LESSON 27

Marbleslides with Exponential Functions

LEARNING OBJECTIVES

- Today I am: using a Desmos activity, *Marbleslides-Exponentials.*
- So that I can: explore how transformations appear in exponential functions.
- I'll know I have it when I can: match equations of exponential functions to their graphs.

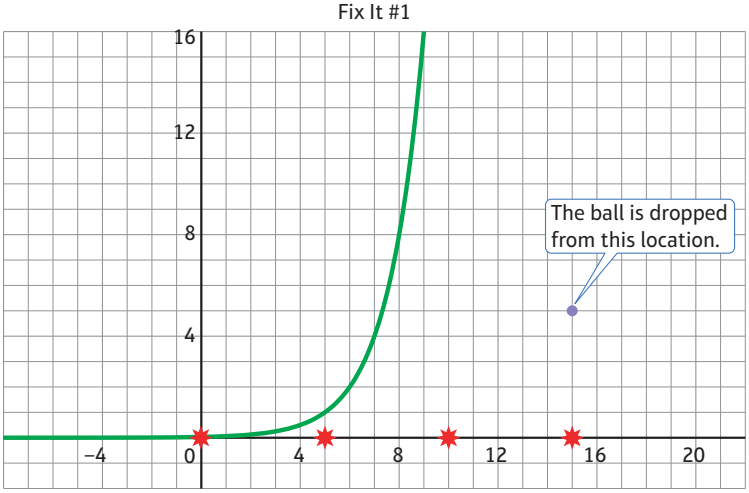
Opening Exercise

Below is a graph based on the Desmos activity *Marbleslides-Exponentials*. The function $y = 2^{(x-5)}$ is the equation of the curve shown. The challenge is to change one of the numbers in the equation so that when the ball is dropped onto the curve, it will roll along the curve and hit all of the stars.

1. Discuss with your group which number should be changed and what its new value should be.

2. Sketch where you believe the new equation will be located.

Fix It #1



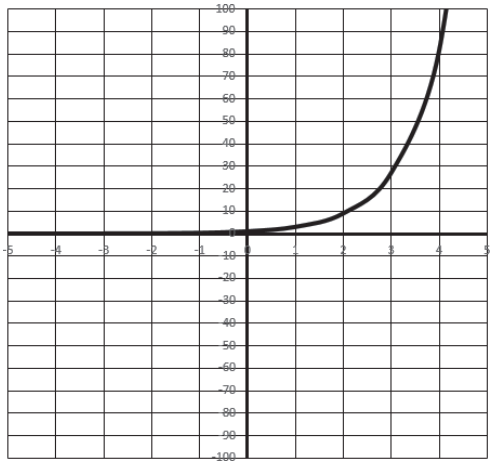
You will need: class code for Desmos *Marbleslides: Exponentials*, Chromebook

3. Go to https://student.desmos.com and type in the class code your teacher gives you. Complete all 24 tasks.

Using your experience from the Desmos activity and your work with transformations in this unit, match each graph to the correct equation. Then summarize how changes in the equation change the graph of an exponential function.

Equations $y =$

A. $f(x) = 3^x + 10$	B. $f(x) = -3^{-x}$	C. $f(x) = 3^{x-2}$
D. $f(x) = 3^x - 10$	E. $f(x) = 3^{x+2}$	F. $f(x) = 2 \cdot 3^x$
G. $f(x) = \frac{1}{2} \cdot 3^x$	H. $f(x) = 3^{-x}$ reflect over y-axis	I. $f(x) = 3^{\frac{1}{2}x}$
J. $f(x) = 3^{2x}$	K. $f(x) = -3^x$ reflect over x-axis	



Parent Graph: $y = 3^x$

$f(x) = 3^x$

$f(x+2) = 3^{x+2}$

horizontal shift left → $y = 3^{(x+2)}$

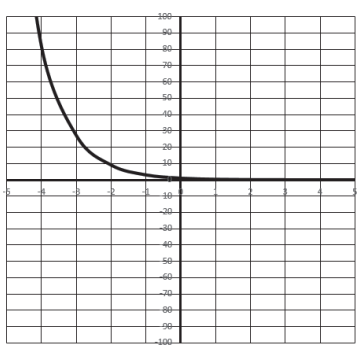
vertical shift up → $y = 3^x + 2$

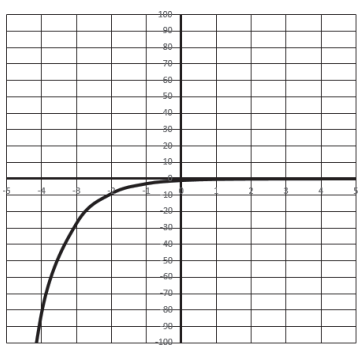
Reflections

4. Equation: H $f(x) = 3^{-x}$

5. Equation: $f(x) = -3^{-x}$

6. Equation: $f(x) = -3^x$





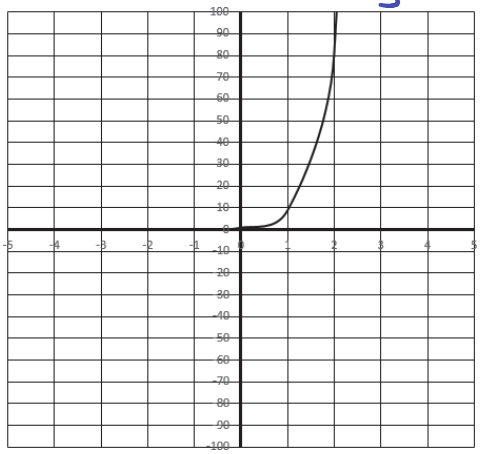
7. The equation of an exponential function shows a reflection by . . .

the negative sign next to the variable or base

Horizontal Stretches and Shrinks

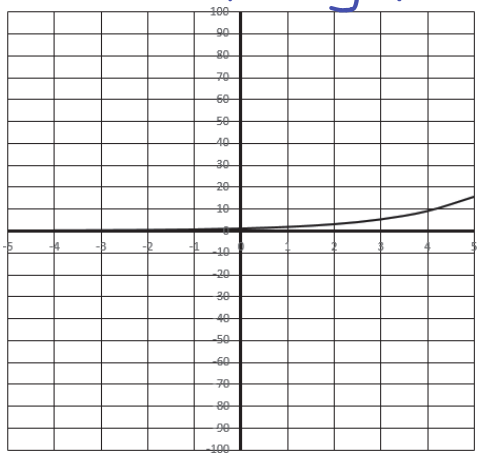
8. Equation: J $f(x) = 3^{2x}$

shrink by a factor of $\frac{1}{2}$



9. Equation: I $f(x) = 3^{\frac{1}{2}x}$

stretch by a factor of 2



10. A. The equation of an exponential function shows a horizontal stretch by . . .

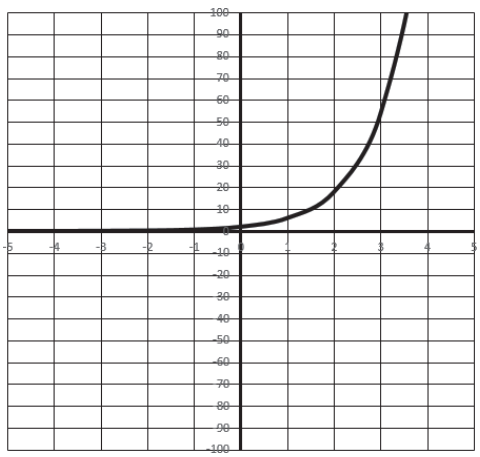
having a constant between 0 and 1

B. The equation of an exponential function shows a horizontal shrink by . . .

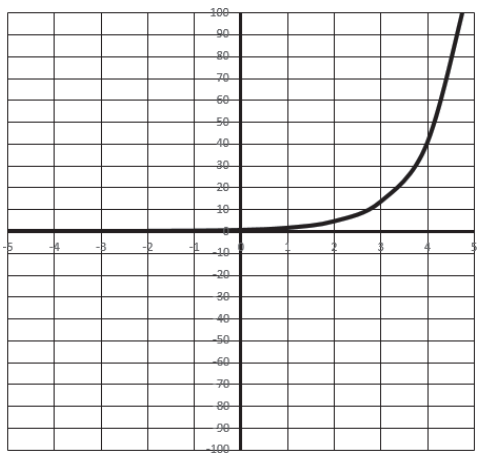
having a constant greater than 1

Vertical Stretches and Shrinks

11. Equation: F $f(x) = 2 \cdot 3^x$



12. Equation: G $f(x) = \frac{1}{2} \cdot 3^x$



13. A. The equation of an exponential function shows a vertical stretch by . . .

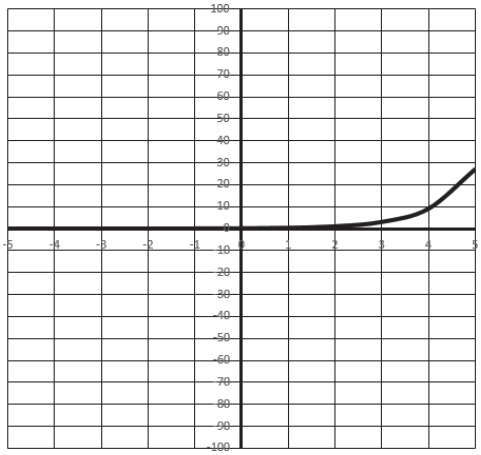
constant > 1

B. The equation of an exponential function shows a vertical shrink by . . .

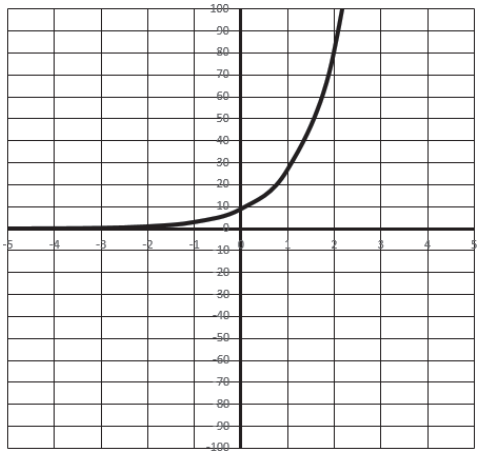
$0 <$ constant < 1

Horizontal Shifts

14. Equation: C $f(x) = 3^{x-2}$



15. Equation: E $f(x) = 3^{x+2}$



16. A. The equation of an exponential function shows a horizontal shift to the right by . . .

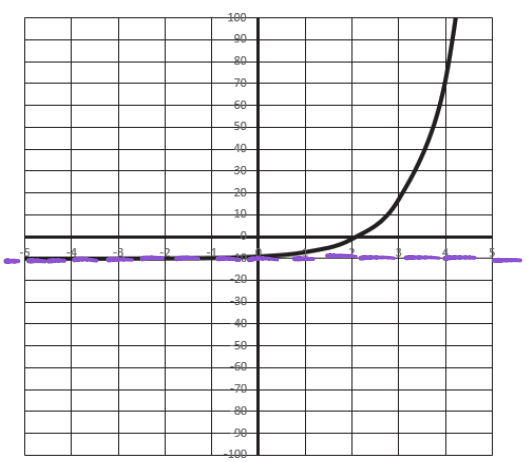
subtract from X

B. The equation of an exponential function shows a horizontal shift to the left by . . .

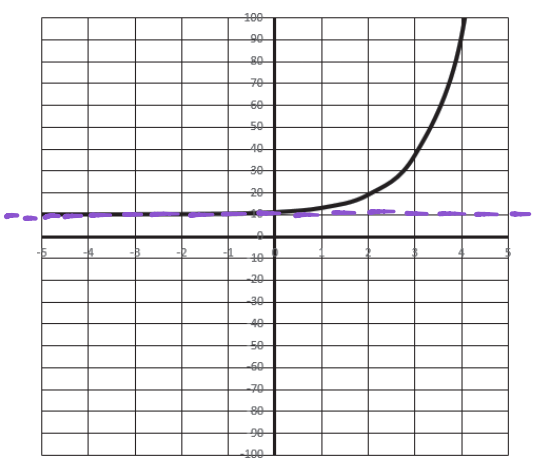
Add to X

Vertical Shifts

17. Equation: D $f(x) = 3^{x}-10$



18. Equation: A $f(x) = 3^{x}+10$



19. A. The equation of an exponential function shows a vertical shift up by . . .

add 10 to f(x)

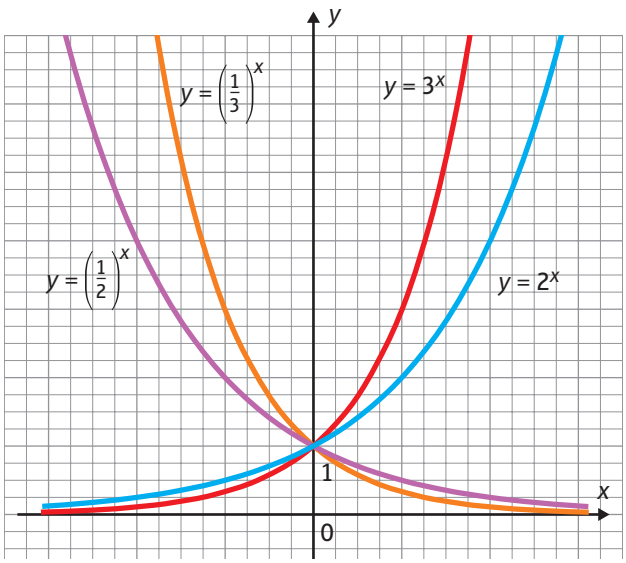
B. The equation of an exponential function shows a vertical shift down by . . .

subtract 10 from f(x)

NAME: _____ PERIOD: _____ DATE: _____

Homework Problem Set

Use the image below for Problems 1–3.



Source: Nat White, University of Wisconsin-Milwaukee

1. What do you notice about these four exponential functions? List at least three similarities or differences.

2. Write an exponential function that would fall between $y = 3^x$ and $y = 2^x$. How do you know?

3. Write an exponential function that would fall between $y = \left(\frac{1}{2}\right)^x$ and $y = \left(\frac{1}{3}\right)^x$. How do you know?

Pay It Forward and Exponential Growth

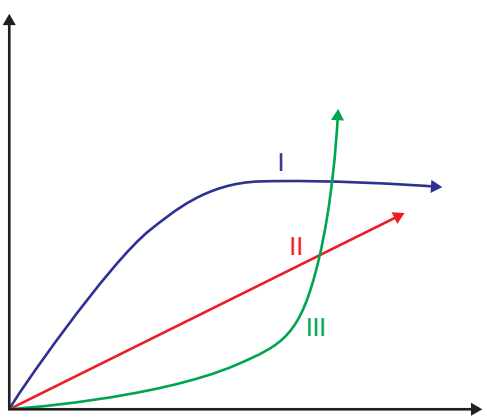
In the popular book and movie, *Pay It Forward*, 12-year-old Trevor McKinney gets a challenging assignment from his social studies teacher. *Think of an idea for world change, and put it into practice!* Trevor came up with an idea that fascinated his mother, his teacher, and his classmates.

He suggested that he would do something really good for three people. Then when they would ask how they can pay him back for the good deeds, he would tell them to "pay it forward"— each doing something good for three other people.

Trevor figured that those three people would do something good for a total of nine others. Those nine would do something good for 27 others, and so on. He was sure that before long there would be good things happening to billions of people all around the world.

Source: Emily Burkett on Algebra 4 All

4. Which of the graphs below do you think is most likely to represent the pattern by which the number of people receiving Pay It Forward good deeds increase as the process continues over time? Explain your choice.



5. Use the table below to look for patterns in the number of good deeds done and the total number of good deeds done.

Set of People	Number of good deeds done by this set of people	Total number of good deeds done
1st set—Trevor alone	3	3
2nd set—People Trevor helped	9	12 (3 from Trevor and 9 from the 2nd set of people)
3rd set		
4th set		
5th set		
6th set		

6. Use Trevor's idea to create an equation to show how many good deeds would occur for each set of people (not the total number of good deeds).

7. Using your equation from Problem 6, how many good deeds would be done by the fourth set of people? Does your answer agree with the values in the table?

8. How many sets of people would be needed to get to 1,000,000 good deeds? Explain your thinking.

9. Elaine came up with the formula below to find the **total** number of good deeds. Use Elaine's formula to see if it agrees with your values in the table.

$$f(x) = \frac{3^{x+1} - 3}{2}, \text{ where } x \text{ is the set number}$$

10. Determine how many sets of people it would take for 1,000,000 **total** good deeds to be achieved.

11. Trevor says it wouldn't take long to reach a billion people. Do you agree with Trevor? Suppose each good deed takes at least a week to complete, how long would it take to complete a billion good deeds?