LESSON 4

Graphs of Piecewise Linear Function

LEARNING OBJECTIVES

- Today I am: graphing elevation and height.
- So that I can: draw a graph to represent a story.
- I'll know I have it when I can: write a story from a graph.

Exploratory Challenge

1. Watch the first 1:08 minutes of the video or until the man reaches the cars. Describe in words the motion of the man.
   "Elevation vs. Time #2" http://www.mrmeyer.com/graphingstories1/graphingstories2.mov

My description of the motion of the man:

2. Share your ideas about the motion of the man with your group.

3. Answer the following questions with your group.
   - How high do you think he was at the top of the stairs? How did you estimate that elevation?

   - Were there intervals of time when his elevation wasn't changing? Was he still moving?

   - Did his elevation ever increase? When?

4. You'll be graphing the man's motion below. Answer the following questions before you start your graph.
   - How should we label the vertical axis? What unit of measurement should we choose (feet or meters)?

   - How should we label the horizontal axis? What unit of measurement should we choose?

5. Sketch a graph of the man's motion. Then watch the remainder of the video to check your work.

Discussion

6. With your group, discuss each of the following questions.

   - Should we measure the man's elevation to his feet or to his head on the graph?

   - The man starts at the top of the stairs. Where would that be located on the graph?

   - The graph shown at the end of the video is incorrect! Can you find the error made in the video?

7. Watch the video, "Elevation vs. Time #3"
   http://www.mrmeyer.com/graphingstories1/graphingstories3.mov

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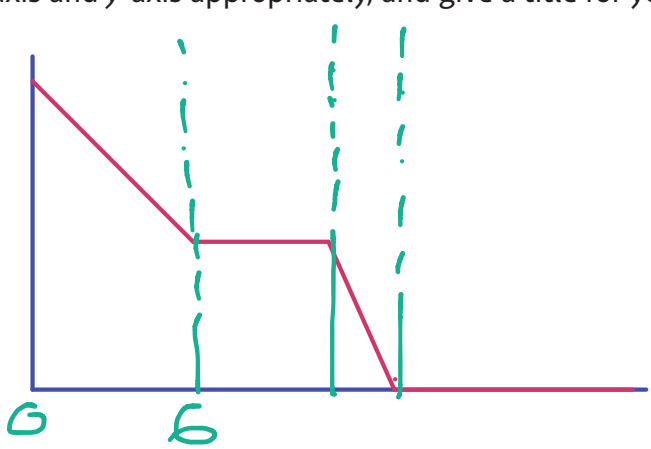
It shows a man climbing down a ladder that is 10 ft high. At time 0 sec., his shoes are at 10 ft. above the floor, and at time 6 sec., his shoes are at 3 ft. From time 6 sec. to the 8.5 sec. mark, he drinks some water on the step 3 ft. off the ground. After drinking the water, he takes 1.5 sec. to descend to the ground, and then he walks into the kitchen. The video ends at the 15 sec. mark.

A. Sketch your own graph for this story. Use straight line segments in your graph to model the elevation of the man over different time intervals. Label your *x*-axis and *y*-axis appropriately, and give a title for your graph.



B. Your picture is an example of a graph of a ***piecewise linear function***. Each linear function is defined over an interval of time, represented on the horizontal axis. List those time intervals.

$[0, 6]$ $(6, 8.5]$ $(8.5, 10]$
$(10, 15]$

C. In your graph in part (A), what does a horizontal line segment represent in the graphing story?

The elevation did not change

D. If you measured from the top of the man's head instead (he is 6.2 ft. tall), how would your graph change?

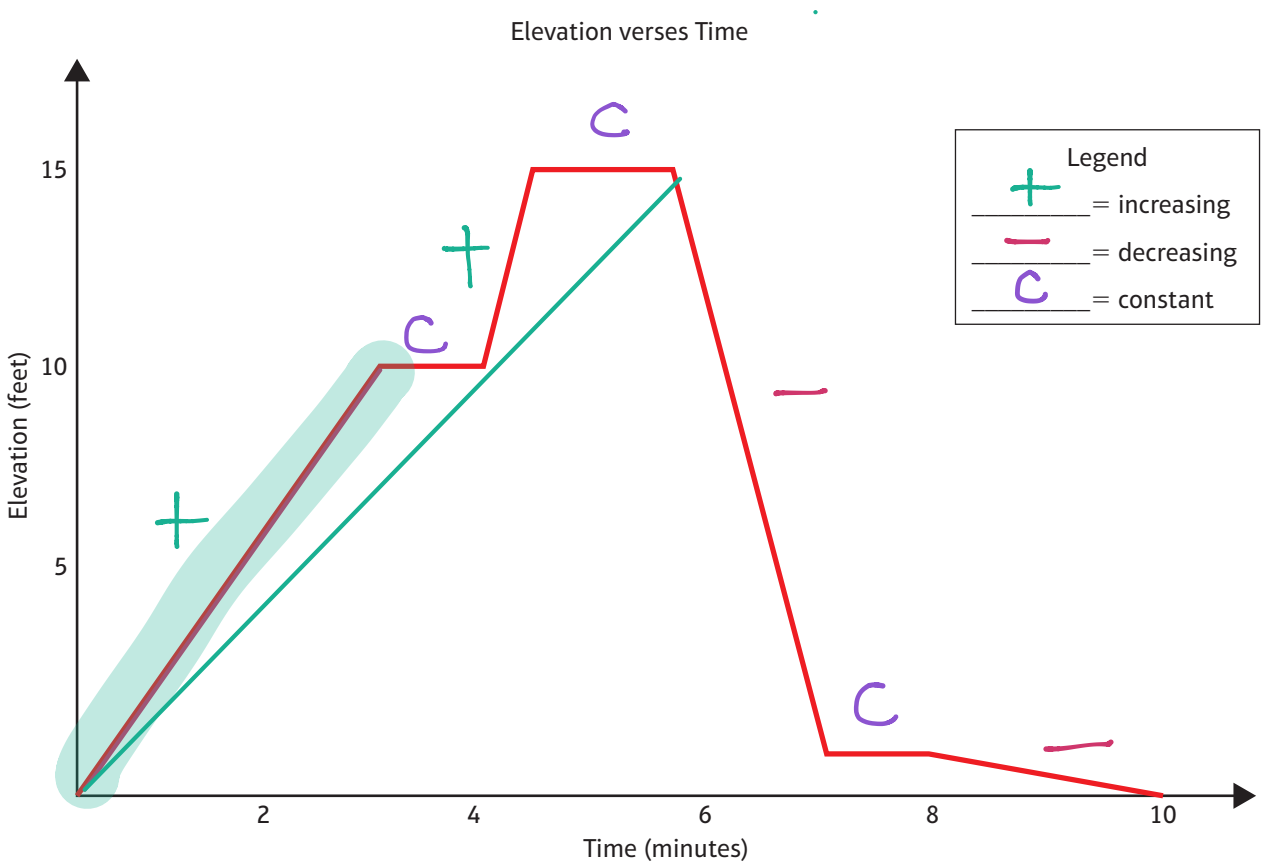
The whole graph will shift up 6.2 ft

E. Suppose the ladder descends into the basement of the apartment. The top of the ladder is at ground level (0 ft.) and the base of the ladder is 10 ft. below ground level. How would your graph change in observing the man following the same motion descending the ladder?

The whole graph will shift down 10 ft.

Class Discussion

8. Here is an elevation-versus-time graph of a person's motion. Describe what the person might have been doing.



You will need: three different colored markers or highlighters

9. Use colored markers or highlighters to show where the graph is ***increasing***. Use a different color to show where the graph is ***decreasing*** and the last color to show where the graph is ***constant***. Fill in the legend on the graph so that someone reading it will know what each color means.

10. What is happening in the story you wrote (Exercise 7) when the graph is increasing, decreasing, constant over time? Highlight the parts of the story with the same colors to show the connection to the graph.

11. What is the domain of the graph? What is the range? Write both in interval notation.

Domain: $[0, 10]$ Range: $[0, 15]$

12. What does it mean for one part of the graph to be steeper than another?

↳ slope, rate of change

13. How does the slope of each line segment relate to the context of the person's elevation?

14. The ***average rate of change*** is defined as the difference in the vertical values divided by the difference in the horizontal values. What is another way to describe the average rate of change?

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

15. A. What was the average rate of change of the person's elevation between time 0 minutes and time 3 minutes? (0, 0) (3, 10)

$$\text{Average rate of change} = \frac{\text{Elevation at 3 minutes} - \text{Elevation at 0 minutes}}{\text{3 minutes} - \text{0 minutes}}$$

$$= \frac{10-0}{3-0} = \frac{10}{3} = \frac{3.\overline{3}\ \text{ft}}{1\ \text{min}}$$

B. What was the average rate of change of the person's elevation between time 0 minutes and time 4 minutes?

C. What was the average rate of change of the person's elevation between time 3 minutes and time 4 minutes? (3, 10) (4, 10)

$$m = \frac{10-10}{4-3} = \frac{0\ \text{ft}}{1\ \text{min}}$$

D. What was the average rate of change of the person's elevation between time 4 minutes and time 5 minutes? (4, 10) (5, 15)

$$m = \frac{15-10}{5-4} = \frac{5\ \text{ft}}{1\ \text{min}}$$

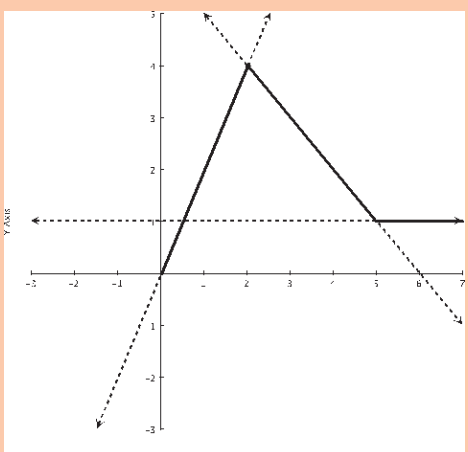
16. Determine the domain restrictions for each graph in the Lesson Summary.

Lesson Summary

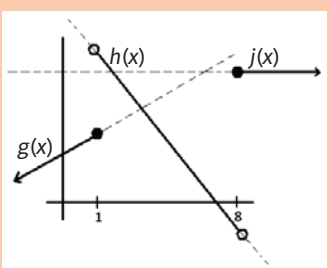
The **average rate of change** is defined as the difference in the vertical values divided by the difference in the horizontal values. This is also called the _____ of the segment.

$$\text{Average rate of change} = \frac{\text{Difference in the vertical values}}{\text{Difference in the horizontal values}} = \frac{f(x_1) - f(x_2)}{x_1 - x_2} = \frac{y_1 - y_2}{x_1 - x_2}$$

PIECEWISE-DEFINED LINEAR FUNCTION: Given non-overlapping intervals on the real number line, a *piecewise linear function* is a function from the union of the intervals on the real number line that is defined by linear functions on each interval.



or

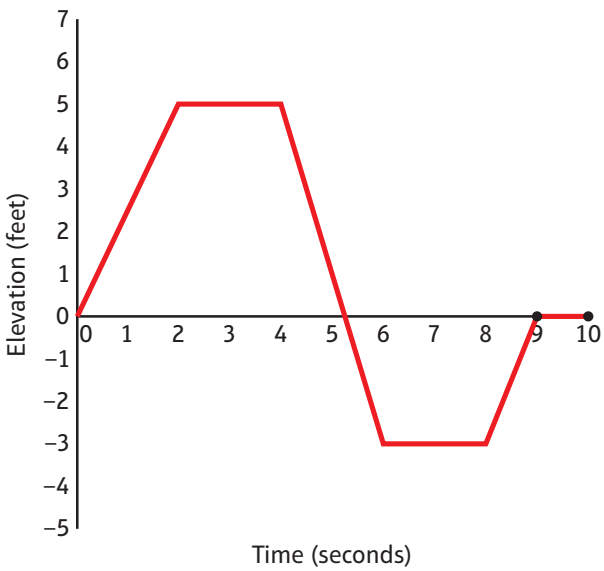


$$f(x) = \begin{cases} y = 2x, & \_\_\_\_ \leq x \leq \_\_\_\_ \\ y = -x + 6, & \_\_\_\_ < x \leq \_\_\_\_ \\ y = 1, & \_\_\_\_ < x \end{cases} \qquad f(x) = \begin{cases} g(x), & \_\_\_\_ < x \leq \_\_\_\_ \\ h(x), & \_\_\_\_ < x < \_\_\_\_ \\ j(x), & \_\_\_\_ \leq x < \_\_\_\_ \end{cases}$$

NAME: _____ PERIOD: _____ DATE: _____

Homework Problem Set

1. A. Create an elevation-versus-time graphing story for the following graph:



My story:

B. What is the domain and range of this graph? Write your answer in interval notation.

DOMAIN: _____

RANGE: _____

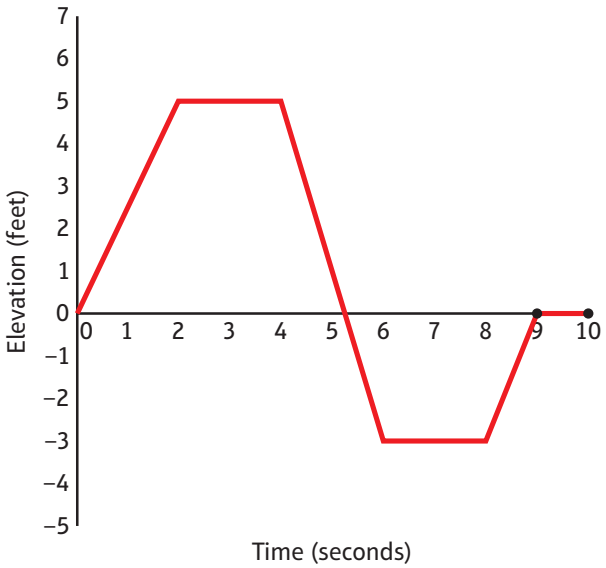
C. Mark the graph to show where it is increasing, decreasing or constant. Use the code below.

Dashed segments (- - - -) for increasing.

Bold segments (▬) for decreasing.

Leave alone segments where the graph is constant.

2. Below are the equations and domain restrictions for the piecewise function at the right. Put them in order so that it accurately describes the graph. Write the equations on the graph.



EQUATIONS

A. $y = -3$

B. $y = \frac{5}{2}x$

C. $y = 3x - 27$

D. $y = 5$

E. $y = -4x + 19$

Order: ____, ____, ____, ____, ____

DOMAIN RESTRICTIONS

F. $2 \leq x < 4$

G. $0 \leq x < 2$

H. $8 \leq x \leq 9$

I. $4 \leq x < 6$

J. $6 \leq x < 8$

Order: ____, ____, ____, ____, ____

3. Sketch an elevation-versus-time graphing story of your own, and then create a story for it.

My Graph:

My Story:

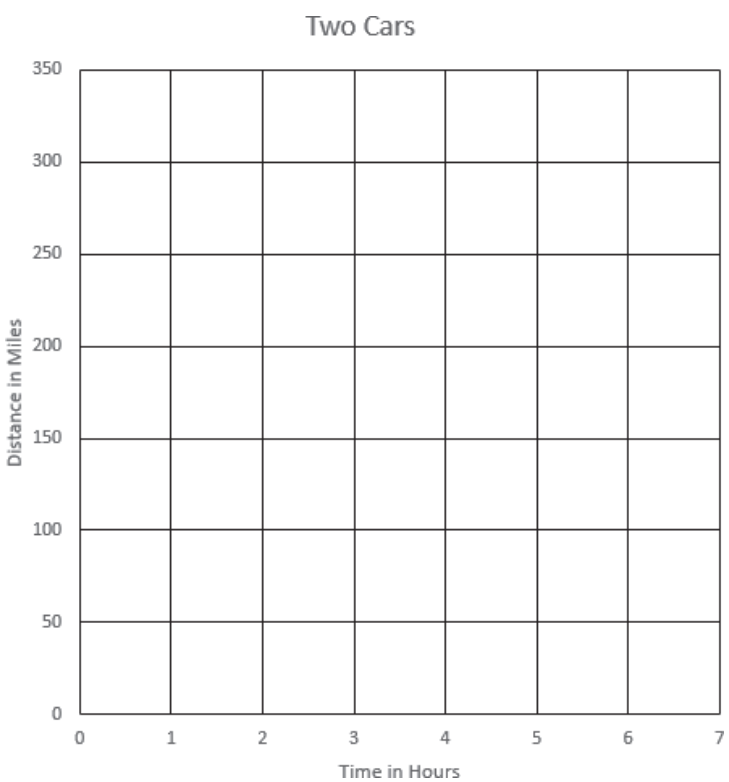
4. Suppose two cars are travelling north along a road.

   Car 1 travels at a constant speed of 50 mph for two hours, then speeds up and drives at a constant speed of 100 mph for the next hour. The car breaks down and the driver has to stop and work on it for two hours. When he gets it running again, he continues driving recklessly at a constant speed of 100 mph.

   Car 2 starts at the same time that Car 1 starts, but Car 2 starts 100 mi. farther north than Car 1 and travels at a constant speed of 25 mph throughout the trip.

   A. Sketch the distance-versus-time graphs for Car 1 and Car 2 on a coordinate plane at the right. Be sure to include a legend.

   B. Approximately when do the cars pass each other?



   C. Tell the entire story of the graph from the point of view of Car 2. (What does the driver of Car 2 see along the way and when?)

5. **Challenge**

   A. Write linear equations representing each car's distance in terms of time (in hours). Note that you will need four equations for Car 1 and only one for Car 2.

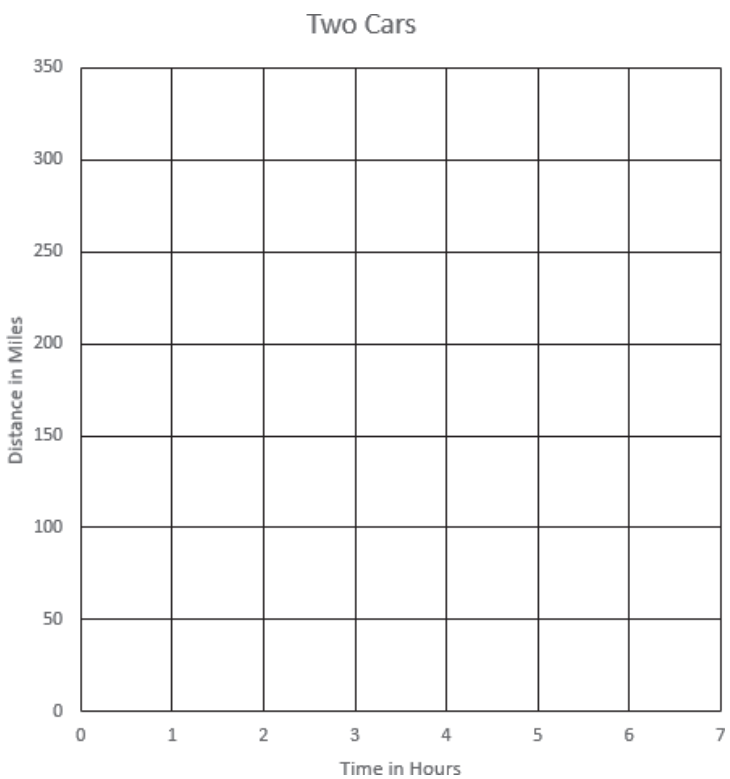
   B. Use these equations to find the exact coordinates of when the cars meet.

6. Suppose that in Problem 4, Car 1 travels at the constant speed of 25 mph the entire time.

   A. Sketch the distance-versus-time graphs for the two cars on a graph below.

   B. Do the cars ever pass each other? Explain.

   C. What is the linear equation for Car 1 in this case?



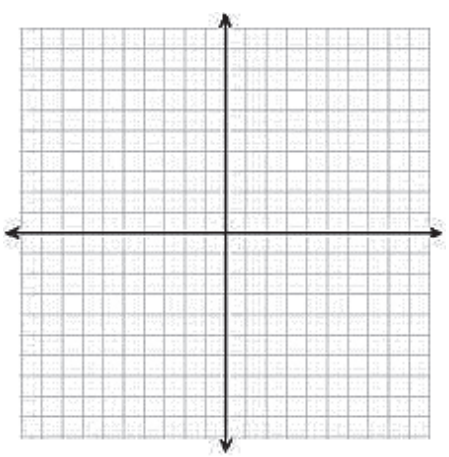
Spiral REVIEW—Undefined and 0 Slope

7. Find the slope between the two points given.

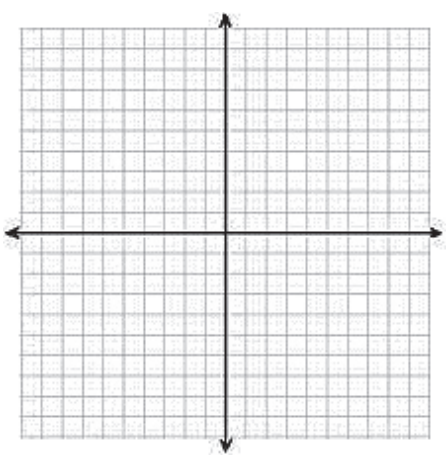
A. (0, 3) and (6, 2)	B. $(-1, -2)$ and (4, 0)
C. (3, 4) and (3, 7)	D. (3, 4) and (5, 4)

8. Graph the points from Problem 7C and 7D above. Then connect them to form a line.

7C. (3, 4) and (3, 7)



7D. (3, 4) and (5, 4)



9. How could you describe the lines in Problem 8?