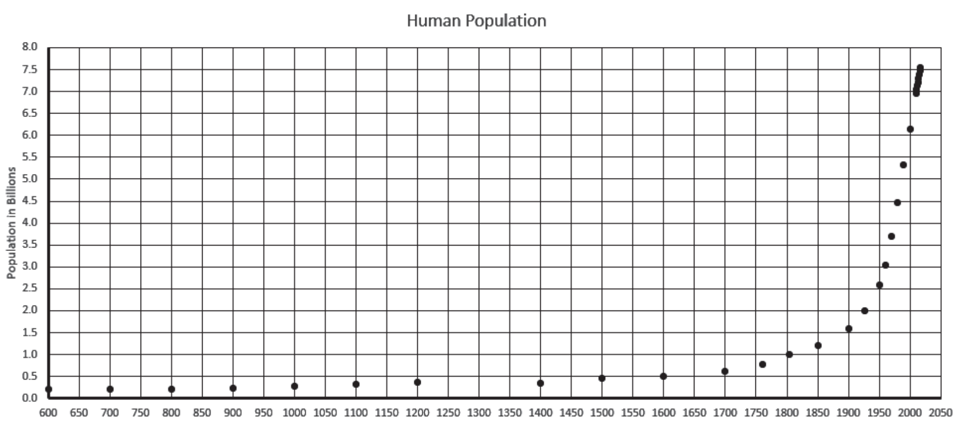
LESSON 30 Modeling with Exponential Functions

LEARNING OBJECTIVES

- Today I am: revisiting the world population graph.
- So that I can: write an equation to fit the data.
- I'll know I have it when I can: describe the graph of the world population and estimate a population using my model.

Opening Exercise

In Lesson 27, you used Desmos' *Marbleslides: Exponentials* to create new equations to hit all the stars. Scientists, economists and others, use tools like Desmos, Excel or a graphing calculator to create complex equations from real world data. In Lesson 21, you examined data for the world's population as shown in the graph below.



Source: http://www.worldometers.info/world-population/world-population-by-year/

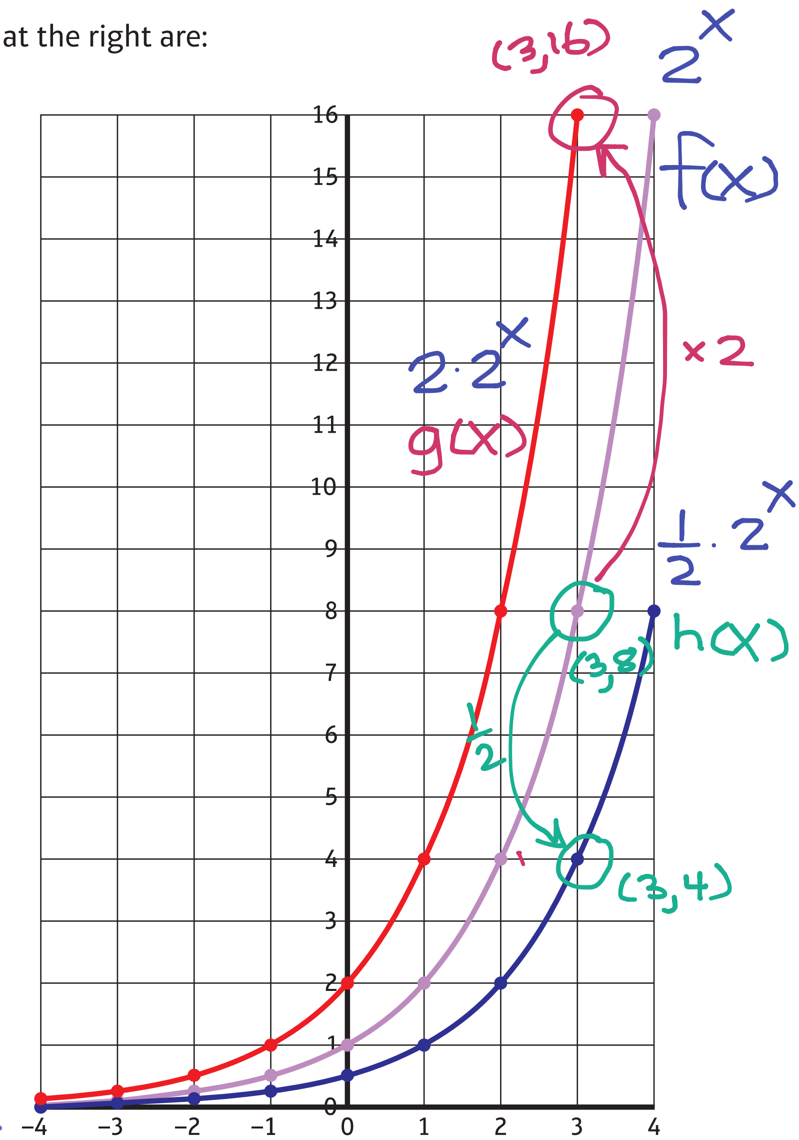
1. Sketch in the curve that you think best fits the data. Don't just connect the data points though. Draw an exponential curve that is the best fit for the data.

It will take a bit more knowledge of transformations to write an accurate equation for the graph on the previous page. We'll further explore transformations of exponential functions and then we'll look at piecewise function to practice describing graphs. Finally we'll put all of this together to write an equation for our world population data and then write about the data using academic language.

Exploration 1—Vertical Stretches and Shrinks

The three exponential functions shown in the grid at the right are:

- $f(x) = 2^x$
- $g(x) = 2 \cdot 2^x$
- $h(x) = \frac{1}{2} \cdot 2^x$



2. Use your knowledge of transformations to determine which graph is which. Try not to use coordinate points. Label each of the graphs.

3. Describe the transformation that takes the graph of $y = f(x)$ to the graph of $y = g(x)$.

   vertical stretch by a factor of 2

4. This exploration is about vertical stretches and shrinks. Use the vertical points where $x = 3$ to show the relationship between the three functions and their equations.

5. Write an equation that would lie between the graph of $f(x)$ and $g(x)$. $m(x) = 1.5(2)^x$

   $f(x) = 2^x$ $g(x) = 2 \cdot 2^x$

6. Write an equation that would lie between the graphs of $f(x)$ and $h(x)$.

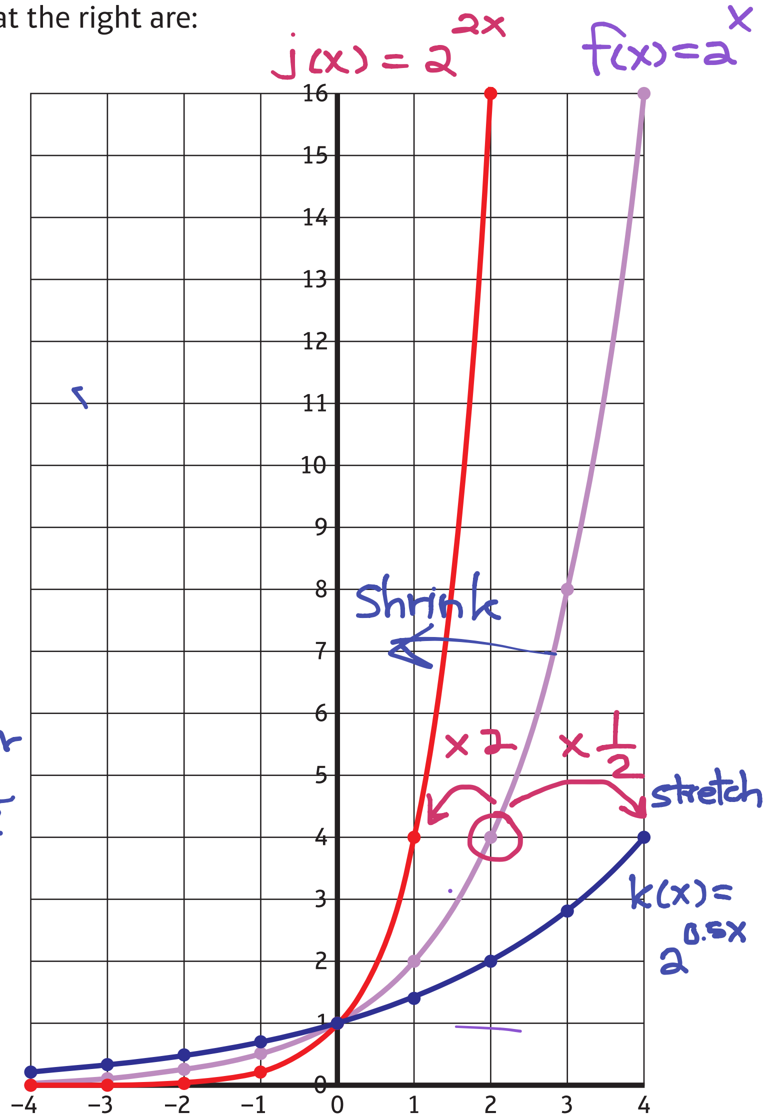
   $f(x) = 1 \cdot 2^x$ $h(x) = \frac{1}{2} \cdot 2^x$

   $n(x) = \frac{3}{4} \cdot 2^x$

Exploration 2—Horizontal Stretches and Shrinks

The three exponential functions shown in the grid at the right are:

- $f(x) = 2^x$
- $j(x) = 2^{(2x)}$
- $k(x) = 2^{(0.5x)}$



7. Use your knowledge of transformations to determine which graph is which. Try not to use coordinate points. Label each of the graphs.

8. Describe the transformation that takes the graph of $y = f(x)$ to the graph of $y = j(x)$.

   $f(x) \rightarrow j(x) = 2^{2x}$

   horizontal shrink w/ factor of $\frac{1}{2}$

9. How can you tell which graph is $j(x)$ and which is $k(x)$?

   k(x) is horizontal stretching

10. This exploration is about horizontal stretches and shrinks. Use the horizontal points where $y = 4$ to show the relationship between the three functions and their equations.

11. Write an equation that would lie between the graph of $f(x)$ and $j(x)$.

    $f(x) = 2^x \rightarrow 2^{1.5x} \rightarrow j(x) = 2^{2x}$

12. Write an equation that would lie between the graphs of $f(x)$ and $k(x)$.

    $f(x) = 2^x \rightarrow 2^{0.7x} \rightarrow k(x) = 2^{0.5x}$

World Population Revisited

Now that you know more about transformations of functions, let's look at the world population graph again and use Desmos to help us find an appropriate exponential model for the data.

You will need: a Chromebook

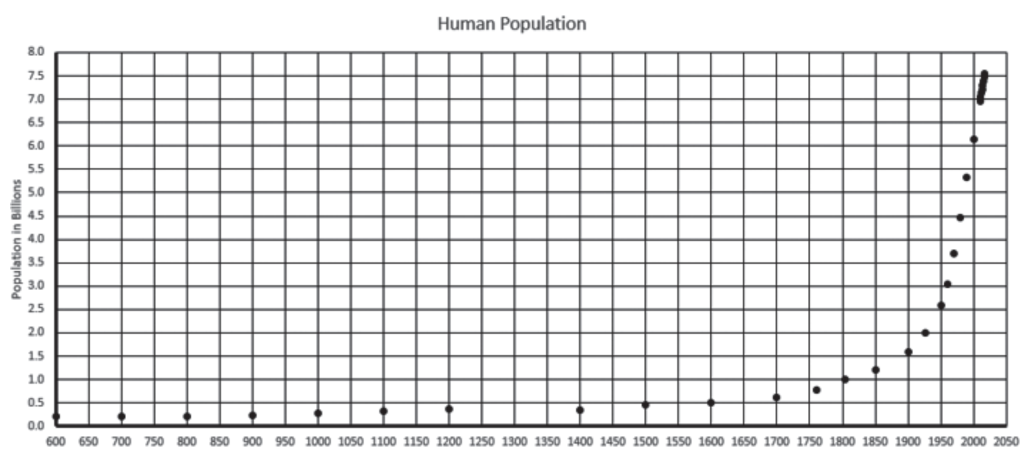
13. Go to https://www.desmos.com/calculator/mmcixv5yyi and use the sliders to create an equation that looks most like the curve you drew in Exercise 6. Write your equation in the space below.

$$f(x) = a \cdot b^{(x-c)} + d$$

$$f(x) = \_\_\_\_\_\_ \cdot \_\_\_\_\_\_^{(x-\_\_)} + \_\_\_\_\_\_$$

14. Use the word bank to help you write an accurate description of the world population data shown in the graph.

Domain	Relative maximum
Range	Relative minimum
Increasing	Intervals
Decreasing	Rapid
Constant	Slow



Source: http://www.worldometers.info/world-population/world-population-by-year/

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Lesson Summary

Transforming Exponential Functions

The parent function of an exponential function is $f(x) = b^x$

- The b is the base.
- The x is the exponent.

A transformed exponential function takes the form $f(x) = a \cdot b^{x-h} + k$

- a is the vertical stretch or shrink factor.
- If $a < 0$, then there is a reflection over the x-axis.
- The b is the base. If $0 < b < 1$, then we have exponential decay. If $b > 1$, then we have exponential growth.
- The h is the horizontal shift left or right
- The k is the vertical shift up or down

NAME: ______________________ PERIOD: ____________ DATE: ____________

Homework Problem Set

Describe the transformation(s) of the parent function, $f(x) = 4^x$, for each of the following functions.

1. $f(x) = -4^{x+3}$

2. $f(x) = \frac{1}{4} \cdot 4^x + 20$

3. $f(x) = 4^{x-1} - 25$

4. $f(x) = 6 \cdot 4^{2x}$

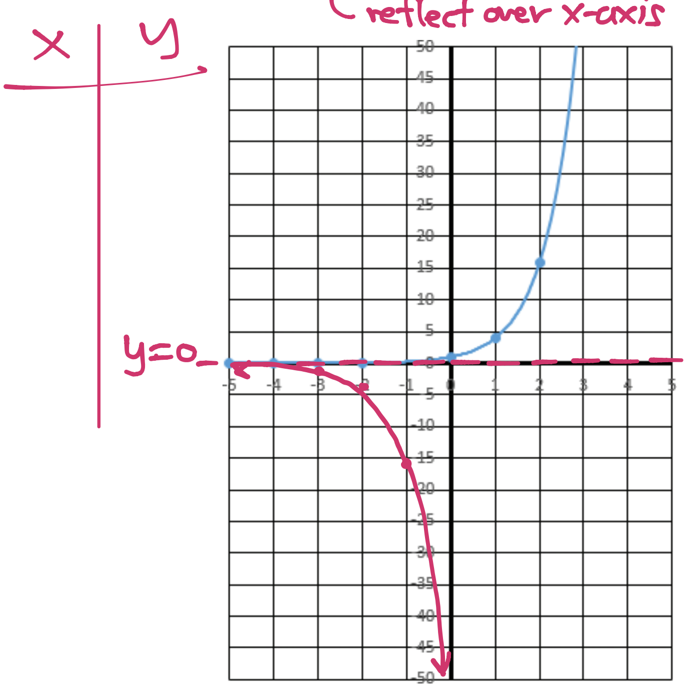
$y = 4^x$

x	y	$\frac{1}{4}f(x)$	f(x)+20
-2	1/16	1/64	$20\frac{1}{64}$
-1	1/4	1/16	$20\frac{1}{16}$
0	1	1/4	20.25
1	4	1	21
2	16	4	24

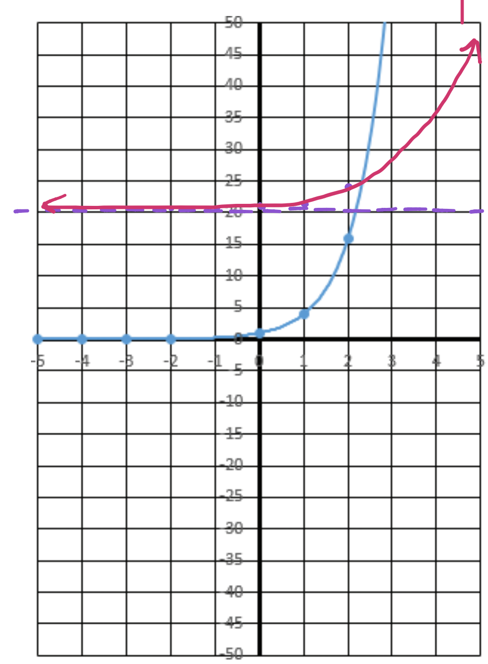
Graph each of the functions. The parent graph $g(x) = 4^x$ is drawn on each grid.

5. $f(x) = -4^{x+3}$

← 3, reflect over x-axis

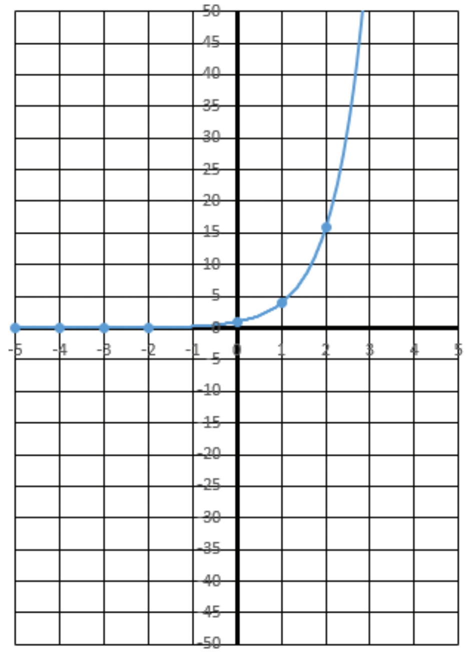


6. $f(x) = \frac{1}{4} \cdot 4^x + 20$

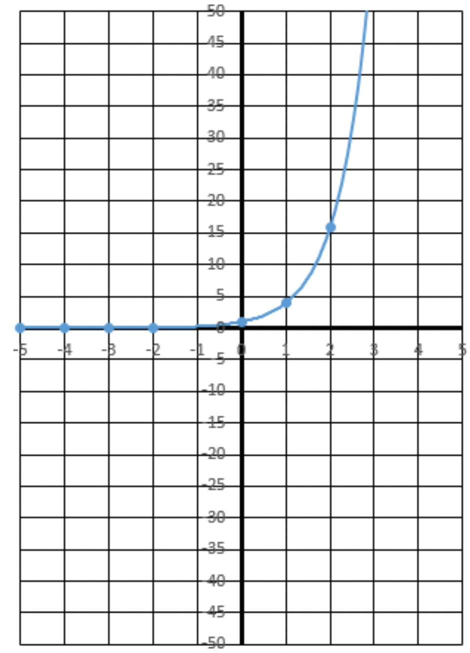


$(-2, 20\frac{1}{64})$
$(-1, 20\frac{1}{16})$
$(0, 20.25)$
$(1, 21)$
$(2, 24)$

7. $f(x) = 4^{x-1} - 25$



8. $f(x) = 6 \cdot 4^{2x}$



Write the equation for the function described below. All are using the parent function $f(x) = 6^x$.

9. shift up 30 and left 5

$f(x+5)+30$

$g(x) = 6^{x+5} + 30$

10. reflect over the *x*-axis and shift right 2

11. vertically stretch by a factor of 7

$g(x) = 7 \cdot 6^x$

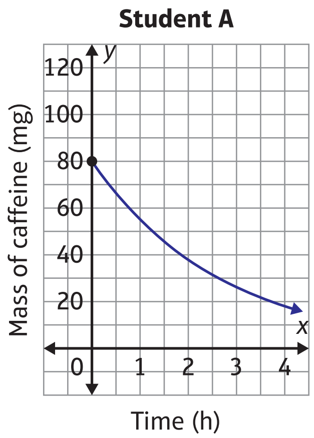
12. reflect over the *y*-axis and horizontally stretch by a factor of 2

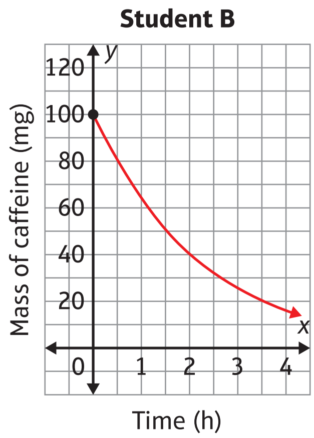
13. shift down 7 and right 3

14. stretch vertically by a factor of 3

Spiral REVIEW—Applications of Exponential Growth

15. Two students drink a soda that contains caffeine. The amount of caffeine was measured over an 8-hour period. The data is shown in the graphs below.





A. How much caffeine was initially ingested by each student? Which feature of the graphs gives this piece of information?

B. Is this situation an example of exponential growth or exponential decay? How do you know?

C. Which student processed the caffeine more quickly? How do you know?

16. The population of Kelowna in 2011 was 117,000. The population growth rate was 3.4%.

A. Use the equation $P(t) = a \cdot b^t$ to write a function that models the population growth of Kelowna, where $P(t)$ is the population of the city t years after 2011.

$a = 117{,}000$ $b = 1.034$

B. According to this model, what is the projected population of Kelowna in the year 2020 to the nearest thousand people?

17. If you have \$10,000 to invest at 2.8% per year and the equation $y = a \cdot b^t$ is an appropriate model for the investment. With $a = 10000$ and $b = 1.028$, How much money would you have after 10 years? (Assuming you don't add or take out any money to the account.)