LESSON 17

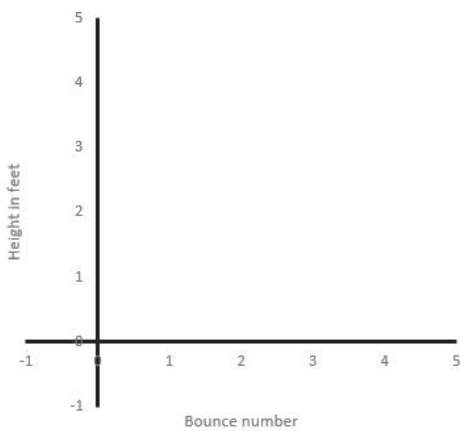
Writing Equations of Functions

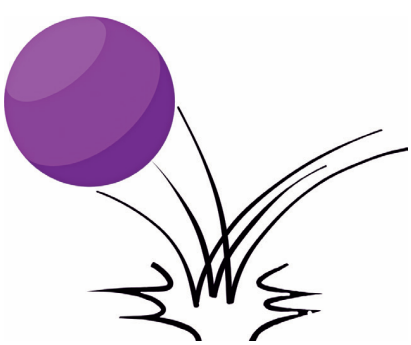
LEARNING OBJECTIVES

- Today I am: analyzing tennis ball bounces.
- So that I can: see how exponential functions can help determine if a tennis ball has a good bounce ratio.
- I'll know I have it when I can: explain why a student's assertion that a pattern is geometric is incorrect.

Opening Exercise

1. Draw a graph of a bouncing ball on the axes provided.





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Discussion

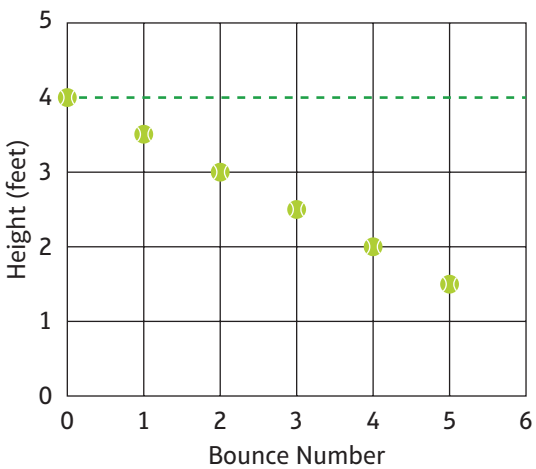
2. What do you know about bouncing balls? What are the differences between the bounce heights of a tennis ball and golf ball? Which types of balls bounce highest?

You will need: the class code for the Desmos activity *Game, Set, Flat* and a Chromebook

In this activity, you will develop your understanding of the exponential relationship that describes a bouncing tennis ball. You will examine successive terms in a sequence to determine if it represents an exponential relationship or not.

3. Use the class code given to you by your teacher to complete the Desmos activity, *Game, Set, Flat*.

4. A. Is the graph at the right showing a "good" tennis ball? How do you know?



   B. What type of function would best describe the data points at the right?

   C. If the discrete data points were connected, what would be the equation of the function?

5. Terence looked down the second column of the table below and noticed that $\frac{3}{1} = \frac{9}{3} = \frac{27}{9} = \frac{81}{27}$. Because of his observation, he claimed that the input-output pairs in this table could be modeled with an exponential function. Explain why Terence is incorrect. Then determine and write a formula for a function that generates the input-output pairs given in the table.

x	$T(x)$
0	1
1	3
4	9
13	27
40	81

1 → 3, 2 → 6

$m = 2 \quad b = 1$

$y = 2x + 1$

Lesson Summary

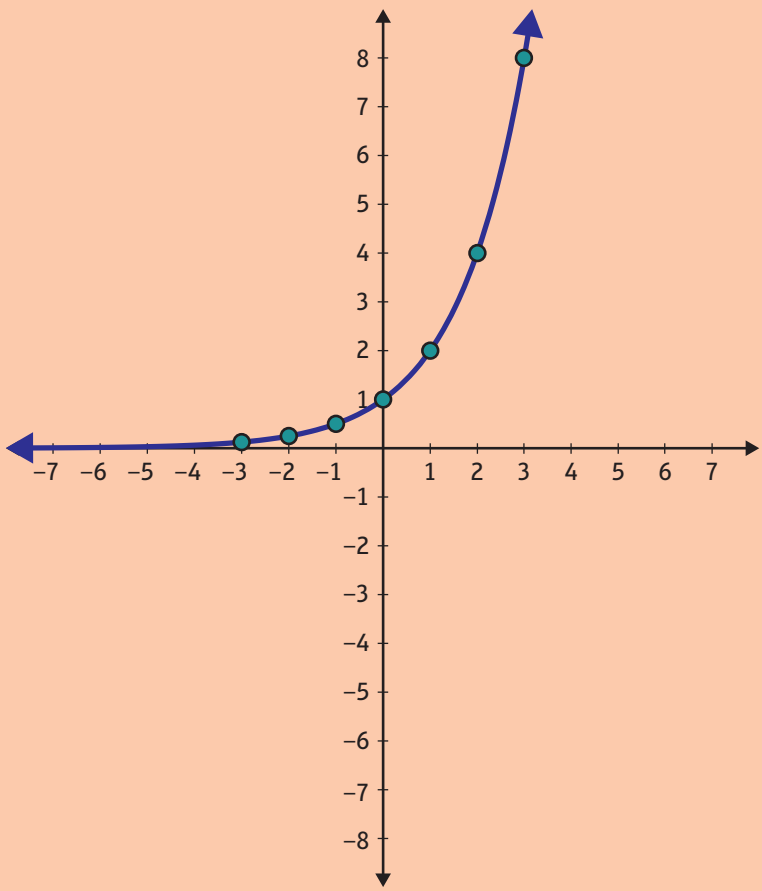
Let's examine ***exponential functions***. They are different than any of the other types of functions we've studied because the independent variable is in the exponent.

$$f(x) = 2^x$$

BASE

Let's look at the graph of this function by plotting some points.

x	2^x
3	8
2	4
1	2
0	1
−1	1/2
−2	1/4
−3	1/8



Recall what a negative exponent means: $f(-1) = 2^{-1} = \frac{1}{2}$

Source: https://www.slideshare.net/jessicagarcia62/exponential-functions-4772163

NAME: _____ PERIOD: _____ DATE: _____

Homework Problem Set

For each table in Problems 1–6, classify the data as describing a linear or exponential relationship or neither. If the relationship is linear or exponential, write a formula for the function that models the data. Then graph the data and connect the data points.

1.

x	*f*(*x*)
0	10
1	5
2	2.5
3	1.25
4	0.625
5	0.3125

$b = \frac{1}{2}$

$y = a \cdot b^x$

Linear, Exponential or Neither?

Equation, if linear or exponential:

$y = 10\left(\frac{1}{2}\right)^x$

2.

x	*f*(*x*)
0	10
1	8
2	6
3	4
4	2
5	0

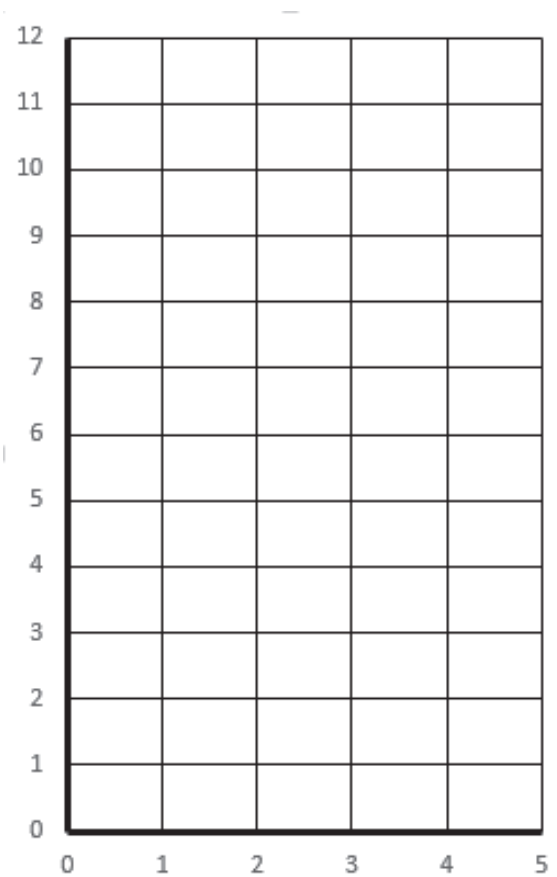
$m = \frac{-2}{1}$

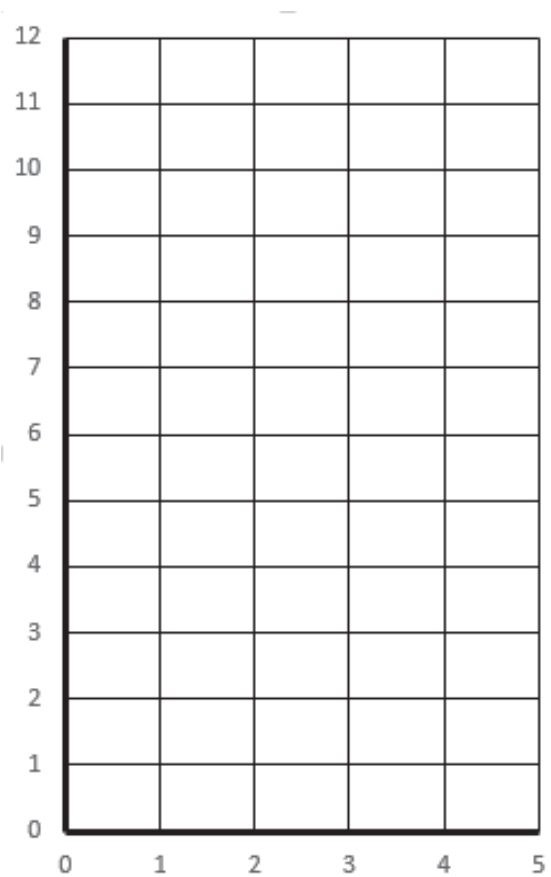
$b = 10$

Linear, Exponential or Neither?

Equation, if linear or exponential:

$y = -2x + 10$





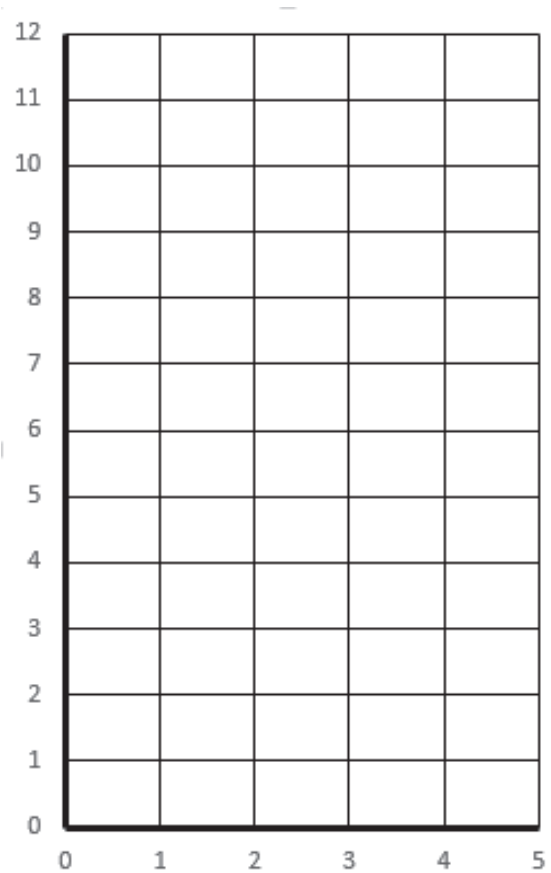
3.

x	$f(x)$
0	2
1	3
2	4
3	5
4	6
5	7

Linear, Exponential or Neither?

Equation, if linear or exponential:

_____



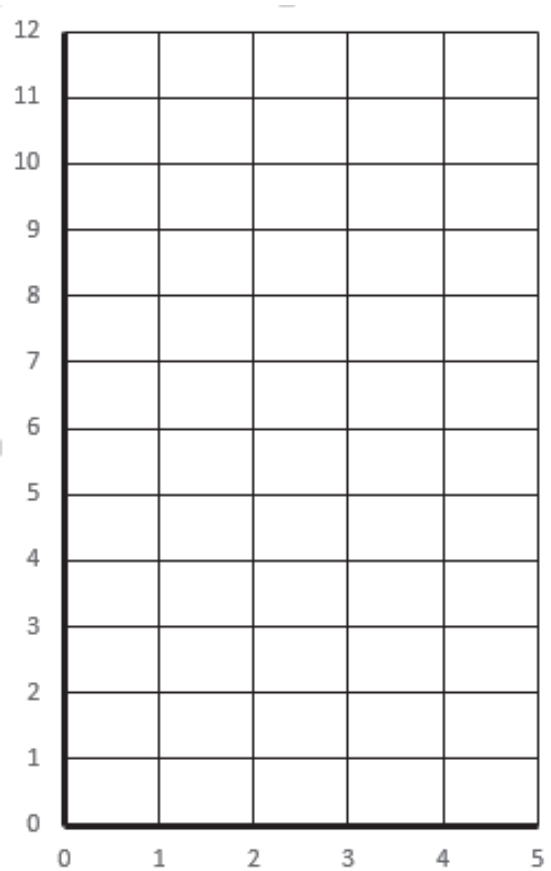
4.

x	$f(x)$
0	2
1	3
2	5
3	8
4	12
5	17

Linear, Exponential or Neither?

Equation, if linear or exponential:

_____



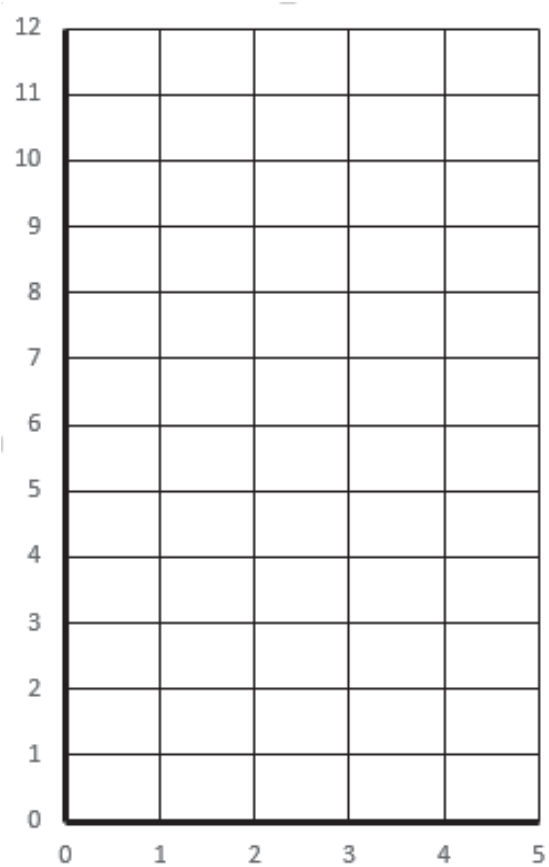
5.

x	$f(x)$
0	1
1	1.5
2	2.25
3	3.375
4	5.0625
5	7.59375

Linear, Exponential or Neither?

Equation, if linear or exponential:

_____



6. Here is a variation on a classic riddle: Jayden has a dog-walking business. He has two plans. Plan 1 includes walking a dog once a day for a rate of $5 per day. Plan 2 also includes one walk a day but charges 1 cent for 1 day, 2 cents for 2 days, 4 cents for 3 days, and 8 cents for 4 days and continues to double for each additional day. Mrs. Maroney needs Jayden to walk her dog every day for two weeks. Which plan should she choose? Show the work to justify your answer.

7. Tim deposits money in a certificate of deposit account. The balance (in dollars) in his account t years after making the deposit is given by $T(t) = 1000(1.06)^t$ for $t \geq 0$.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   A. Explain, in terms of the structure of the expression used to define $T(t)$, why Tim's balance can never be \$999.

   B. By what percent does the value of $T(t)$ grow each year? Explain by writing a recursive formula for the sequence $T(1)$, $T(2)$, $T(3)$, etc.

   C. By what percentages does the value of $T(t)$ grow every two years? (Hint: Use your recursive formula to write $T(n + 2)$ in terms of $T(n)$.)

8. A river has an initial minnow population of 40,000 that is growing at 5% per year. Due to environmental conditions, the amount of algae that minnows use for food is decreasing, supporting 1,000 fewer minnows each year. Currently, there is enough algae to support 50,000 minnows.


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   A. Is the minnow population increasing linearly or exponentially?

   B. Is the amount of algae decreasing at a linear or an exponential rate?

   C. In what year will the minnow population exceed the amount of algae available?

9. **Challenge** Your mathematics teacher asks you to sketch a graph of the exponential function $f(x) = \left(\frac{3}{2}\right)^x$ for x, a number between 0 and 40 inclusively, using a scale of 10 units to one inch for both the x- and y-axes.

   A. What are the dimensions (in feet) of the roll of paper needed to sketch this graph?

   B. How many more feet of paper would you need to add to the roll in order to graph the function on the interval $0 \leq x \leq 41$?

   C. Find an m so that the linear function $g(x) = mx + 2$ is greater than $f(x)$ for all x such that $0 \leq x \leq 40$, but $f(41) > g(41)$.

Spiral REVIEW—Looking for Patterns

Determine a pattern that could be used to get from the first term to the last term in each sequence.

10.

Term Number	1	2	3	4
Term	10			40

Pattern description: _____

11.

Term Number	1	2	3	4
Term	10			80

Pattern description: _____

12.

Term Number	1	2	3	4
Term	10			270

Pattern description: _____

13.

Term Number	1	2	3	4
Term	10			1

Pattern description: _____