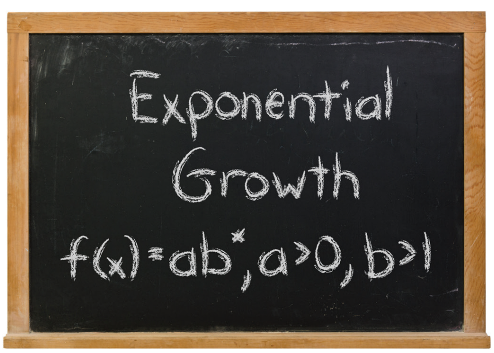
LESSON 28

Focus on Exponential Transformations

LEARNING OBJECTIVES

- Today I am: graphing a series of exponential functions using tables of values.
- So that I can: see that the pattern of transformations works with exponential functions, too.
- I'll know I have it when I can: graph $y = \frac{1}{2} \cdot 6^{x+1} - 1$.

In the last lesson, we examined some exponential transformations with the Desmos activity *Marbleslides*. To really get a good grasp of exponential functions and their transformations, we need to do a deeper investigation into how their coordinate points change.



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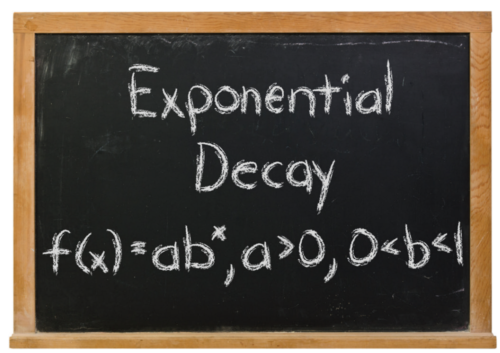

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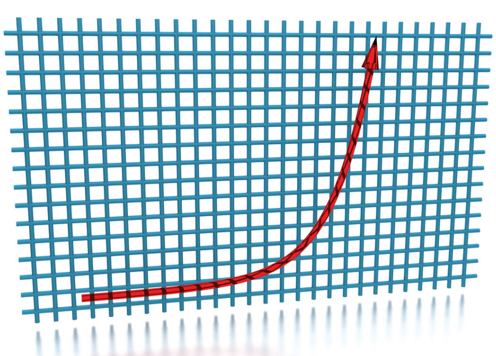
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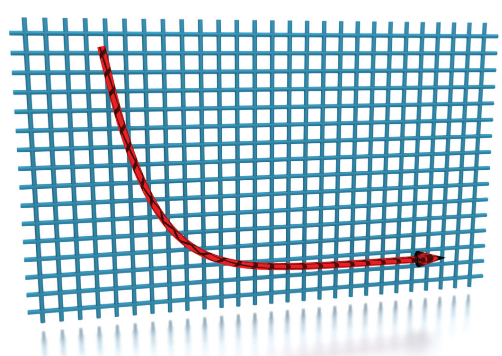
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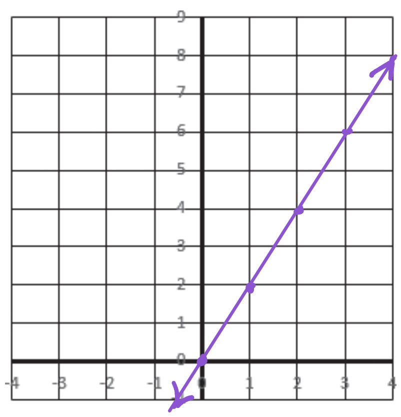
Opening Exercise—Linear Compared to Exponential Functions

Complete the tables of values and then graph the following functions. Answer the questions that follow.

asymptote

1. $f(x) = 2x$

x	y
-2	-4
-1	-2
0	0
1	2
2	4
3	6

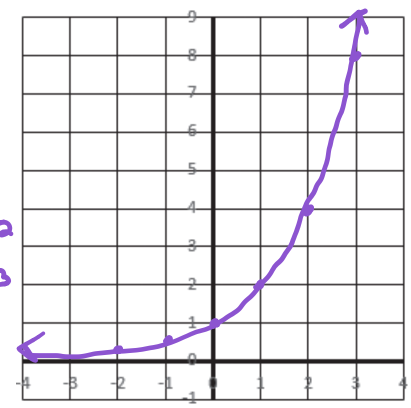


2. $g(x) = 2^x$

x	y
-2	$1/4$
-1	$1/2$
0	1
1	2
2	4
3	8

2^2

2^3



Discussion

3. How is the **graph** in Exercise 1 different from the one in Exercise 2?

4. How is the **equation** in Exercise 1 different from the one in Exercise 2?

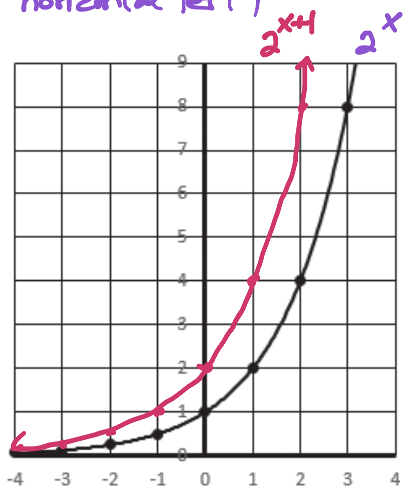
5. Exercise 2 is the parent graph for exponential functions in the form $f(x) = b^x$, where "b" represents the base. What is b in this problem? Does it show exponential decay or exponential growth? How do you know?

Exploration 1: Graphing $f(x) = b^{x-h} + k$, where $b = 2$

Now let's look at transformations of the parent graph, $g(x) = 2^x$. The parent graph is shown on each grid.

6. $h(x) = 2^{x+1}$ horizontal left 1

x	y
-2	1/2
-1	1
0	2
1	4
2	8



$x-(-1)$
3^{x+1}

The generic equation is $y = b^{x-h} + k$, what is . . .

$h =$ -1 $k =$ 0

Describe what changed from the graph of $g(x) = 2^x$:

Horizontal shift left 1

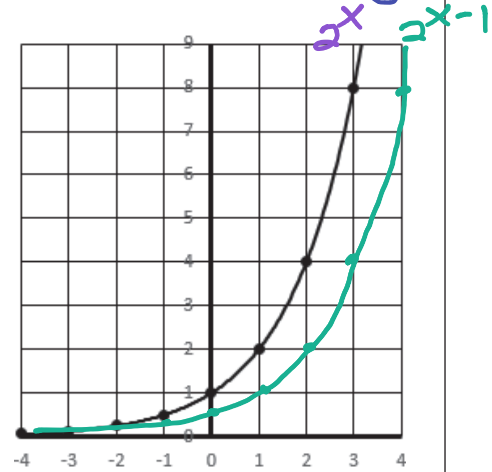
Choose one point on the graph of $g(x) = 2^x$ and show where that point was transformed on the graph above.

g(x) h(x)

(1, 2) $\xrightarrow{-1}$ (0, 2)

7. $i(x) = 2^{x-1}$ horizontal shift right 1

x	y
-2	
-1	
0	
1	
2	
3	
4	



The generic equation is $y = b^{x-h} + k$, what is . . .

$h =$ 1 $k =$ 0

Describe what changed from the graph of $g(x) = 2^x$:

Horizontal shift right 1

Choose one point on the graph of $g(x) = 2^x$ and show where that point was transformed on the graph above.

g(x) i(x)

(1, 2) $\xrightarrow{+1}$ (2, 2)

8. In both Exercises 6 and 7, the functions, including the parent function, get closer and closer to the x-axis. Will any of these functions touch the x-axis? How do you know?

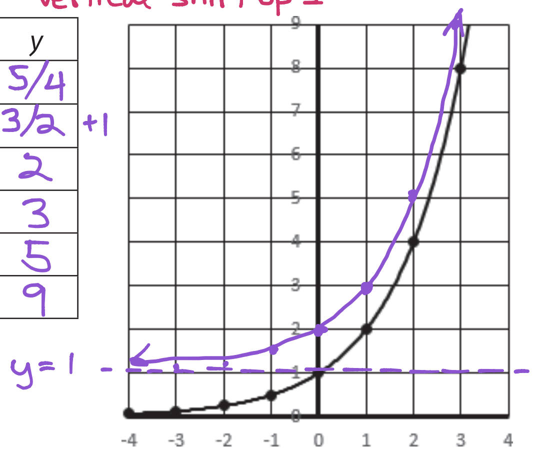
2^{x+1} $f(x+1)$

9. $j(x) = 2^x + 1$ $f(x)+1$

vertical shift up 1

x	y
-2	5/4
-1	3/2
0	2
1	3
2	5
3	9

+1



The generic equation is $y = b^{x-h} + k$, what is . . .

$h =$ 0 $k =$ 1

Describe what changed from the graph of $g(x) = 2^x$:

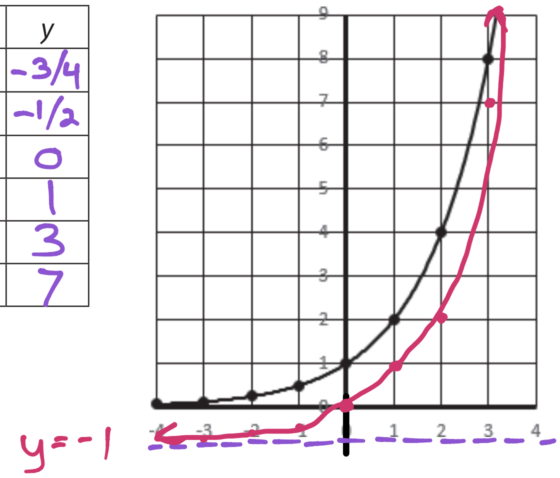
shift up 1 unit

Choose one point on the graph of $g(x) = 2^x$ and show where that point was transformed on the graph above.

$g(x) \rightarrow (1, 2)$
$+1$
$j(x) \rightarrow (1, 3)$

10. $k(x) = 2^x - 1$

x	y
-2	-3/4
-1	-1/2
0	0
1	1
2	3
3	7



The generic equation is $y = b^{x-h} + k$, what is . . .

$h =$ 0 $k =$ -1

Describe what changed from the graph of $g(x) = 2^x$:

shift down 1

Choose one point on the graph of $g(x) = 2^x$ and show where that point was transformed on the graph above.

$g(x) \rightarrow (1, 2)$
$1-$
$k(x) \rightarrow (1, 1)$

Reflection

11. If you see a function $f(x) = b^x$, what shape do you expect the graph to be?

12. If $h > 0$, what happens to the graph of the exponential function?

13. If $h < 0$, what happens to the graph of the exponential function?

14. If $k > 0$, what happens to the graph of the exponential function?

15. If $k < 0$, what happens to the graph of the exponential function?

16. For Exercises 9 and 10, the parent graph gets close to the x-axis or $y = 0$, but doesn't cross it.

A. What line does $j(x) = 2^x + 1$ get close to?

B. What line does $k(x) = 2^x - 1$ get close to?

An ***asymptote*** is a line that a function gets closer and closer to, but never reaches. Exponential functions have horizontal asymptotes of the form $\boxed{y = \text{number}}$, where the number is the value that the function can never reach.

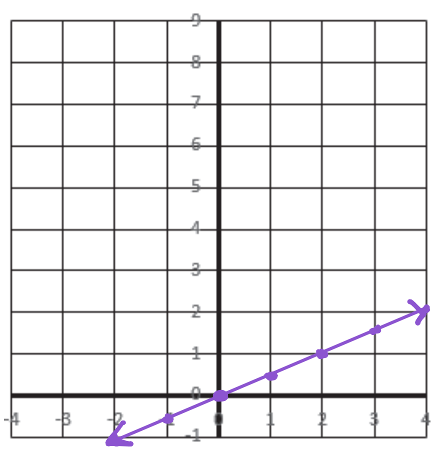
The answers in Exercise 16 are asymptotes of the two functions, $j(x) = 2^x + 1$ and $k(x) = 2^x - 1$. The parent graph, $g(x) = 2^x$, has a horizontal asymptote of $y = 0$.

Linear Compared to Exponential Functions

Complete the tables of values and then graph the following functions. Answer the questions that follow.

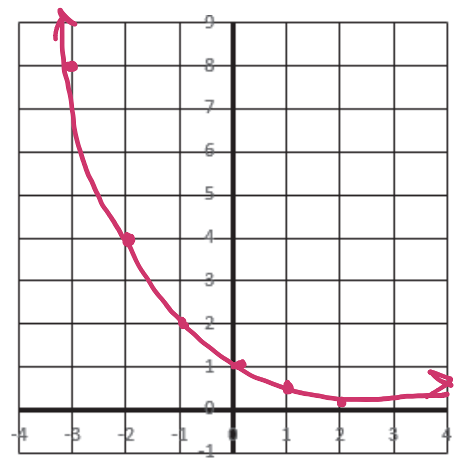
17. $f(x) = \frac{1}{2}x$ $m = \frac{1}{2}$

x	y
−3	−3/2
−2	−1
−1	−1/2
0	0
1	1/2
2	1



18. $g(x) = \left(\frac{1}{2}\right)^x = 2^{-x}$

x	y
−3	8
−2	4
−1	2
0	1
1	1/2
2	1/4



19. How is the **graph** in Exercise 17 different from the one in Exercise 18?

20. How is the **equation** in Exercise 17 different from the one in Exercise 18?

21. Exercise 18 is the parent graph for exponential functions in the form $f(x) = b^x$, where "b" represents the base. What is b in this problem? Does it show exponential decay or exponential growth? How do you know?

$y = \left(\frac{1}{2}\right)^x$ $y = 2^{-x}$

exponential decay

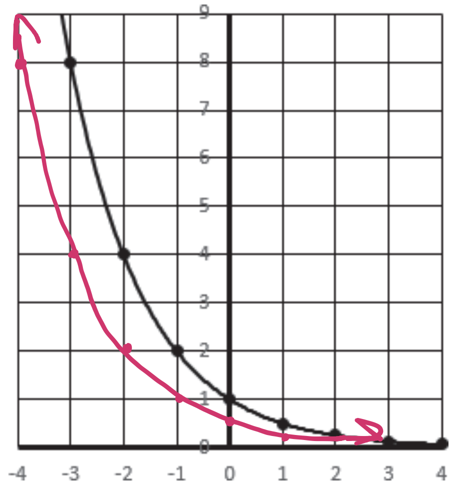
because $0 < b < 1$

Exploration 2: Graphing $f(x) = b^{x-h} + k$, where $b = \frac{1}{2}$

The parent graph $g(x) = \left(\frac{1}{2}\right)^x$ is shown on each grid.

22. $h(x) = \left(\frac{1}{2}\right)^{x+1}$ horizontal shift left 1

	x-1	y
	~~−4~~	
−4	~~−3~~	8
−3	~~−2~~	4
−2	~~−1~~	2
−1	~~0~~	1
0	~~1~~	1/2
1	~~2~~	1/4



The generic equation is $y = b^{x-h} + k$, what is . . .

$h =$ <u>-1</u> $k =$ <u>0</u>

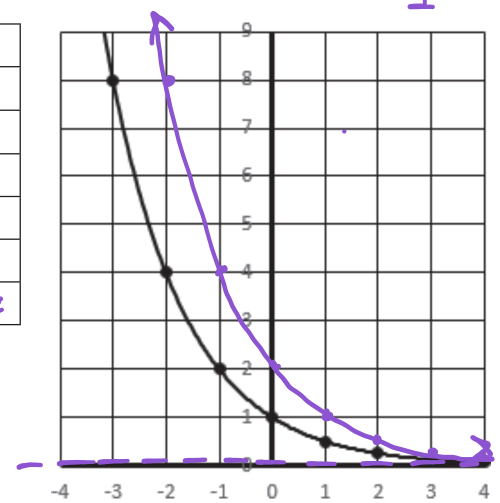
Describe what changed from the graph of $g(x) = \left(\frac{1}{2}\right)^x$:

Choose one point on the graph of $g(x) = \left(\frac{1}{2}\right)^x$ and show where that point was transformed on the graph above.

What is the asymptote for the function $h(x) = \left(\frac{1}{2}\right)^{x-1}$? $y=0$

23. $i(x) = \left(\frac{1}{2}\right)^{x-1}$ horizontal shift right 1

x	y
−2	8
−1	4
0	2
1	1
2	1/2
3	1/4



The generic equation is $y = b^{x-h} + k$, what is . . .

$h =$ <u>1</u> $k =$ <u>0</u>

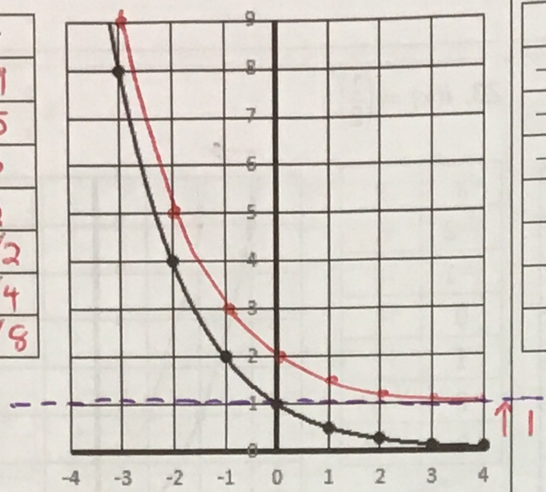
Describe what changed from the graph of $g(x) = \left(\frac{1}{2}\right)^x$:

Choose one point on the graph of $g(x) = \left(\frac{1}{2}\right)^x$ and show where that point was transformed on the graph above.

What is the asymptote for the function $h(x) = \left(\frac{1}{2}\right)^{x-1}$? $y=0$

24. $j(x) = \left(\frac{1}{2}\right)^x + 1$

x	y
−3	9
−2	5
−1	3
0	2
1	3/2
2	5/4
3	9/8



The generic equation is $y = b^{x-h} + k$, what is . . .

$h =$ 0 $k =$ 1

Describe what changed from the graph of $g(x) = \left(\frac{1}{2}\right)^x$:

vertical shift up 1

Choose one point on the graph of $g(x) = \left(\frac{1}{2}\right)^x$ and show where that point was transformed on the graph above.

g(x) → (-1, 2)

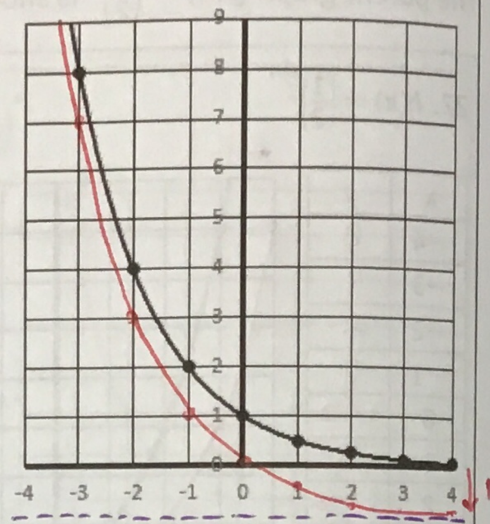
j(x) → (-1, 3)

What is the asymptote for the function $j(x) = \left(\frac{1}{2}\right)^x + 1$?

y = 1

25. $k(x) = \left(\frac{1}{2}\right)^x - 1$

x	y
−3	7
−2	3
−1	1
0	0
1	-1/2
2	-3/4
3	-7/8



The generic equation is $y = b^{x-h} + k$, what is . . .

$h =$ 0 $k =$ −1

Describe what changed from the graph of $g(x) = \left(\frac{1}{2}\right)^x$:

vertical shift down 1

Choose one point on the graph of $g(x) = \left(\frac{1}{2}\right)^x$ and show where that point was transformed on the graph above.

g(x) → (-1, 2)

k(x) → (-1, 1)

What is the asymptote for the function $k(x) = \left(\frac{1}{2}\right)^x - 1$?

y = -1

Discussion

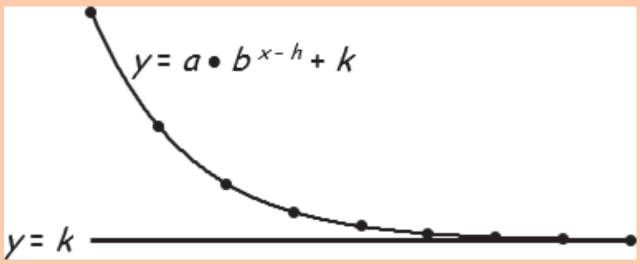
26. Look back on the Discussion questions in Exercises 10–14. Will any of your answers be different due to the change in the base of the exponential function?

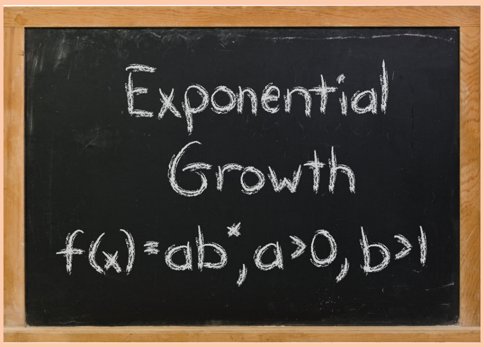
27. Sometimes exponential functions are described as having a pattern of “repeated multiplication.” Do you agree or disagree? Explain.

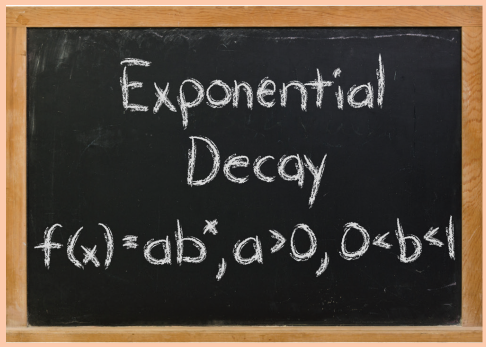
28. Sometimes linear functions are described as having a pattern of “repeated addition.” Do you agree or disagree? Explain.

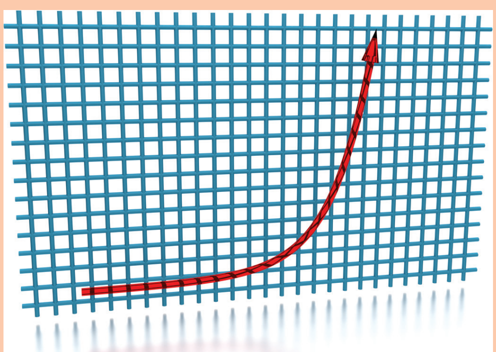
Lesson Summary

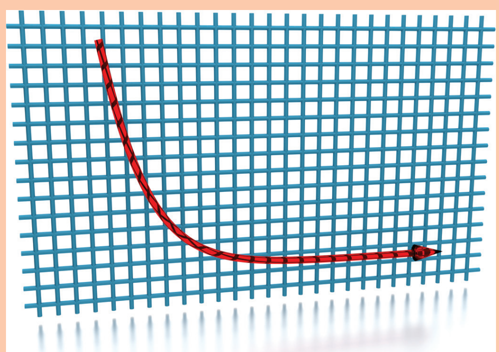
In exponential functions, an asymptote is a line that a function gets close to but never touches. The asymptote for exponential functions, $y = a \cdot b^{x-h} + k$ will be a horizontal line $y = k$.











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Homework Problem Set

Write the parent function for each of the following. Then, describe in words what the transformation would be for the new graph.

1. $f(x) = 3^{x-2}$

2. $g(x) = \left(\frac{1}{2}\right)^{x} - 4$

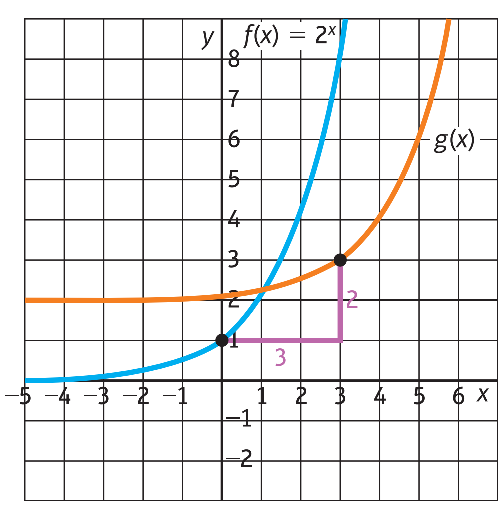
3. $h(x) = 3 \cdot 2^{x} + 1$

4. $j(x) = -5^{x+2}$

5. $k(x) = \frac{1}{3} \cdot 4^{x} + 7$

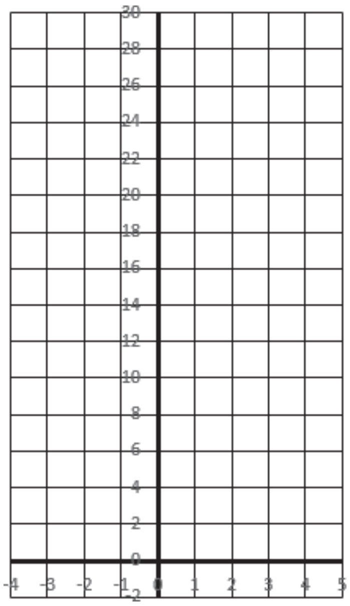
6. $l(x) = \left(\frac{2}{3}\right)^{x-9} - 10$

7. The graph for $f(x) = 2^x$ is shown below. The function $g(x)$ is also shown. What is the equation for $g(x)$?



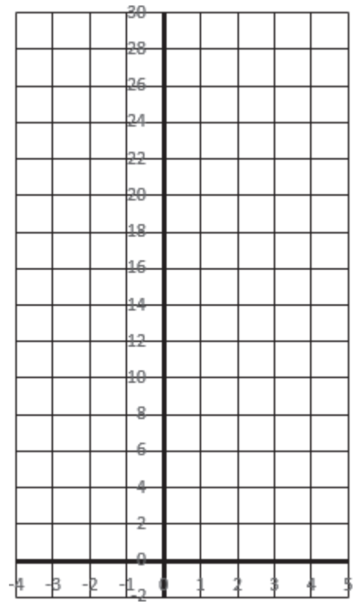
Graph each of the following exponential functions. Then state the domain and range.

8. $y = 3^{x-2}$



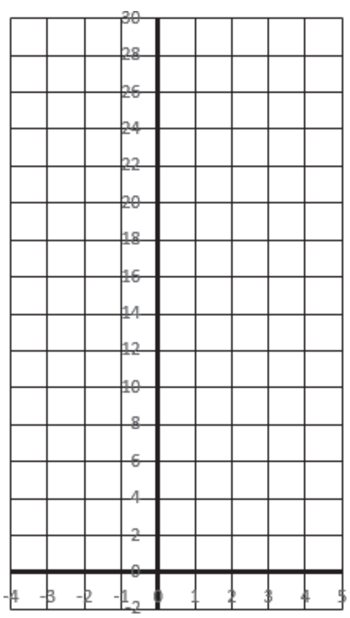
Domain:
Range:

9. $y = 3^{x} - 2$



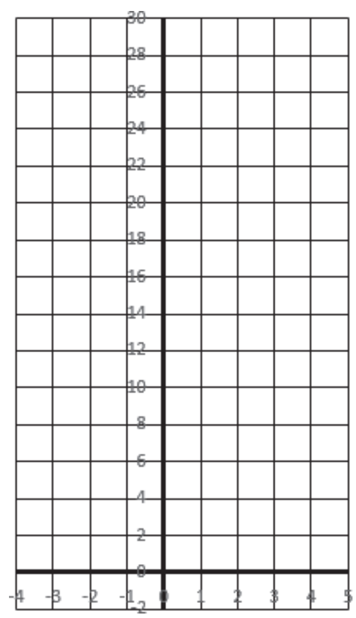
Domain:
Range:

10. $y = \frac{1}{4}^{x+2} - 1$



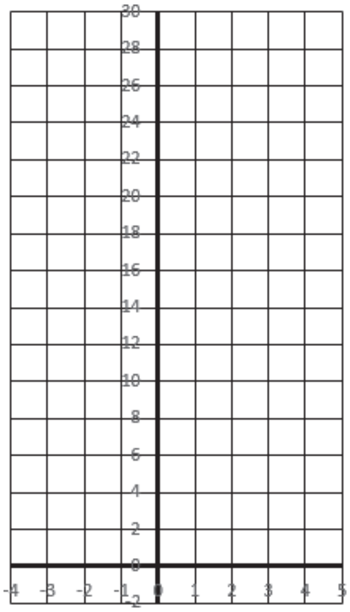
Domain:
Range:

11. $y = 2 \cdot \frac{1}{2}^{x}$



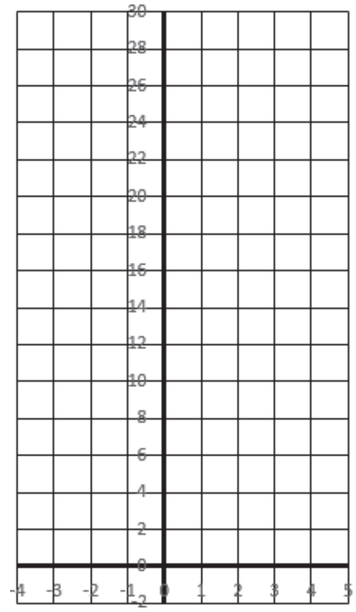
Domain:
Range:

12. $y = 4 \cdot \frac{1}{2}^{x} + 1$



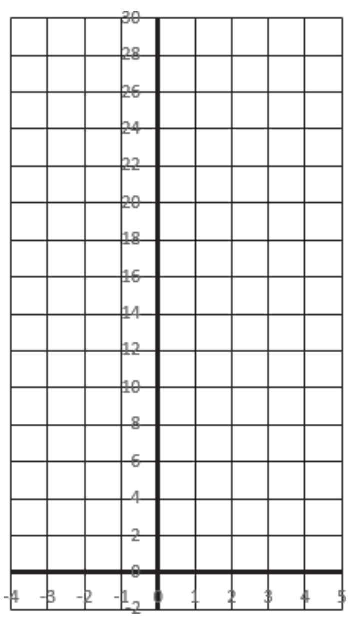
Domain:
Range:

13. $y = 5 \cdot 2^{x-1}$



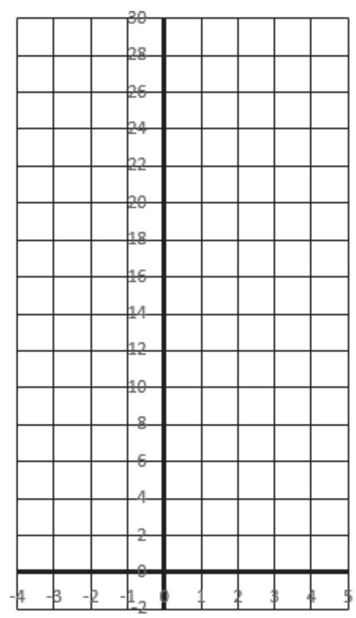
Domain:
Range:

14. $y = 4^{x+2} + 2$



Domain:
Range:

15. $y = \frac{1}{2} \cdot 6^{x+1} - 1$



Domain:
Range: