)$
20. $f(10)$

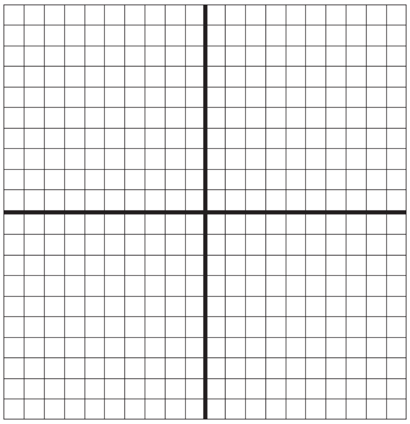
Spiral REVIEW—Graphing Linear Functions

Graph each of the following.

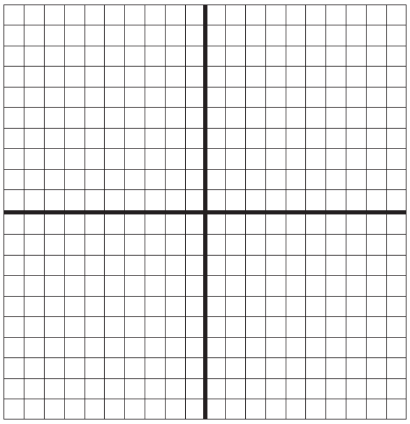
21. $f(x) = x + 3$



22. $f(x) = -x$



23. $f(x) = 2$



Spiral REVIEW—Evaluating and Graphing a Piecewise Function

Determine the following values for the function, $f(x) = \begin{cases} x+3, & \text{if } x > 1 \\ -x, & \text{if } x \leq 1 \end{cases}$**.**

24. $f(1)$

25. $f(2)$

26. $f(-2)$

27. $f(0)$

28. $f(-1)$

29. $f(3)$

30. Graph the function, $f(x) = \begin{cases} x+3, & \text{if } x > 1 \\ -x, & \text{if } x \leq 1 \end{cases}$. You may want to use the values in Problems 24–29 and the graphs in Problems 21 and 22 to help you draw the piecewise graph.