LESSON 21

Filling Up—The Power of Exponential Growth

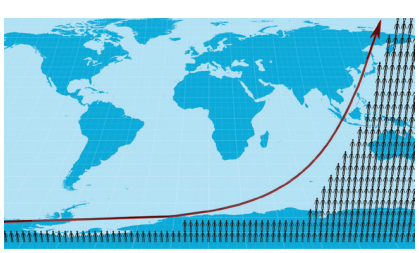
LEARNING OBJECTIVES

- Today I am: watching a YouTube video on the world's population growth.
- So that I can: understand ways to model real-world data.
- I'll know I have it when I can: make an accurate prediction about the price of movie tickets in 1948.

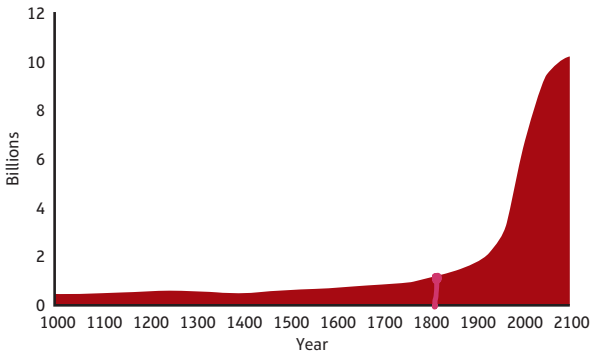
Opening Exercise

It was just over two centuries ago that the global population was 1 billion—in 1804. But better medicine and improved agriculture resulted in higher life expectancy for children, dramatically increasing the world population, especially in the West. As higher standards of living and better health care are reaching more parts of the world, the rates of fertility—and population growth—have started to slow down, though the population will continue to grow for the foreseeable future. U.N. forecasts suggest the world population could hit a peak of 10.1 billion by 2100 before beginning to decline. But exact numbers are hard to come by—just small variations in fertility rates could mean a population of 15 billion by the end of the century.

1. Understanding how much the population of the world has changed can be difficult to fully comprehend by reading a paragraph or seeing a bunch of numbers. Watch the YouTube video *7 Billion: How did we get so big so fast?* on human population at https://www.youtube.com/watch?v=VcSX4ytEfcE. This 2.5 minute video does a good job of helping us grasp the enormity of our growing population.



Statisticians and economists use mathematical models to make predictions and see trends in the data. Graphs are one type of mathematical model but they can be difficult to use for predictions.



2. A. Use the graph above to estimate the population of the world in 1806.

About 1 billion.

B. Does this agree with the information in the water-dripping graphic?

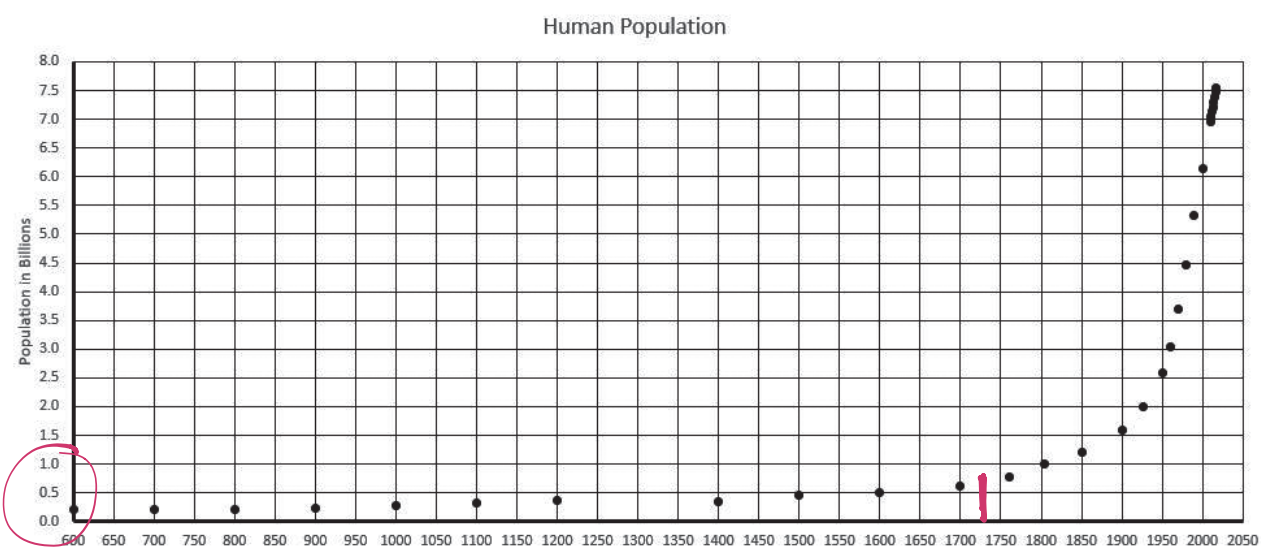
Yes

C. How accurate can you get with the graph? What could be done to get a more accurate value for 1806 from the graph?

Smaller intervals, include grids

3. The graph below contains the same world population data. Estimate the world population in 1725. Explain how you determined the population.

0.75 billion.



Source: http://www.worldometers.info/world-population/world-population-by-year/

4. Amelie states that a piecewise function would best represent this data. Explain what Amelie may be thinking. Do you agree with her?

5. Elijah wants to use an exponential function for this data. What information would Elijah need to write the exponential function? Do you think an exponential function is an accurate model for this data? Explain your thinking.

$y = a \cdot b^x$ ← growth
↑ initial population

You will need: Desmos class code for *Predicting Movie Ticket Prices*, Chromebook

6. Complete the Desmos activity *Predicting Movie Ticket Prices*.
7. Is there another way the data could have been graphed that may have been easier to understand? Explain your thinking.

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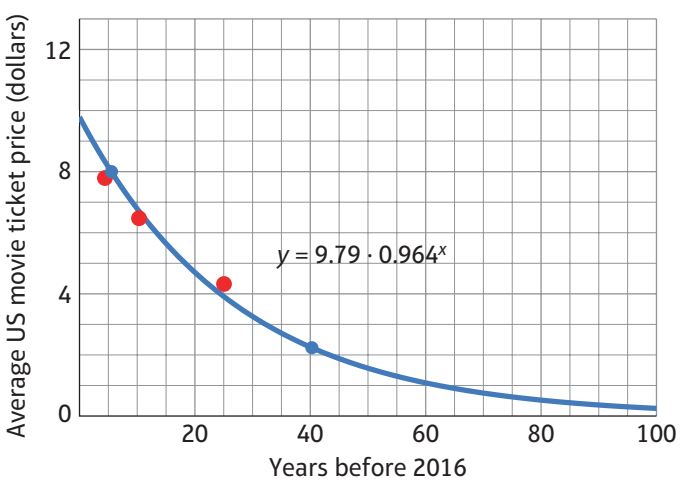
8. Ayden states that although the exponential function seemed like a good choice, he wasn't able to get a very accurate prediction for the ticket prices in 1948. His prediction was 80 cents or about twice as large as the actual price. Give Ayden at least two suggestions to improve his prediction. His model is shown below.



Lesson Summary

Models include diagrams, physical replicas, mathematical representations, analogies, and computer simulations. Although models do not correspond exactly to the real world, they bring certain features into focus while obscuring others. All models contain approximations and assumptions that limit the range of validity and predictive power, so it is important for you to recognize their limitations.

The three stages of modeling are:

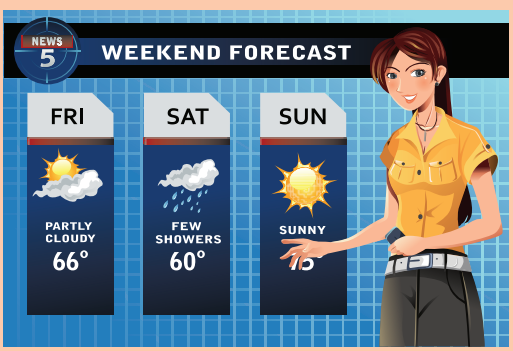
- Defining the problem
- Developing possible solutions
- Improving designs

Source: http://ngss.nsta.org/high-school-engineering-design.aspx

Example of Mathematical Modeling

The NOAA Weather Forecast Model provides a good example of a computational model.

- Scientists use information about the past to build their climate models.
- Scientists test their climate models by using them to forecast past climates.
- When scientists can accurately forecast past climates, they can be more confident about using their models to predict future climates.



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Source: https://www.nationalgeographic.org/activity/using-models-make-predictions/

NAME: _____ PERIOD: _____ DATE: _____

Homework Problem Set

For each problem below, determine what type of model could best help you find the answer. Some models to consider are: tables, graphs, equations, or diagrams. Then solve each problem.

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1. A bucket is put under a leaking ceiling. The amount of water in the bucket doubles every minute. After 8 minutes, the bucket is full. After how many minutes is the bucket half-full?

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2. A three-bedroom house in Burbville sold for $190,000. If housing prices are expected to increase 1.8% annually in that town, what is the price of the house in 5 years?

3. Mrs. Davis is making a poster of math formulas for her students. She takes the 8.5 in. $\times$ 11 in. paper she printed the formulas on to the photocopy machine and enlarges the image so that the length and the width are both 150% of the original. She enlarges the image a total of 3 times before she is satisfied with the size of the poster. Write an explicit formula for the sequence that models the area of the poster, A, after n enlargements. What is the area of the final image compared to the area of the original, expressed as a percent increase and rounded to the nearest percent?

4. Two band mates have only 7 days to spread the word about their next performance. Jack thinks they can each pass out 100 fliers a day for 7 days, and they will have done a good job in getting the news out. Meg has a different strategy. She tells 10 of her friends about the performance on the first day and asks each of her 10 friends to tell a friend on the second day and then everyone who has heard about the concert to tell a friend on the third day, and so on, for 7 days. Make an assumption that students are not telling someone who has already been told.

   A. Over the first 7 days, Meg's strategy will reach fewer people than Jack's. Show that this is true.

   B. If they had been given more than 7 days, would there be a day on which Meg's strategy would begin to inform more people than Jack's strategy? If not, explain why not. If so, on which day would this occur?

   C. Knowing that she has only 7 days, how can Meg alter her strategy to reach more people than Jack does?

5. On June 1, a fast-growing species of algae is accidentally introduced into a lake in a city park. It starts to grow and cover the surface of the lake in such a way that the area it covers doubles every day. If it continues to grow unabated, the lake will be totally covered, and the fish in the lake will suffocate. At the rate it is growing, this will happen on June 30.

   A. When will the lake be covered halfway?

   B. On June 26, a pedestrian who walks by the lake every day warns that the lake will be completely covered soon. Her friend just laughs. Why might her friend be skeptical of the warning?

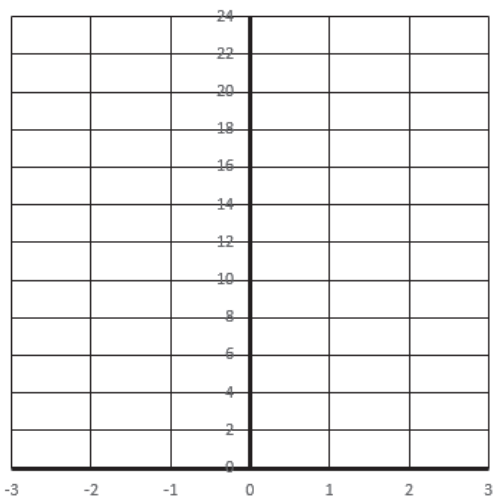
   C. On June 29, a cleanup crew arrives at the lake and removes almost all of the algae. When they are done, only 1% of the surface is covered with algae. How well does this solve the problem of the algae in the lake?

Spiral REVIEW

Use a table of values to graph each equation below.

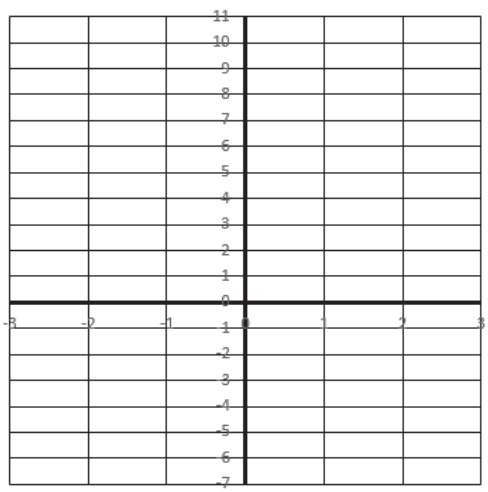
6. $f(x) = 3 \cdot 2^x$

x							
$f(x)$							



7. $f(x) = 3x + 2$

x							
$f(x)$							



8. $f(x) = 3\sqrt{x} + 2$

x	$f(x)$

x	$f(x)$