## LESSON 22

Graphs of Absolute Value

LEARNING OBJECTIVES

➤ Today I am: exploring absolute value graphs on Desmos.

➤ So that I can: see how absolute value graphs are a special piecewise function graph.

➤ I'll know I have it when I can: write a piecewise function that is equivalent to an absolute value graph.
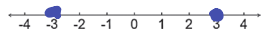
Throughout this year, we've looked at different types of linear and exponential functions. Now we'll turn our attention to a special **piecewise linear** function—the **absolute value function**.

Warm-Up

dec   *inc*
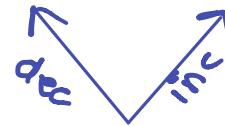
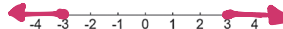Graph each absolute value equation or inequality below.

1. $|x| = 3$

2. $|x + 1| = 3$

or

3. $|x| \geq 3$

and

4. $|x| \leq 3$

$x = 3, -3$

$X + 1 = 3$ $X + 1 = -3$

$X = 2$ $X = -4$

$X \geq 3$ or $X \leq -3$

$X \leq 3$ and $X \geq -3$

In each of these warm–up exercises, there was only one variable, *x*, but suppose there are two variables with one in an absolute value symbol.

5. Let's look at the simplest absolute value function, $y = |x|$. This is the **parent** graph for all absolute value graphs.

A. List several coordinate points that would make the equation $y = |x|$ true. Choose an x, input, value and then determine the y, output, value. Be sure to choose points that will fit on the grid.
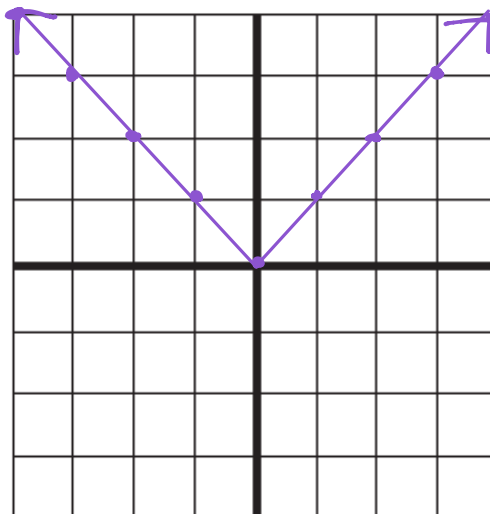
B. Graph your points on the grid below. Is this a linear function? Check with other students to see what shape seems to be appearing.

x	y
-4	4
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3
4	4

$|-4| = 4$
$|-3| = 3$

$y = |x - 3|$

Exploratory Activity

You will need: a Chrome book and a class code for Desmos

6. Go to student.desmos.com and enter your class code: _____ . You will need to enter some of your answers below as well as on the computer.

7. What is the shape of an absolute value graph?

8. For the equation $y = |x| + k$, what does the k do to the graph? How does it **transform** the parent graph?

$y = |x| + k$ ↑ k units   } vertical shift
$y = |x| - k$ ↓ k units

9. For the equation $y = |x - h|$, what does the h do to the graph?

$y = |x - h|$ → h units   } horizontal shift
$y = |x + h|$ ← h units

10.   For the equation $y = a|x|$, what does the a do to the graph? What happens when $a < 0$?

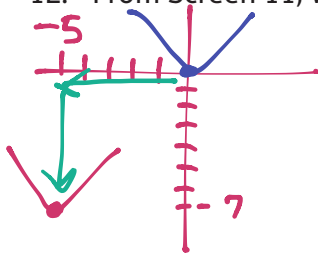$\hookrightarrow$ change the steepness

a > 1   (more steep)

0 < a < 1 (less steep, wider)

11.   What is the **vertex** of an absolute value graph?

the lowest/highest point.

vertex

12.   From Screen 11, what is the equation of an absolute value graph that has a vertex at $(-5, -7)$?

-5

$y = |x+5| - 7$

left 5, down 7

-7

13.   From Screen 14, what vertex, steepness and opening direction did you choose? What is the equation for your graph?

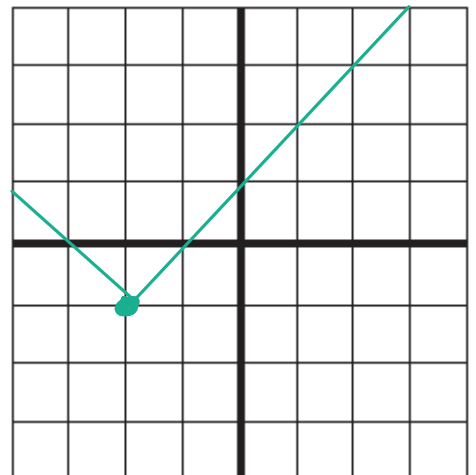14.   Graph the absolute function $y = |x + 2| - 1$. You may want to create a table of values.

vertex: (-2, -1)

translation — horizontal shift left 2
           — vertical shift down 1

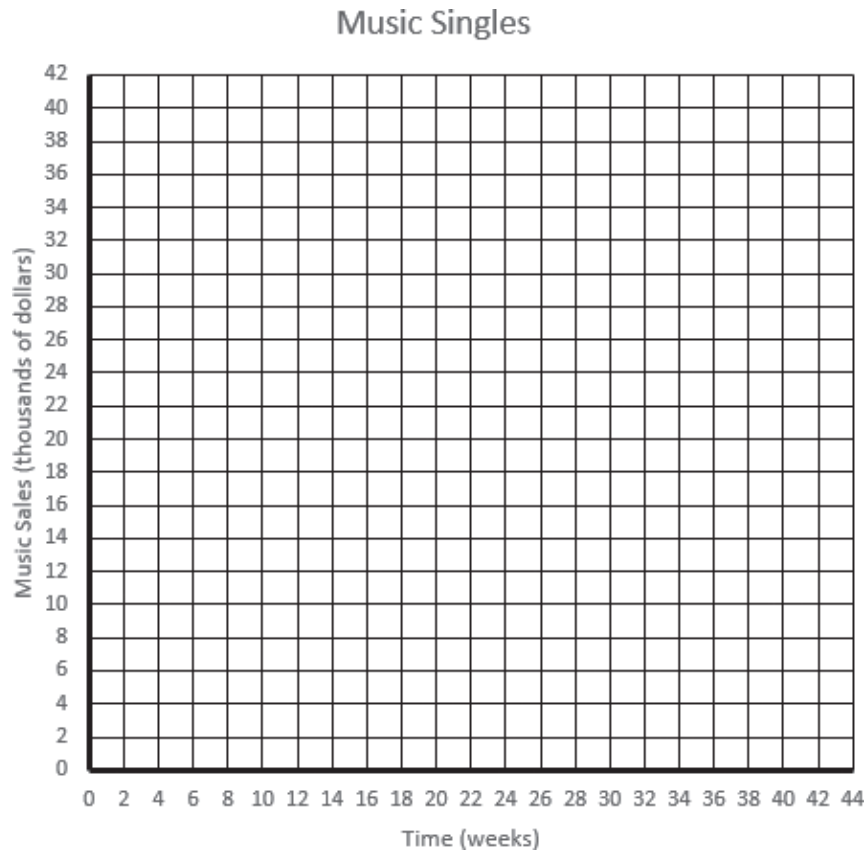Domain: (-∞, ∞)

Range: [-1, ∞)

Applications of Absolute Value Functions

15. A musical group's new single is released. Weekly sales s (in thousands) increase steadily for a while and then decrease. The function $s = -2|t - 20| + 40$, where t is the time (in weeks), represents the sales.

    Equation source: McDougal Littell

    A. Graph this function.

    Music Singles

    B. What was the maximum sales of singles sold in one week?

    C. What restraints should be placed on this problem? What is a reasonable domain and range?

16. A rainstorm begins as a drizzle, builds up to a heavy rain, and then drops back to a drizzle.
    The rate r (in inches per hour) at which it rains is given by the function $r = -\frac{1}{2}|t - 1| + \frac{1}{2}$,
    where t is the time (in hours).
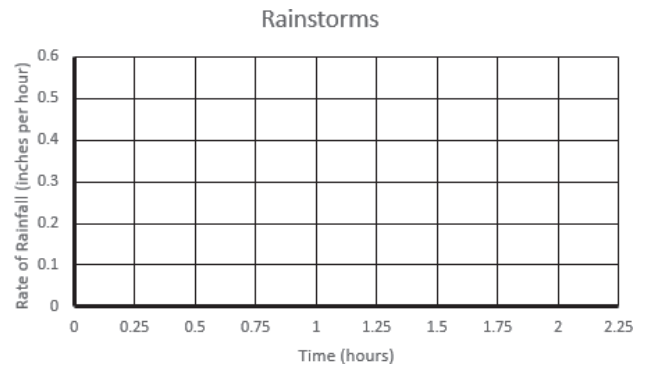
    Equation source: McDougal Littell

    A. Graph this function.

    B. How long does it rain?

    C. When does it rain the hardest?

    D. What restraints should be placed on this problem? What is a reasonable domain and range?

Rainstorms

Discussion

Determine if each statement is True or False. Be prepared to justify your answer.

17. $|a - b| = |b - a|$

18. $|ab| = |a| \cdot |b|$

19. $|a + b| = |a| + |b|$
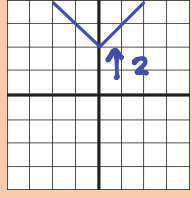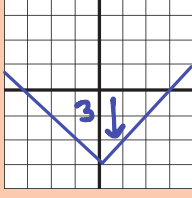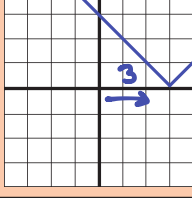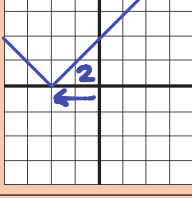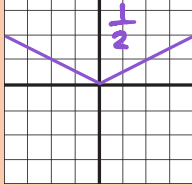
20. $|a| = b$ has two solutions.
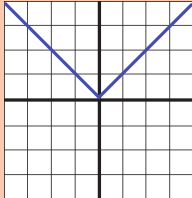
21. The equation $|a| = -5$ is equivalent to $a = 5$ or $a = -5$.

22. Complete the Lesson Summary on the following page and sketch a graph of each equation.

Lesson Summary

The absolute value function $y = |x|$ is the **parent graph** for all absolute value functions.

We can change the location of this V-shaped graph by changing the values added or subtracted from the equation. We can change the "steepness" of the graph with multiplication.

Transformation	Example with Equation	Example in Words	Example with Graph
None	$y = |x|$	This is the parent graph.	
Shift Up	$y = |x| + 2$	Shift the parent graph up __2__.	
Shift Down	$y = |x| - 3$	Shift the parent graph __down__ 3.	
Shift Right	$y = |x - 3|$	Shift the __parent__ graph to the right 3.	
Shift Left	$y = |x + 2|$	Shift the parent graph to the __left__ 2.	
Change the steepness	$y = \frac{1}{2}|x|$	Make the parent graph less steep by a factor of $\frac{1}{2}$.	

NAME: _____ PERIOD: _____ DATE: _____

Homework Problem Set

1. Suppose a musical piece calls for an orchestra to start at fortissimo (about 90 decibels), decrease in loudness to pianissimo (about 50 decibels) in four measures, and then increase back to fortissimo in another four measures. The sound level s (in decibels) of the musical piece can be modeled by the function $s = 10|m - 4| + 50$ where m is the number of measures.

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Equation source: McDougal Littell

A. Graph the function.

Sounds Levels
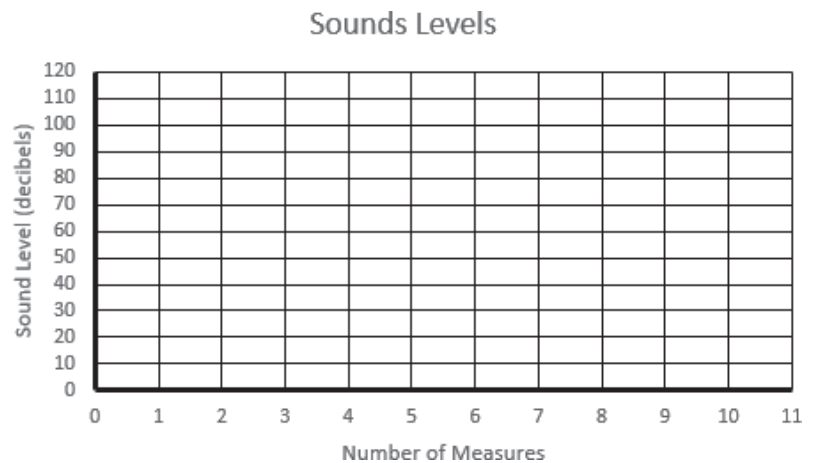
B. After how many measures should the orchestra be at the loudness of mezzo forte (about 70 decibels)?

C. Determine a reasonable domain and range for this situation.

Graph each equation and state the domain, range and vertex.

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2. $y = |x - 2| - 4$

right 2   down 4

Domain: $(-\infty, \infty)$

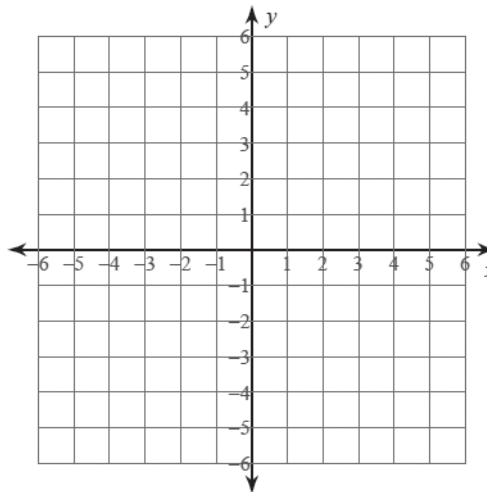Range: $[-4, \infty)$

Vertex: $(2, -4)$

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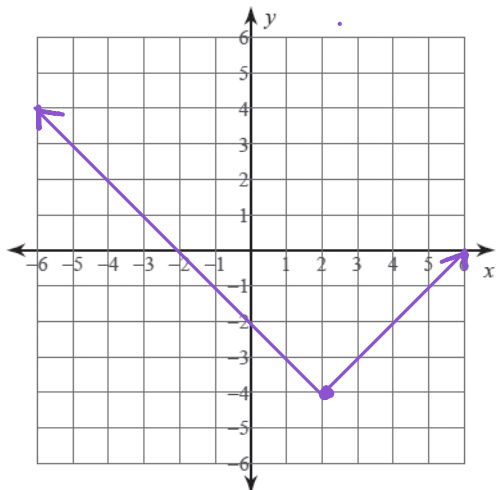
3. $y = |x + 1|$
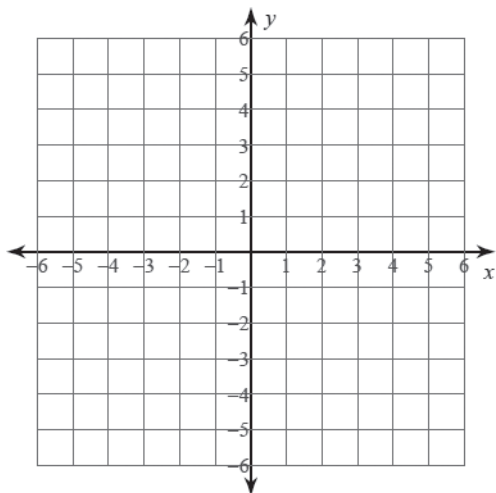
Domain:

Range:

Vertex:

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4. $y = |x| + 1$

Domain:
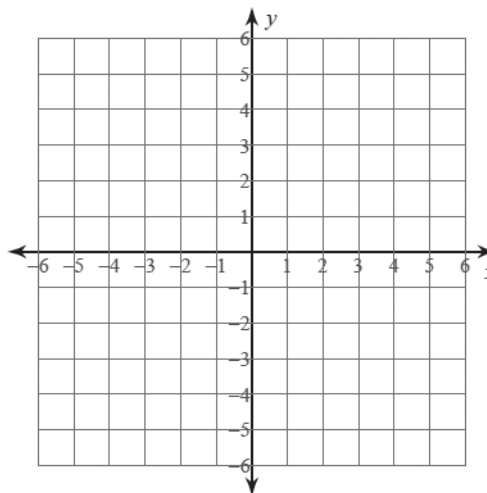
Range:

Vertex:

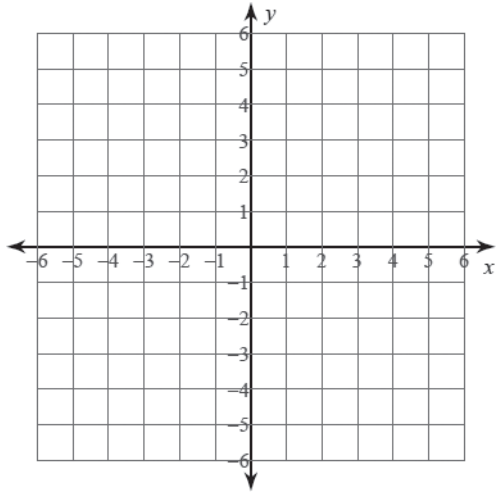5. $y = |x| + 2$

Domain:

Range:

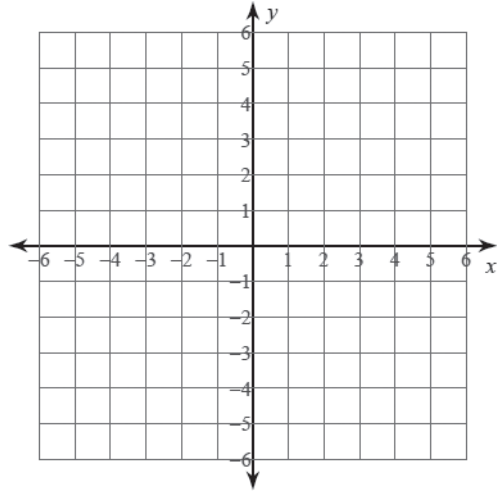Vertex:

6. $y = |x + 2|$

   Domain:

   Range:

   Vertex:

7. $y = |x + 1| + 3$

   Domain:

   Range:

   Vertex:
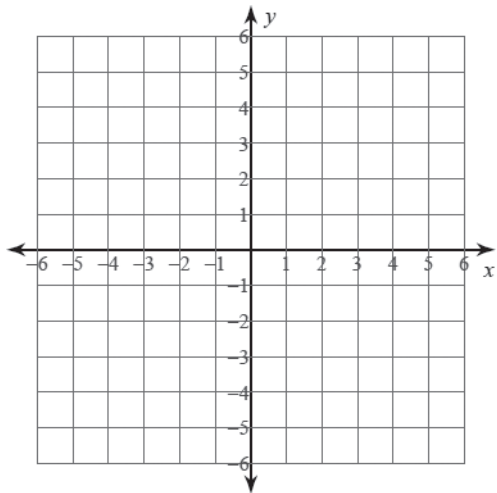
8. $y = -|x - 2| - 2$

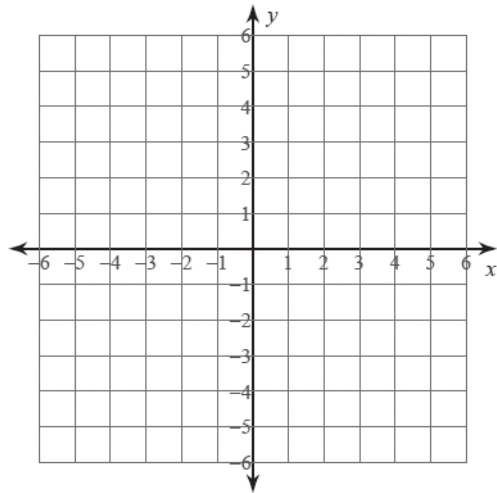   Domain:

   Range:

   Vertex:

9. $y = -|x + 1| + 4$ ← up 4

   left 1

   reflect over x-axis
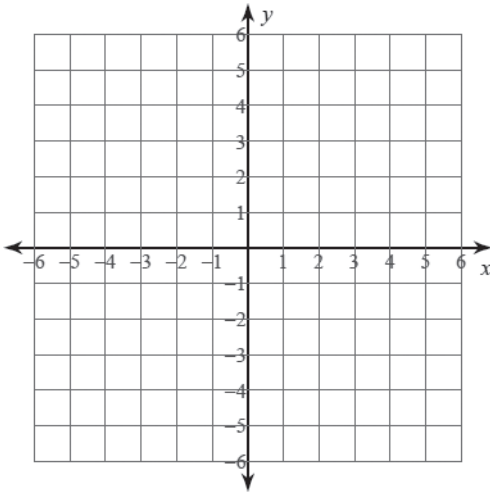
   Domain:

   Range:

   Vertex:

10. $y = -|x + 4| + 2$
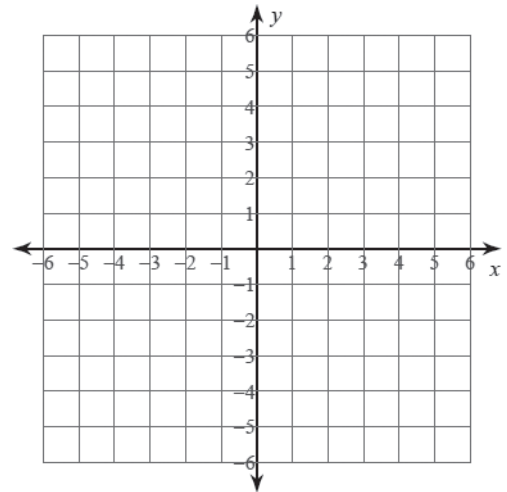
Domain:

Range:

Vertex:

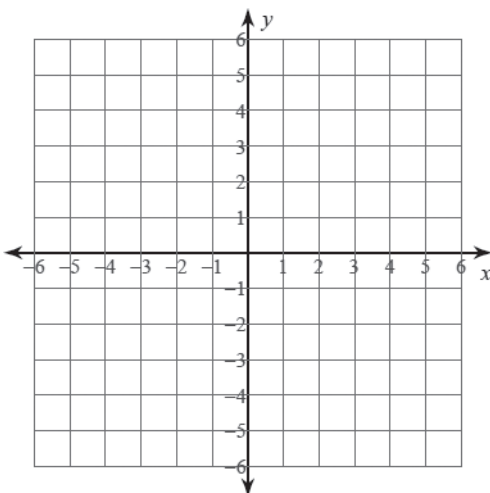11. $y = -|x - 1| + 1$

Domain:

Range:

Vertex:

12. $y = -|x - 2| + 4$

Domain:

Range:

Vertex:

13. $y = -|x - 1| - 1$

Domain:

Range:

Vertex: