# LESSON 19 Applications of Exponential Growth and Decay

**LEARNING OBJECTIVES**

- Today I am: using an exponential growth model.
- So that I can: check if an article about the Disneyland measles outbreak is accurate.
- I'll know I have it when I can: examine equations to see if they show exponential growth or decay.

## Opening Exercise

1. Read the following excerpt from an article by Tara Haelle on the *Forbes* website on January 20, 2015. Highlight information that would help you decide if her last claim is correct.

"... the number of measles cases linked to the outbreak ... at Disneyland has continued increasing, most recently to 52 ... Let's say all the visitors at Disneyland ... had been unvaccinated and had never had measles. The first case of measles ... would have conservatively infected 12 people ... on Dec 17. With an incubation period of 10–12 days ... each of those 12 people would have infected 12 others each by about Dec. 28."

2. Use the calendar below to check Tara's claim that "by the end of the incubation period we would have thousands more cases"? Is Tara correct?

| Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday |
|---|---|---|---|---|---|---|
| | | Dec 17  Measles Outbreak 12 | 18 | 19 | 20 | 21 |
| 22 | 23 | 24 | 25 | 26 | 27 | 28 $12^2$ How many cases? 144 |
| 29 | 30 | 31 | **Jan 1** | 2 | 3 | 4 |
| 5 | 6 | 7 | 8 $12^3$ How many cases 1728 | 9 | 10 | 11 |
| 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| 19 $12^4$ How many cases? 20736 | 20 | 21 | | | | |

Mickey Mouse icon © Gary Watts/Shutterstock.com

3. How many cases would you expect to see on Day 48?

$12^5 = 248{,}832$

4. The Center for Disease Control and Prevention (CDC) documented 102 cases on Day 48. Why is the CDC number so much lower than you might expect?

Vaccination

Engineers, biologists, economists and many others use exponential functions to make predictions. You've seen how the measles outbreak in 2015 can be modeled with an exponential function. Throughout this lesson, you'll explore other applications of exponential growth and decay.

5. A rare coin appreciates (increases in value) at a rate of 5.2% a year. The initial value of the coin is $500.


© Tony Baggett/Shutterstock.com

A. Complete the table below to show how the value of the coin changes over time.

| Number of years, *t*, since the coin was valued | Coin value after *t* years | 5.2% appreciation of current coin value |
|---|---|---|
| 0 | $500 | $500 • (0.052) = $26 |
| 1 | $500 + $26 = $526 | $526 • (0.052) ≈ $27.35 |
| 2 | $526 + ______ = $553.35 | $553.35 • (0.052) ≈ $ ______ |
| 3 | $553.35 + ______ = $ ______ | |
| 4 | | |
| 5 | | |

B. What is the common ratio, *r*, for the sequence of coin values?

$$\frac{526}{500} = 1.052$$

C. What is a short cut to finding the remaining values of the coin values in the table? Is there a faster way than finding 5.2% of the current coin value and then adding it to the last coin value?

*Use 1.052 instead 0.052 (add to total)

D. Write a rule that describes this geometric sequence. Use 526 as the first term so that the number of years is equivalent to the term number.

$f(t) = f(1) \cdot r^{t-1}$

$$f(t) = 526(1.052)^{t-1}$$

E. Use your rule to predict the coin's value after 5 years. Confirm your answer with the value in the table.

$$f(5) = 526(1.052)^{5-1}$$
$$= 644.24$$

We can also approach writing the rule from another perspective:

*

> The explicit formula ***f*(*t*)** = ***ab*^t** models exponential ***growth***, where ***a*** represents the initial value (zero term) of the sequence, $\boxed{b > 1}$ represents the ***growth*** factor per unit of time, and ***t*** represents units of time.

6. Using this new formula to model exponential growth, the equation for this situation would be

$$\boxed{g(t) = 500 \cdot (1.052)^t}$$

Confirm that $f(5)$ from Exercise 5E and $g(5)$ are equivalent.

$g(5) = 500 \cdot (1.052)^5$

$= 644.24$

7. A. What are the similarities and differences between using the rule for a geometric sequence and the new explicit formula for modeling exponential growth?

* They both have the same growth rate 1.052
* They have diff starting values

B. Let's look closer at the two equations below. What does $\frac{f(1)}{r}$ represent? Why is that? What does $r^t$ represent?

| | | |
|---|---|---|
| $f(t) = a \cdot b^t$ | $f(t) = f(1) \cdot r^{t-1}$ | Geometric sequence formula. |
| | $f(t) = f(1) \cdot \frac{r^t}{r^1}$ | Use the exponent rule $c^{n-m} = \frac{c^n}{c^m}$. |
| | $f(t) = \frac{f(1)}{r} \cdot r^t$ | Rearrange the expression. |

8. Malik bought a new car for $15,000. As he drove it off the lot, his best friend, Will, told him that the car's value just dropped by 15% and that it would continue to depreciate 15% of its current value each year. If the car's value is now $12,750 (according to Will), what will its value be after 5 years?



A. Complete the table below to determine the car's value after each of the next five years. Round each value to the nearest cent.

| Number of years, *t*, passed since driving the car off the lot | Car value after *t* years | 15% depreciation of current car value | Car value minus the 15% depreciation |
|---|---|---|---|
| 0 | $12,750.00 | $1,912.50 | $10,837.50 |
| 1 | 10,837.50 | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |

B. What is the common ratio, *r*, for the sequence of car values? How does this relate to the 15% depreciation each year?

$1 - 0.15 = 0.85$

9. Use both the sequence rule and explicit formula to model Malik's car value after *t* years.

Sequence method: $f(t) = f(1) \cdot r^{t-1}$

$f(t) = 10837.50(0.85)^{t-1}$

$t \geq 1$

use this

*

Explicit formula: $g(t) = ab^t$

$g(t) = 12750(0.85)^t$

$t \geq 0$

10. Use both of your equations from Exercise 9 to find the value of Malik's car at year 5.

$g(5) = 12750(0.85)^5$

$= 5657.24$

11. What is the difference between sequences and these types of applications? Why didn't we use the explicit rule $g(t) = ab^t$ for sequences?

Since the value of the car decreases, we call this exponential decay.

* * *

The explicit formula $\boldsymbol{f(t)} = \boldsymbol{ab^t}$ models exponential ***decay***, where $\boldsymbol{a}$ represents the initial value (zero term) of the sequence, $\boxed{\boldsymbol{0 < b < 1}}$ represents the **decay** factor per unit of time, and $\boldsymbol{t}$ represents units of time.

decimals / fractions

**Which of these exponential functions represents growth and which represents decay? How do you know?**

12. $h(t) = 3 \cdot (0.7)^t$

$b = 0.7$

what's left

decay

13. $j(t) = \frac{2}{3} \cdot (1.2)^t$

$b = 1.2$

Growth

* 14. $k(t) = -2 \cdot 3^t$

$b = 3$

Decay

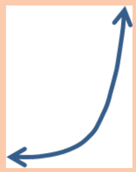
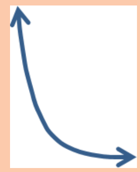
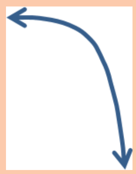
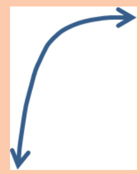
**Identify the initial value in each formula below, and decide whether the formula models exponential growth or exponential decay. Justify your responses.**

| | Initial Value | Exponential Growth or Decay? | Reasoning |
|---|---|---|---|
| 15. $f(t) = 2\left(\frac{2}{5}\right)^t$ | 2 | Growth or Decay | $0 < \frac{2}{5} < 1$ |
| 16. $f(t) = 2\left(\frac{5}{3}\right)^t$ | 2 | Growth or Decay | $\frac{5}{3} > 1$ |
| 17. $f(t) = \frac{2}{3}(3)^t$ | 2/3 | Growth or Decay | $3 > 1$ |
| 18. $f(t) = \frac{2}{3}\left(\frac{1}{3}\right)^t$ | 2/3 | Growth or Decay | $0 < \frac{1}{3} < 1$ |
| 19. $f(t) = \frac{3}{2}\left(\frac{2}{3}\right)^t$ | 3/2 | Growth or Decay | $0 < \frac{2}{3} < 1$ |

# Lesson Summary

## Graphs of Exponential Functions

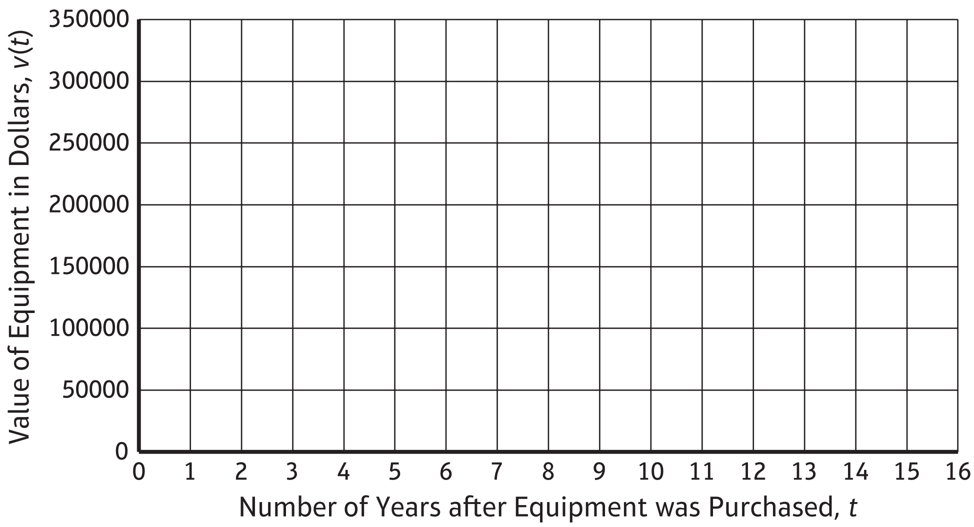
$$f(x) = a \cdot b^x$$

| Parameters | $1 < b$ | $0 < b < 1$ |
|---|---|---|
| $0 < a$ |  |  |
| $a < 0$ |  |  |

NAME: ______________________ PERIOD: ____________ DATE: ____________

# Homework Problem Set

1. A construction company purchased some equipment costing $300,000. The value of the equipment depreciates (decreases) at a rate of 14% per year.




A. Write a formula that models the value of the equipment each year.

B. What is the value of the equipment after 9 years?

C. Graph the points $(t, v(t))$ for integer values of $0 \le t \le 15$.

D. Estimate when the equipment will have a value of $50,000

2. The number of newly reported cases of HIV (in thousands) in the United States from 2000 to 2010 can be modeled by the following formula:

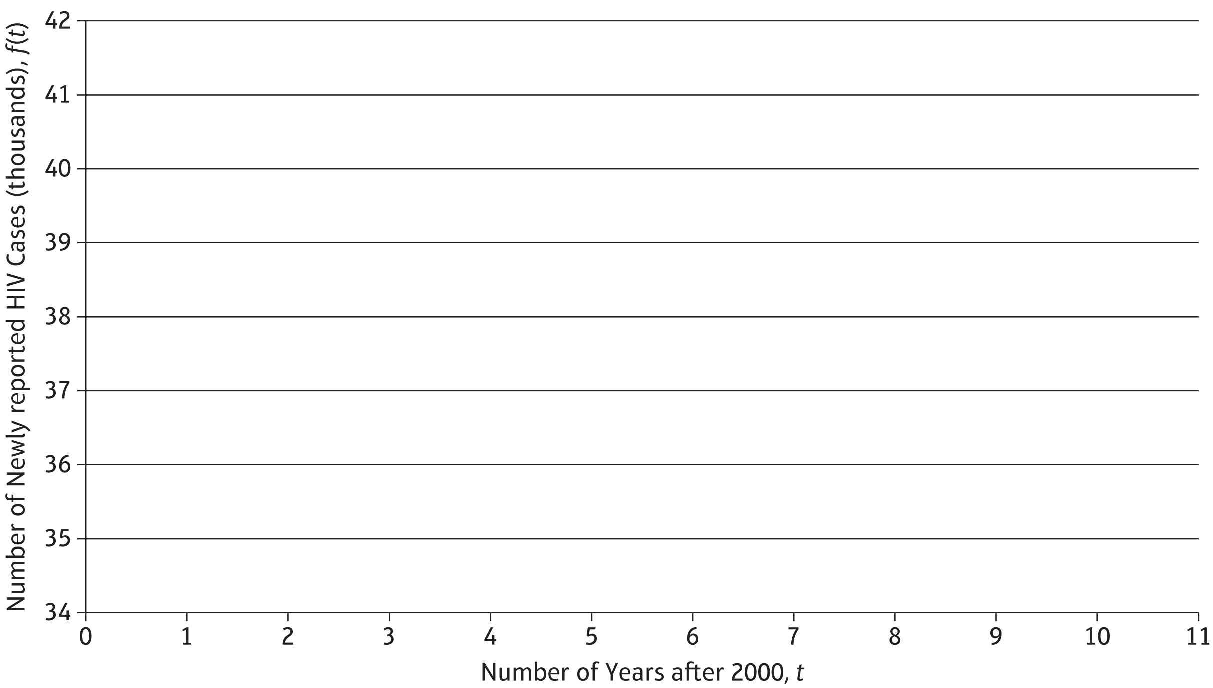
   $f(t) = 41(0.9842)^t$, where $t$ is the number of years after 2000



   A. Identify the growth factor.

   B. Calculate the estimated number of new HIV cases reported in 2004.

   C. Graph the points $(t, f(t))$ for integer values of $0 \leq t \leq 10$.



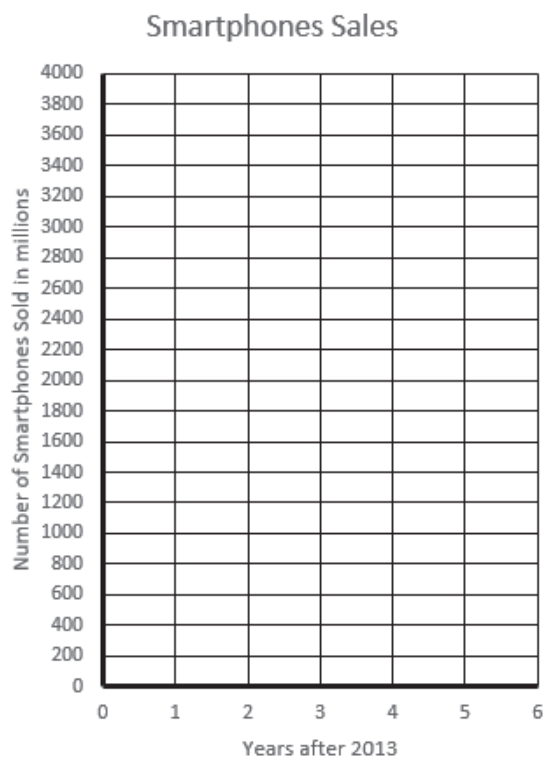

   D. During what year did the number of newly reported HIV cases drop below 36,000?

3. In 2013, a research company found that smartphone shipments (units sold) were up 32.7% worldwide from 2012, with an expectation for the trend to continue. If 959 million units were sold in 2013, how many smartphones can be expected to sell in 2018 at the same growth rate? (Include the explicit formula for the sequence that models this growth.)

© Jiw Ingka/Shutterstock.com

   A. Identify the growth factor.

   B. Calculate the estimated number of smartphones expected to be sold in 2018 at the same growth rate.

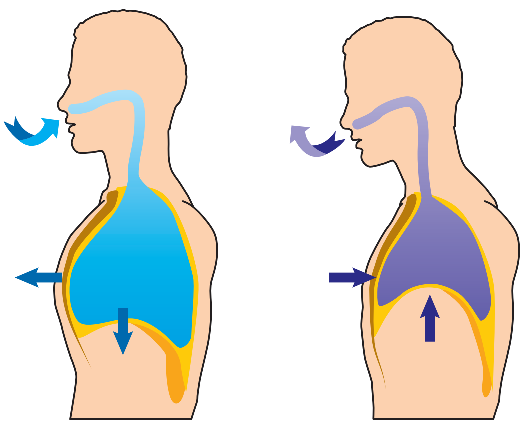



   C. Graph the points $(t, f(t))$ for integer values of $0 \leq t \leq 5$.

   D. Can this trend continue? Explain your thinking.

4. When you breathe normally, about 12% of the air in your lungs is replaced with each breath. Write an explicit formula for the sequence that models the amount of the original air left in your lungs, given that the initial volume of air is 500 ml. Use your model to determine how much of the original 500 ml remains after 50 breaths.

The movements of the chest during breathing.

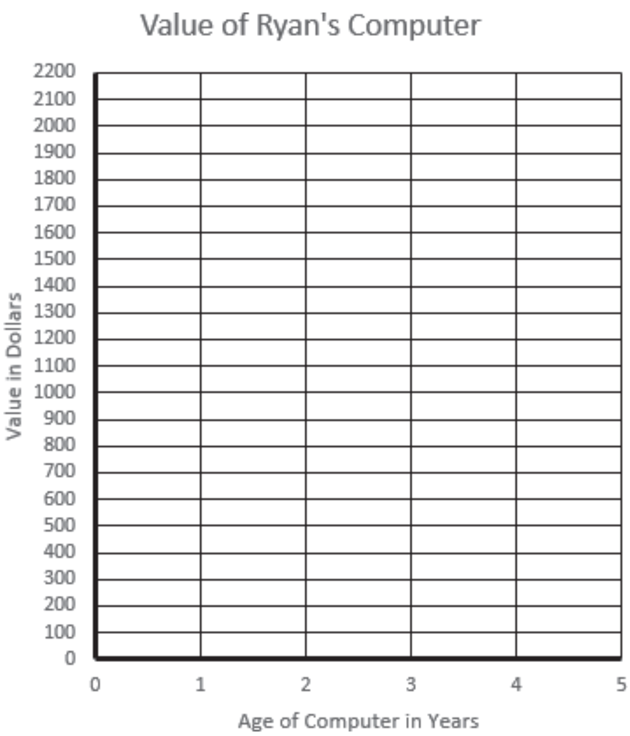





5. Ryan bought a new computer for $2,100. The value of the computer decreases by 50% each year.

   A. Identify the decay factor.

   B. Calculate the value of Ryan's computer after 5 years.




   C. Graph the points ($t$, $f(t)$) for integer values of $0 \leq t \leq 5$.

   D. When will the value drop below $300?