LESSON 5

Aunt Lucy's Gift

LEARNING OBJECTIVES

- Today I am: analyzing four gift plans.
- So that I can: write the terms of a sequence and graph the points.
- I'll know I have it when I can: identify if a sequence has a linear pattern or not.

Opening Exercise

Aunt Lucy has decided to begin distributing her vast fortune to her nieces and nephews. She has given them four plans from which to choose. Once they choose a plan, they cannot change to another plan.

Plan 1	Plan 2	Plan 3	Plan 4
Receive \$10 the first year, \$20 the second year, \$30 the third year, \$40 the fourth year, and so on.	Receive \$100 the first year, \$90 the second year, \$80 the third year, \$70 the fourth year, and so on.	Receive \$10 the first year, 1 and ½ times as much the second year, 1 and ½ times as much the third year, and so on.	Receive \$1 the first year, \$2 the second year, \$4 the third year, \$8 the fourth year and so on.

1. With your group, determine which plan is the best. Be prepared to explain your choice to the class.

Sequences

A *sequence* can be thought of as an ordered list of elements. These elements can be numbers, pictures, letters, geometric figure, or just about any object you like. The elements of the list are called the *terms of the sequence*. Commas are often used to separate the terms of a sequence.

2. List the first five terms of each sequence of Aunt Lucy's Gift.

Plan 1: 10, 20, 30, 40, 50

Plan 2: 100, 90, 80, 70, 60

Plan 3: 10, 15, 22.5, 33.75, 50.63

Plan 4: 1, 2, 4, 8, 16

f(1) f(2) f(3) f(4) f(5)

Plan 1	**Plan 2**
Receive $10 the first year, $20 the second year, $30 the third year, $40 the fourth year, and so on.	Receive $100 the first year, $90 the second year, $80 the third year, $70 the fourth year, and so on.
Plan 3	**Plan 4**
Receive $10 the first year, 1 and ½ times as much the second year, 1 and ½ times as much the third year, and so on.	Receive $1 the first year, $2 the second year, $4 the third year, $8 the fourth year and so on.

Usually the terms are *indexed* (and, therefore, ordered) by a subscript starting at 1: $a_1, a_2, a_3, a_4, \ldots$. The ellipsis indicates a regular pattern; that is, the next term is a_5, the next is a_6, and so on.

"a sub 3"

3. What is a_6 for the first plan?

the sixth term in the sequence.

$a_6 = 60$

4. Which terms in the second plan are 0?

$a_{10}, a_{11}, a_{12}, \ldots\ldots$

$f(n) = 2n + 5$

$y = 2x + 5$

Explicit Formulas—Identifying the Pattern

When a function is expressed as an algebraic function only in terms of numbers and the index variable n, then the function is called the *explicit form of the sequence* (or *explicit formula*). Sequences can be indexed starting with any integer. To avoid confusion, this module adopts the convention that indices start at 1. That way, the first term in the list is always $f(1)$ or a_1, and there is no confusion about what the 100^{th} term is.

"f of 1"

function

a sub 1

$a_1 = f(1) \rightarrow 1^{st}$ term

$a_{100} = f(100) \rightarrow 100^{th}$ term

 $n = 1, 2, 3, 4, 5, \ldots\ldots$

5. The explicit formula for the first plan is $f(n) = 10n$, where n is the year the gift was given. Check that this formula holds for the pattern stated by Aunt Lucy.

$f(1) = 10 \cdot 1 = 10$
$f(2) = 10 \cdot 2 = 20$
$f(3) = 10 \cdot 3 = 30$

6. With your group, determine the explicit formula for the second plan.

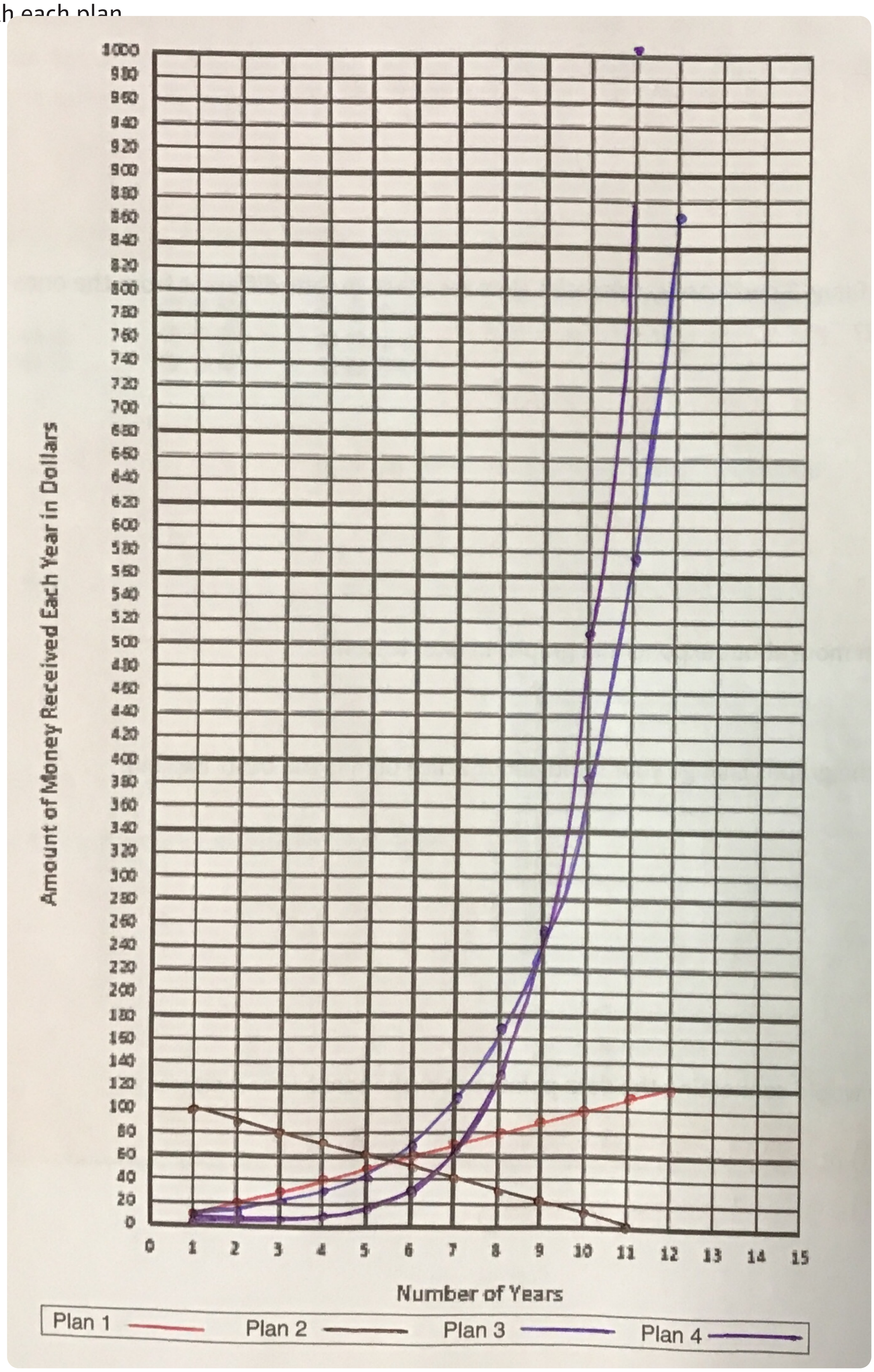
$f(n) = 110 - 10n$

7. What is the pattern in Plan 3? Discuss with your group ways to write a formula for that plan. Is it possible to write an explicit formula for this plan?

$f(n) = 10(1.5)^{n-1}$

8. Josh wrote the explicit formula for Plan 4 as $f(n) = 2^n$, but Kent wrote it as $f(n) = 2^{n-1}$. Is there any way that both could be correct? Explain your thinking.

9. One way to see the pattern in a sequence is to graph the data points. For each of Aunt Lucy's plans, graph the amount received each year. Create a legend to make it clear which set of data goes with each plan.



Discussion

10. Which of Aunt Lucy's plans is linear? How do you know?

11. Both Plans 3 and 4 are *exponential*. How are these graphs different from the ones in Plans 1 and 2?

We'll learn more about exponential graphs in later lessons.

12. Did the graphs change your mind about which plan is the best? Explain.

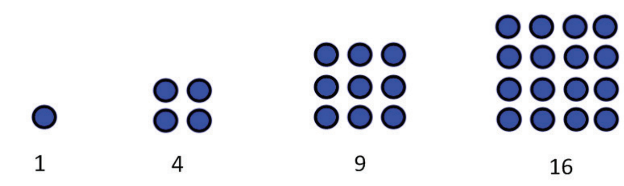
13. Why would connecting the data points not make sense in this situation?

Lesson Summary

A sequence is an ordered list of terms. Often it is possible to write an equation to describe the sequence. A graph of the data can make it easier to see patterns and decide if the data is linear or not.

Example 1:

The sequence below is formed by the square numbers.

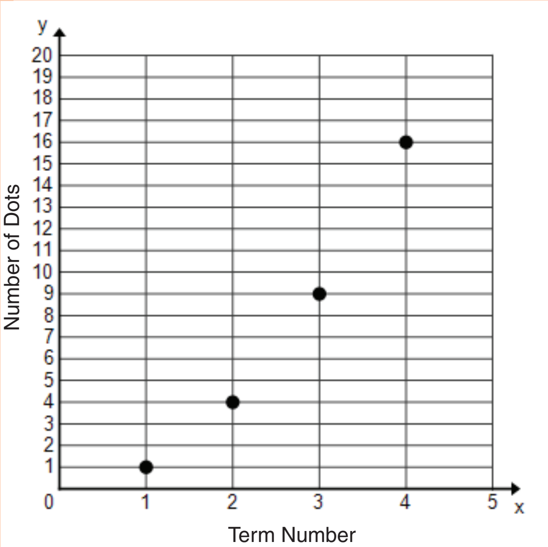


The formula for this sequence is $f(n) = n^2$ starting with $n = 1$.

$a_1 = 1$ $a_2 = 4$ $a_3 = 9$ $a_4 = 16$ ← "a sub" Notation

$f(1) = 1$ $f(2) = 4$ $f(3) = 9$ $f(4) = 16$ ← Function Notation

The graph of this data is shown at the right.



14. Complete the Lesson Summary.

It is not always clear what is the next term in a sequence. Sometimes many possibilities exist when you only have the first few terms.

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Example 2: Mrs. Rosenblatt gave her students what she thought was a very simple task:

> What is the next number in the sequence 2, 4, 6, 8, . . .?

Cody: *I am thinking of a plus 2 pattern, so it continues* 10, 12, 14, 16, ...

Ali: *I am thinking of a repeating pattern, so it continues* 2, 4, 6, 8, 2, 4, 6, 8, ...

Suri: *I am thinking of the units digits in the multiples of two, so it continues* 2, 4, 6, 8, 0, 2, 4, 6, 8, ...

Sequence Notation

Meaning	**"a sub" Notation**	**Function Notation**
First term		
100^{th} term		
Any term (_____ term)		
Previous term	a_{n-1}	
_____________ term		$f(n+1)$

NAME: ______________ PERIOD: ________ DATE: ________

Homework Problem Set

For each problem, write the corresponding "*a* sub" or function notation.

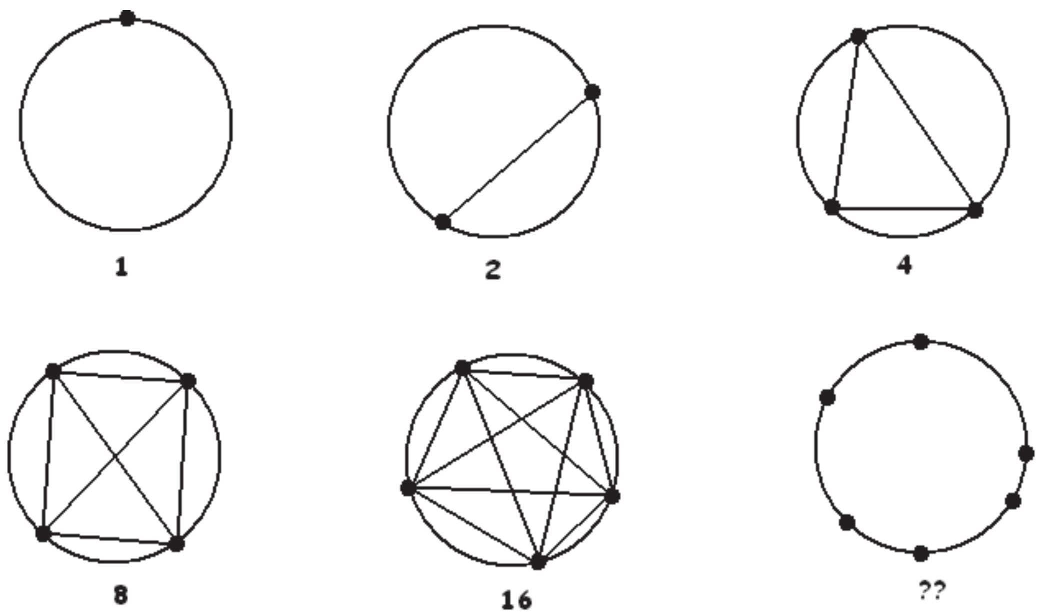
Meaning	"*a* sub" Notation	Function Notation
1. 7^{th} term of the sequence	a_7	
2. 10^{th} term of the sequence		$f(10)$
3. 1^{st} term of the sequence added to the 2^{nd} term of the sequence		$f(1) + f(2)$
4. n^{th} term of the sequence	a_n	
5. The n^{th} term subtracted from the $(n-1)^{th}$ term of the sequence		$f(n-1) - f(n)$

6. Consider a sequence generated by the formula $f(n) = 6n - 4$ starting with $n = 1$. Generate the terms $f(1)$, $f(2)$, $f(3)$, $f(4)$, and $f(5)$.

7. Consider a sequence given by the formula $f(n) = \frac{1}{3^{n-1}}$ starting with $n = 1$. Generate the first 5 terms of the sequence. Remember $a^0 = 1$ as long as $a \neq 0$.

8. Consider a sequence given by the formula $f(n) = (-1)^n \times 3$ starting with $n = 1$. Generate the first 5 terms of the sequence.

9. **Challenge** Here is the classic puzzle that shows that patterns need **not** hold true. What are the numbers counting?



A. Based on the sequence of numbers, predict the next number.

B. Write a formula based on the pattern.

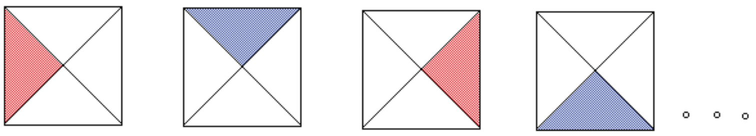
C. Find the next number in the sequence by actually counting.

D. Based on your answer from Part C, is your model from Part B effective for this puzzle?

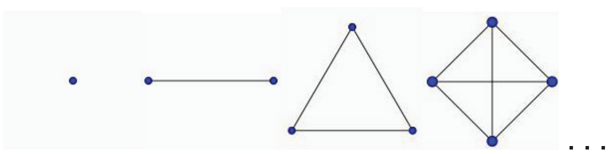
Sequences That Aren't Numbers

For each sequence below, write the next two terms and explain in words how the pattern continues.

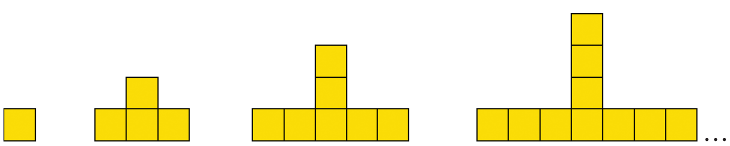
10.



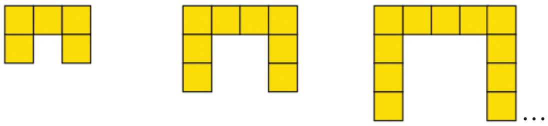
11.



12.



13.



14.

a a b c a a b c a a b c a...

15. **Challenge** O, T, T, F, F, S, S, E, . . .

16. **Challenge** M, T, W, T, . . .

17. The Lesson Summary Example 2 stated:

Mrs. Rosenblatt gave her students what she thought was a very simple task:
What is the next number in the sequence 2, 4, 6, 8, . . .?

Cody: *I am thinking of a plus* 2 *pattern, so it continues* 10, 12, 14, 16, ...

Ali: *I am thinking of a repeating pattern, so it continues* 2, 4, 6, 8, 2, 4, 6, 8, ...

Suri: *I am thinking of the units digits in the multiples of two, so it continues* 2, 4, 6, 8, 0, 2, 4, 6, 8, ...

A. Are each of these valid responses? Explain your thinking.

B. What is the hundredth number in the sequence in Cody's scenario? Ali's? Suri's?

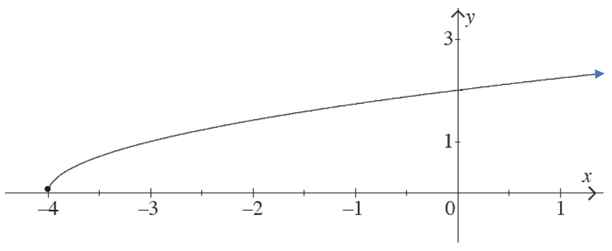
C. What is an explicit formula for the n^{th} number in the sequence in Cody's scenario?

Spiral REVIEW—Domain and Range

For Problems 18–21, determine the domain and range for each graph shown.

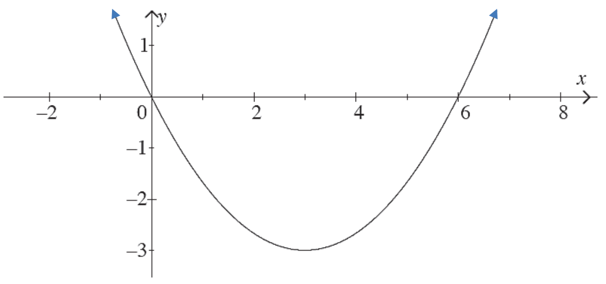
18. Domain: ____________________

Range: ____________________



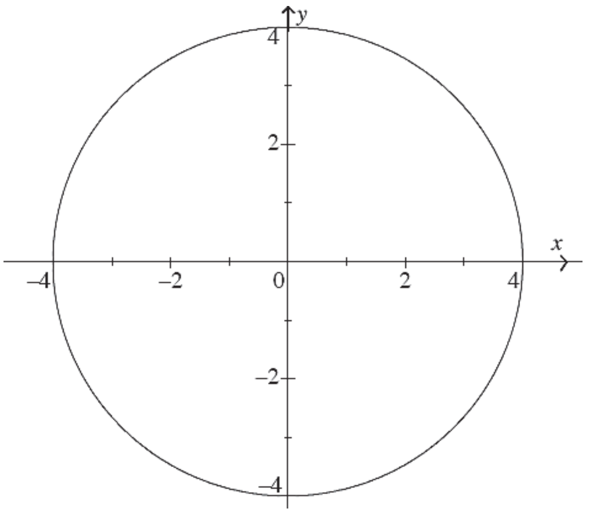
19. Domain: ____________________

Range: ____________________



20. Domain: ____________________

Range: ____________________



21. Domain: ____________________

Range: ____________________