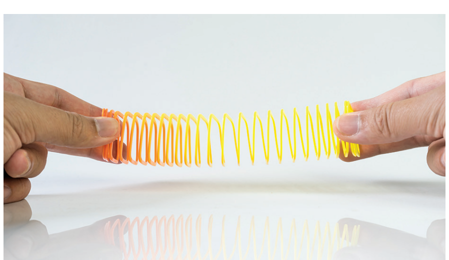
LESSON 25 Stretches and Shrinks

LEARNING OBJECTIVES

- Today I am: examining changes to an image of a hot air balloon.
- So that I can: determine if the change is vertical or horizontal.
- I'll know I have it when I can: graph the transformation of a piecewise function with a vertical stretch or a horizontal stretch.

Opening Exercise

In the last lesson, we looked briefly at horizontal and vertical stretches and shrinks. Let's take a closer look at these transformations.



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1. For each image below, decide if the original image, at the right, was shrunk or stretched. Also, determine if the transformation happened vertically or horizontally. For example, you might say the original was horizontally stretched or vertically shrunk.

vertical stretch

Image A	Image B	Image C	Image D
Vertical shrink (compress)  Same as original	Horizontal shrink 	Horizontal stretch same 	

Hot air balloon modified from © Nadia Buravleva/Shutterstock.com

2. What stayed the same in each image when compared to the original hot air balloon?

Exploration 1: Vertical Stretches and Shrinks

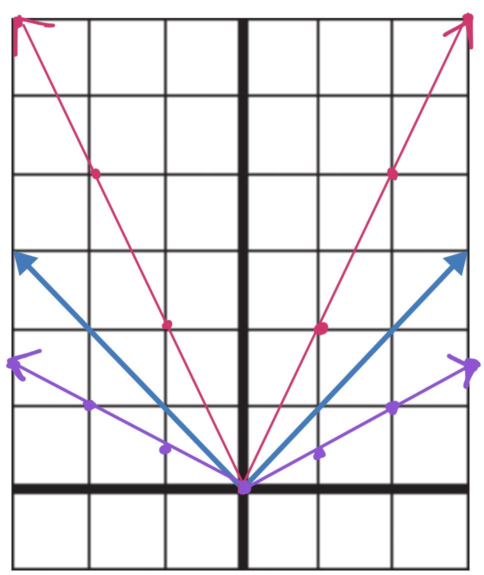
3. The graph of the absolute value function, $f(x)$, is shown below. Use the equation and the table to graph the vertical stretch and shrink of the absolute value function.

A. $g(x) = 2f(x) = 2|x|$

x	$g(x) = 2f(x) = 2\|x\|$	**(x, g(x))**
3	$g(3) = 2f(3) = 2\|3\| = 6$	(3, 6)
2	$g(2) = 2f(2) = 2\|2\| = 4$	(2, 4)
1	$g(1) = 2f(1) = 2\|1\| = 2$	(1, 2)
0	$g(0) = 2f(0) = 2\|0\| = 0$	(0, 0)
−1	$g(-1) = 2f(-1) = 2\|-1\| = 2$	(−1, 2)
−2	$g(-2) = 2\|-2\| = 4$	(−2, 4)
−3	$g(-3) = 2\|-3\| = 6$	(−3, 6)

B. $h(x) = \frac{1}{2}f(x) = \frac{1}{2}|x|$

x	$h(x) = \frac{1}{2}f(x) = \frac{1}{2}\|x\|$	**(x, g(x))**
3	$h(3) = \frac{1}{2}f(3) = \frac{1}{2}\|3\| = \frac{3}{2}$	$\left(3, \frac{3}{2}\right)$
2	$h(2) = \frac{1}{2}f(2) = \frac{1}{2}\|2\| = 1$	(2, 1)
1	$h(1) = \frac{1}{2}f(1) = \frac{1}{2}\|1\| = \frac{1}{2}$	$(1, \frac{1}{2})$
0	$h(0) = \frac{1}{2}\|0\| = 0$	(0, 0)
−1	$h(-1) = \frac{1}{2}\|-1\| = \frac{1}{2}$	$(-1, \frac{1}{2})$
−2	$h(-2) = \frac{1}{2}\|-2\| = 1$	(−2, 1)
−3	$h(-3) = \frac{1}{2}\|-3\| = \frac{3}{2}$	$(-3, \frac{3}{2})$



$2f(x)$ vertical stretch by a factor of 2.

$f(x)$

$\frac{1}{2}f(x)$ vertical shrink by a factor of 1/2

* y-values are moving closer / further away from x-axis

Reflection on Exploration 1

4. Thinking about the original function, $f(x)$, what changed to produce $g(x)$? to produce $h(x)$?

The y-values changed
$(\times 2)$ or $(\times \frac{1}{2})$

5. Thinking about the original function, $f(x)$, what stayed the same when you graphed $g(x)$? $h(x)$?

The x-values.

6. Which graph, $g(x)$ or $h(x)$, was the vertical stretch? What happened to those y-values?

$g(x) \rightarrow 2f(x)$
stretch

7. Which graph, $g(x)$ or $h(x)$, was the vertical shrink? What happened to those y-values?

$h(x) \rightarrow \frac{1}{2}f(x)$
shrink.

8. Look back at the Opening Exercise. Do you still agree with the answers you gave in Exercises 1 and 2?

Exploration 2: Horizontal Stretches and Shrinks

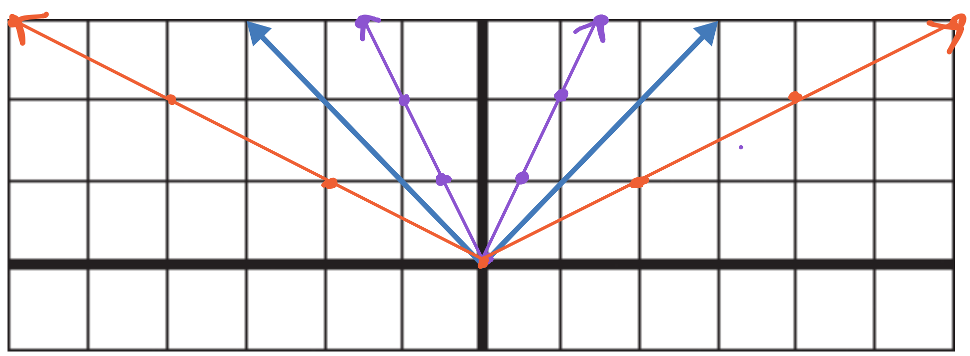
9. The graph of the absolute value function, $f(x)$, is shown below. Use the equation and the table to graph the horizontal stretch and shrink of the absolute value function.

A. $j(x) = f(2x) = \lvert 2x \rvert$

x	$j(x) = f(2x) = \lvert 2x \rvert$	**(x, g(x))**
$\frac{3}{2}$	$j\left(\frac{3}{2}\right) = f\left(2\cdot\frac{3}{2}\right) = \left\lvert 2\cdot\frac{3}{2}\right\rvert = 3$	$\left(\frac{3}{2}, 3\right)$
1	$j(1) = f(2\cdot 1) = \lvert 2\cdot 1\rvert = 2$	$(1, 2)$
$\frac{1}{2}$	$j\left(\frac{1}{2}\right) = \left\lvert 2\cdot\frac{1}{2}\right\rvert = 1$	$\left(\frac{1}{2}, 1\right)$
0	$j(0) = \lvert 2\cdot 0\rvert = 0$	$(0, 0)$
$-\frac{1}{2}$	$j\left(-\frac{1}{2}\right) = \left\lvert 2\cdot -\frac{1}{2}\right\rvert = 1$	$\left(-\frac{1}{2}, 1\right)$
-1	$j(-1) = \lvert 2\cdot -1\rvert = 2$	$(-1, 2)$
$-\frac{3}{2}$	$j\left(-\frac{3}{2}\right) = \left\lvert 2\cdot -\frac{3}{2}\right\rvert = 3$	$\left(-\frac{3}{2}, 3\right)$

B. $k(x) = f\left(\frac{1}{2}x\right) = \left\lvert \frac{1}{2}x \right\rvert$

x	$k(x) = f\left(\frac{1}{2}x\right) = \left\lvert \frac{1}{2}x \right\rvert$	**(x, g(x))**
6	$k(6) = f\left(\frac{1}{2}\cdot 6\right) = \left\lvert \frac{1}{2}\cdot 6\right\rvert = 3$	(6, 3)
4	$k(4) = \left\lvert \frac{1}{2}\cdot 4\right\rvert = 2$	(4, 2)
2	$k(2) = \left\lvert \frac{1}{2}\cdot 2\right\rvert = 1$	(2, 1)
0	$k(0) = \left\lvert \frac{1}{2}\cdot 0\right\rvert = 0$	(0, 0)
-2	$k(-2) = \left\lvert \frac{1}{2}\cdot -2\right\rvert = 1$	$(-2, 1)$
-4	$k(-4) = \left\lvert \frac{1}{2}\cdot -4\right\rvert = 2$	$(-4, 2)$
-6	$k(-6) = \left\lvert \frac{1}{2}\cdot -6\right\rvert = 3$	$(-6, 3)$



* x-values are moving closer/further away y-axis

$\lvert 2x \rvert \rightarrow$ shrink

(horizontal)

$\left\lvert \frac{1}{2}x \right\rvert \rightarrow$ stretch

Reflection on Exploration 2

10. Thinking about the original function, $f(x)$, what changed to produce $j(x)$? to produce $k(x)$?

x-values changed

11. Thinking about the original function, $f(x)$, what stayed the same when you graphed $j(x)$? $k(x)$?

y-values

12. Which graph, $j(x)$ or $k(x)$, was the horizontal stretch? What happened to those x-values?

$$k(x) = f(\tfrac{1}{2}x)$$
$$= \left|\tfrac{1}{2}x\right|$$

twice further away from y-axis

13. Which graph, $j(x)$ or $k(x)$, was the horizontal shrink? What happened to those x-values?

$$j(x) = f(2x)$$
$$= |2x|$$

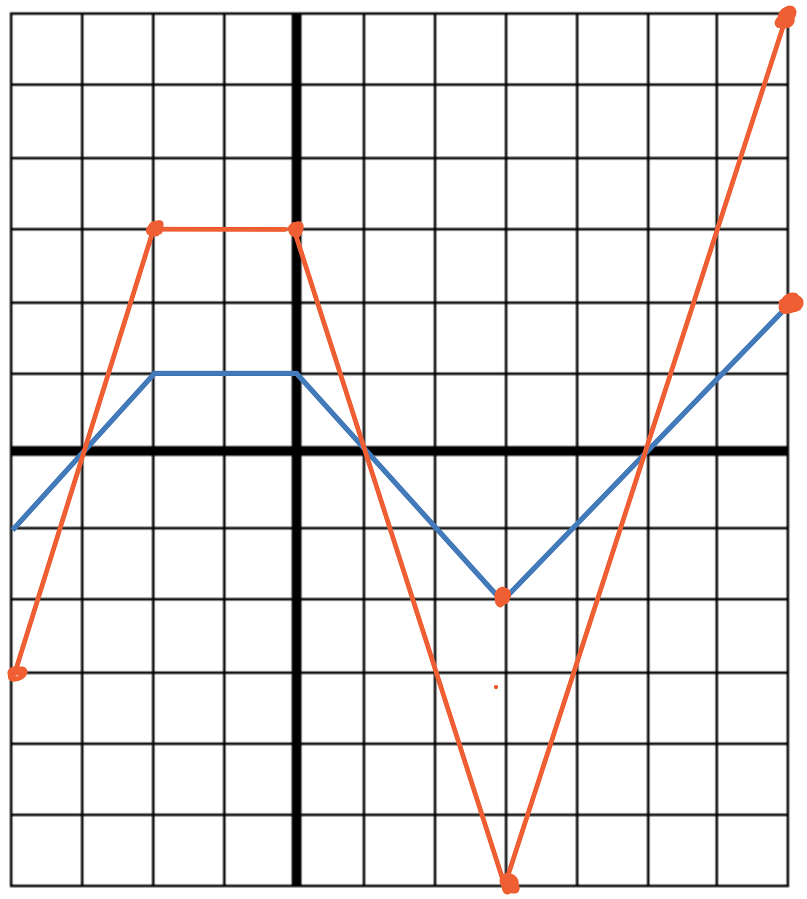
half the distance away from y-axis

Exploration 3: Horizontal and Vertical Stretches and Shrinks with Piecewise Functions

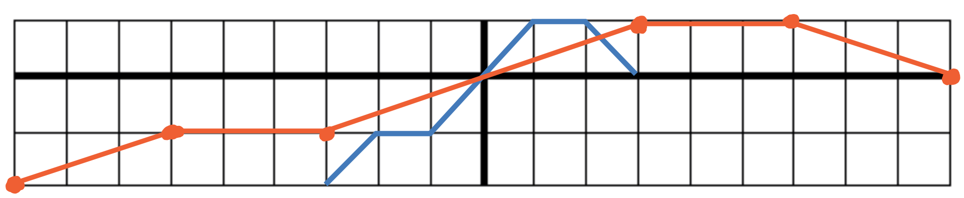
14. A piecewise function, $f(x)$, is shown at the right. Draw a new function, $g(x) = 3f(x)$.

vertical stretch by a factor of 3

(x-axis)

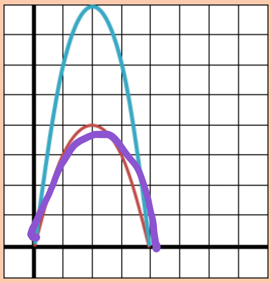
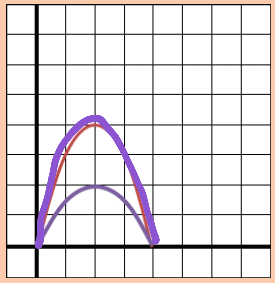
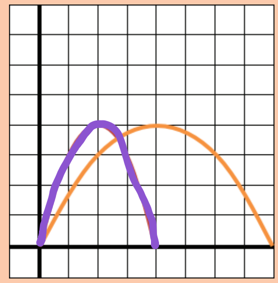
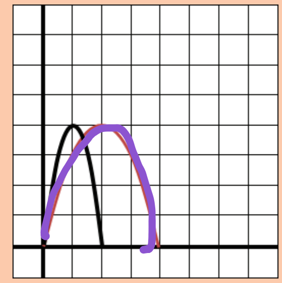


15. A piecewise function, $f(x)$, is shown below. Draw a new function, $k(x) = f\left(\frac{1}{3}x\right)$.



Horizontal stretch with a factor of 3

Lesson Summary

			
Vertical Stretch	Vertical Shrink	Horizontal Stretch	Horizontal Shrink
$v(x) = a \cdot f(x)$		$h(x) = f(a \cdot x)$	
$a > 1$	$0 < a < 1$	* $0 < a < 1$	* $a > 1$
y-values are multiplied by a		x-values are divided by a	
Coordinate points get further from the x-axis.	Coordinate points get closer to the x-axis.	Coordinate points get further from the y-axis.	Coordinate points get closer to the y-axis.

$$y = x^2$$

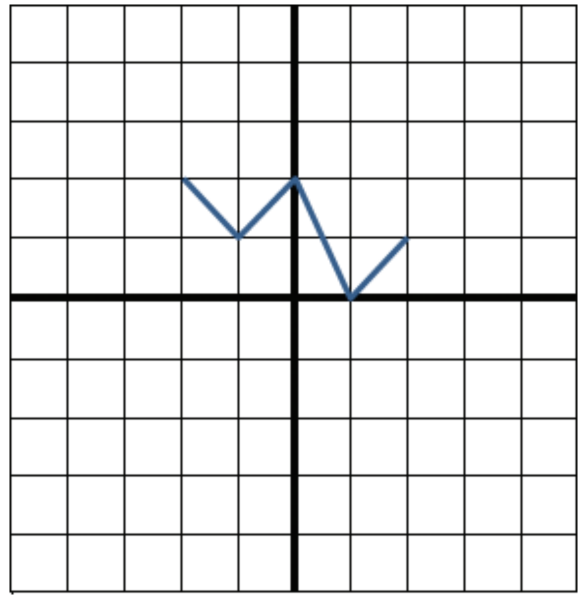
$$y = -2(x-3)^2 + 4$$

- Horizontal shift right 3
- Reflect over x-axis
- vertical stretch w/ factor of 2
- vertical shift up 4

NAME: ______________________ PERIOD: __________ DATE: __________

Homework Problem Set

For Problems 1–3, use the graph given at the right to complete the tables. Then sketch each function.



Parent $y = f(x)$	
x	y
-2	2
-1	1
0	2
1	0
2	1

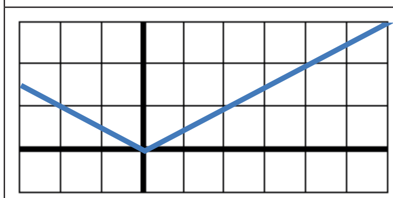
1. $y = 2f(x)$	
x	y

2. $y = f(2x)$	
x	y

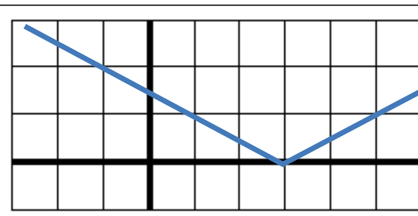
3. $y = f(\frac{1}{2}x)$	
x	y

4. Which graph below is showing the transformation $g(x) = \frac{1}{2}f(x-3)$ for the absolute value parent graph.

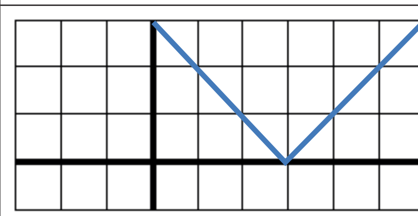
A.



B.



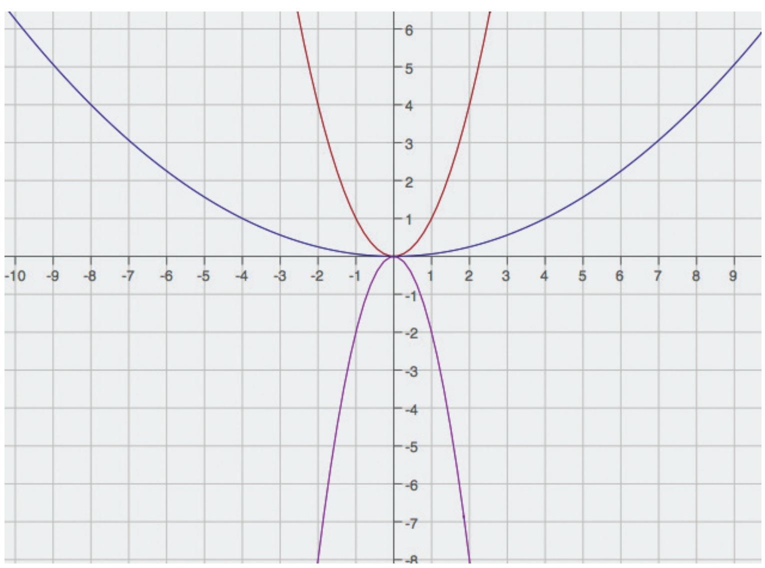
C.



5. For the images of the butterflies below, tell what transformation was done to each image.

Original	Image A	Image B
		

6. The red curve is the parent graph. For the other two graphs shown, identify the equation that matches the curve. Explain how you determined which equation goes with each graph.

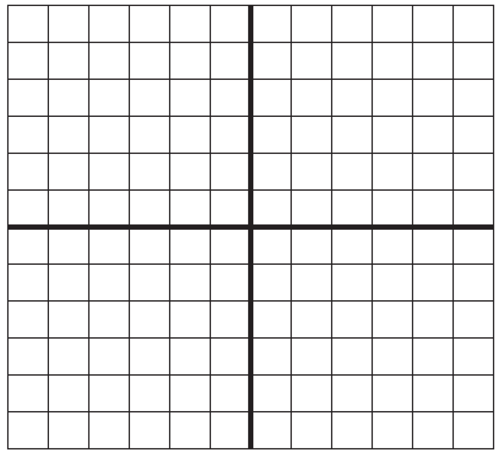


A. $g(x) = \left(\frac{1}{4}x\right)^2$

B. $h(x) = -2x^2$

7. Graph the functions below in the same coordinate plane. Be sure to clearly label the functions.

   A. $f(x) = |x|$ B. $g(x) = 2|x|$ C. $h(x) = |3x|$ D. $k(x) = 2|3x|$



8. Explain how the graphs of $g(x) = 2|x|$ and $h(x) = |3x|$ are related.

9. Write a function, g, in terms of another function, f, such that the graph of g is a vertical shrink of the graph f by a factor of 0.75.

10. A teacher wants the students to write a function based on the parent function $f(x) = \sqrt[3]{x}$. The graph of f is stretched vertically by a factor of 4 and shrunk horizontally by a factor of $\frac{1}{3}$. Mike wrote $g(x) = 4\sqrt[3]{3x}$ as the new function, while Lucy wrote $h(x) = 3\sqrt[3]{4x}$. Which one is correct? Justify your answer.

Spiral REVIEW—Describing Transformations

For each equation below, describe the transformations of the parent graph, $f(x)$.

11. $g(x) = -f(x)$

12. $h(x) = f(x+4) - 1$

13. $j(x) = 2f(x) + 1$

14. $k(x) = \frac{1}{4}f(3x)$

15. $m(x) = f(3x+2)$

16. $n(x) = f(-x) + 5$