LESSON 6

Does the Order Matter?

LEARNING OBJECTIVES

- Today I am: writing out exponent multiplication.
- So that I can: develop rules for exponents.
- I'll know I have it when I can: solve a problem like $\left(a^3b^5\right)^0 = \dfrac{a^3}{a^{\_\_}\, b^{\_\_}}$.

Opening Activity

You will need: *Does the Order Matter?* sorting cards

Source: adapted from 5E Lesson Plan: Using Order of Operations to Evaluate and Simplify Expressions, Pat Tyree

1. Rearrange the cards so they make sense.

Discussion

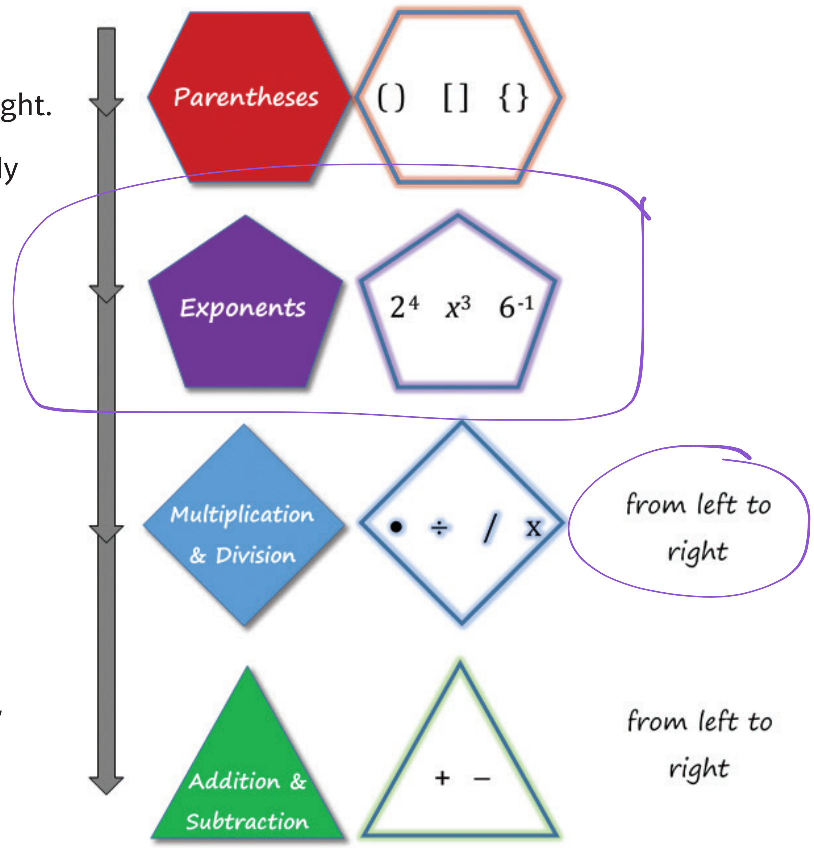
2. When does the order of actions matter in math?

3. What kind of math problem has multiple steps?

4. Does order matter in sequences or can a sequence be in any order?

One way to remember the correct order in math expressions is with a mnemonic (memory aid) like the one shown on the right.

5. How does the phrase "Please Excuse My Dear Aunt Sallie" or PEMDAS help with order of operations?



Since exponents appear often in mathematical sequences and expressions, let's review the exponent rules.

Exploratory Activity

6. A. With your partner, fill in the table below about exponents and then describe the Product Rule.

Expression	Repeated Multiplication	Simplified Form
exponent base $3^2 \cdot 3^4 \cdot 3$ power	$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$	3^7
$4^4 \cdot 4^2 \cdot 4^3$		4^9
$9^6 \cdot 9^3$		9^9
$a^7 \cdot a^9 \cdot a^2$		a^{18}

B. The Product Rule for Exponents: $a^m \cdot a^n =$ a^{m+n}

$5^2 \cdot 5^7 = 5^9$

7. A. With your partner, fill in the table below about exponents and then describe the pattern you see.

Expression with Exponents	Simplified Expression without Exponents
2^3	8
2^2	4
2^1	2
2^0	1
2^{-1}	$\frac{1}{2}$
$2^{-2} = \frac{1}{4} = \frac{1}{2^2}$	$\frac{1}{4}$

Ex:

$$\frac{x^{-5}}{y^{-6}} = \frac{y^6}{x^5}$$

$$\frac{x^{-5}}{y^6} = \frac{1}{x^5 \cdot y^6}$$

B. The Negative Exponent Rule: $x^{-m} = \frac{1}{x^m}$

$$\frac{1}{x^{-m}} = x^m$$

8. A. With your partner, fill in the table below about exponents and then describe the Division Rule.

Expression	Repeated Multiplication	Simplified Form
$\frac{3^2}{3^4}$	$\frac{\cancel{3}\cdot\cancel{3}}{\cancel{3}\cdot\cancel{3}\cdot 3\cdot 3} = \frac{1}{3^2}$	$\frac{1}{3^2}$
$\frac{4^4}{4^3}$	$\frac{\cancel{4}\cdot\cancel{4}\cdot\cancel{4}\cdot 4}{\cancel{4}\cdot\cancel{4}\cdot\cancel{4}} = 4$	4
$\frac{9^3}{9^6}$	$\frac{\cancel{9}\cdot\cancel{9}\cdot\cancel{9}}{\cancel{9}\cdot\cancel{9}\cdot\cancel{9}\cdot 9\cdot 9\cdot 9} = \frac{1}{9^3}$	$\frac{1}{9^3}$
$\frac{a^{17}}{a^9}$		a^8

B. The Division Rule for Exponents: $\frac{x^m}{x^n} = x^{m-n}$

$$\frac{x^5}{x^3} = x^{5-3} = x^2$$

$$\frac{x^3}{x^5} = \frac{1}{x^{5-3}} = \frac{1}{x^2}$$

$$x^{3-5} = x^{-2} = \frac{1}{x^2}$$

x^{-}

9. A. With your partner, fill in the table below about exponents and then describe the Zero Exponent Rule.

Expression	Repeated Multiplication	Simplified Form
$\frac{3^4}{3^4} = 3^{4-4} = 3^0$	$\frac{\cancel{3}\cdot\cancel{3}\cdot\cancel{3}\cdot\cancel{3}}{\cancel{3}\cdot\cancel{3}\cdot\cancel{3}\cdot\cancel{3}} = 1$	1
$\frac{4^3}{4^3} = 4^0$	$\frac{\cancel{4}\cdot\cancel{4}\cdot\cancel{4}}{\cancel{4}\cdot\cancel{4}\cdot\cancel{4}} = 1$	1
$\frac{9^2}{9^2}$		
$\frac{a^3}{a^3}$		

B. The Zero Exponent Rule: $x^0 =$ 1, $x \neq 0$.

10. A. With your partner, fill in the table below about exponents and then describe the Power of a Product or Quotient Rule.

Expression	Repeated Multiplication	Simplified Form
$(3 \cdot 2)^2$	$(3\cdot 2)(3\cdot 2) = 3^2 \cdot 2^2$	
$(5 \cdot 3)^3$	$(5\cdot 3)(5\cdot 3)(5\cdot 3) = 5^3 \cdot 3^3$	
$(abc)^5$	$a^5 b^5 c^5$	
$(2x)^4$	$2^4 x^4$	
$\left(\frac{a}{c}\right)^6$	$\left(\frac{a}{c}\right)\left(\frac{a}{c}\right)\left(\frac{a}{c}\right)\left(\frac{a}{c}\right)\left(\frac{a}{c}\right)\left(\frac{a}{c}\right)$	$\frac{a^6}{c^6}$

B. The Power of a Product or Quotient Rule for Exponents:

$(xy)^m =$ $x^m y^m$ and $\left(\frac{x}{y}\right)^m$ $\frac{x^m}{y^m}$

11. A. With your partner, fill in the table below about exponents and then determine the Power to a Power Rule.

Expression	Repeated Multiplication	Simplified Form
$(3^4)^2 = 3^8$	$(3^4)(3^4)$	3^8
$(5^2)^3 = 5^6$	$(5^2)(5^2)(5^2)$ $2+2+2$	5^6
$(ab^3c^2)^5$	$a^5 b^{15} c^{10}$	
$\left(\frac{2a^2}{3b^4}\right)^3$	$\frac{2^3 a^6}{3^3 b^{12}} = \frac{8a^6}{27b^{12}}$	
$\left(\frac{a^2}{c^3}\right)^6$ $6\cdot 2$ $6\cdot 3$	$\frac{a^{12}}{c^{18}}$	

B. The Power to a Power Rule for Exponents: $(x^m)^n = $ $x^{m\cdot n}$

Simplifying Practice

12. Rewrite 2^{n-1} using the reverse of the quotient rule.

13. What is the simplified version of the expression $32 \cdot (2)^{n-1}$?

14. Simplify $24 \cdot (2)^{n-1}$ and $25 \cdot (2)^{n-1}$.

15. Write a rule for simplifying any expression of the form $b(2)^{n-1}$.

$b(2)^{n-1} = $ __________

16. Simplify $\left(\frac{1}{2}\right)^{n-1}$.

17. A. Simplify $32 \cdot \left(\frac{1}{2}\right)^{n-1}$.

    B. Do you need to revise your rule from Exercise 15? Explain.

18. Simplify each of the following.

    A. $81 \cdot (3)^{n-1}$

    B. $81 \cdot \left(\frac{1}{3}\right)^{n-1}$

    C. $50 \cdot (5)^{n-1}$

    D. $50 \cdot \left(\frac{1}{5}\right)^{n-1}$

    E. $24 \cdot (4)^{n-1}$

    F. $24 \cdot \left(\frac{1}{4}\right)^{n-1}$

19. Complete each rule and example in the Lesson Summary.

Lesson Summary

	General Rule	Example of the Rule
Product Rule	$a^m \cdot a^n = a^{m+n}$	$a^7 \cdot a^6 = a^{13}$
Zero Exponent Rule	$a^0 = \underline{1}$, $a \neq 0$	$3^0 = \underline{1}$
Division Rule	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{4^6}{4^4} = 4^{2}$
Power of a Product Rule	$(a \cdot b)^n = a^n \cdot b^n$	$(2 \cdot 5)^2 = 2^2 \cdot 5^2$ $= 4 \cdot 25 = 100$
Power of a Quotient Rule	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{5}{3}\right)^3 = \frac{5^3}{3^3} = \frac{125}{27}$
Negative Exponent Rule	$\frac{1}{a^n} = a^{-n}$ $\frac{1}{a^{-n}} = a^{n}$	$\frac{1}{4^2} = 4^{-2}$ $\quad 4^{-2} = \frac{1}{4^2}$ $\frac{1}{3^{-2}} = 3^{2} = 9$
Power to a Power Rule	$\left(\frac{a^n}{b^p}\right)^m = \frac{a^{n \cdot m}}{b^{p \cdot m}}$ $(a^m \cdot b^p)^n = a^{m \cdot n} \cdot b^{p \cdot n}$	$\left(\frac{2^3}{3^4}\right)^2 = \frac{2^6}{3^8}$ $(2^5 \cdot 3^6)^3 = 2^{15} \cdot 3^{18}$

NAME: ____________________ PERIOD: __________ DATE: __________

Homework Problem Set

Simplify each expression so that there are no negative exponents.

1. $\left(xy^2\right)^0$

2. $\left(x^2\right)^3$

3. $\left(\dfrac{2y}{x}\right)^2$ $\quad \dfrac{2^2y^2}{x^2} = \dfrac{4y^2}{x^2}$

4. $\left(abc^{-1}\right)^{-1}$

$\dfrac{a^{-1}b^{-1}c^{1}}{1} = \dfrac{c}{ab}$

5. $\dfrac{a^2b^4c^{-2}}{ab^3c^2}$

$\dfrac{a^2b^4}{a^1b^3c^4} = \dfrac{ab}{c^4}$

6. $\left(\dfrac{a^{-2}}{b^3}\right)^{-4} = \dfrac{a^8}{b^{-12}} = a^8b^{12}$

7. $\left(\dfrac{m^4}{n^{-2}}\right)^3$

8. $\dfrac{m^{-1}n^0p^2}{m^2n^{-1}p^3}$

$\dfrac{m^{-3}}{n^{-1}p} = \dfrac{n}{m^3p}$

9. $\left(mnp^3\right)^{-2}$

$m^{-2}n^{-2}p^{-6} = \dfrac{1}{m^2n^2p^6}$

10. $\dfrac{d^3e^2f}{d^{-3}e^{-2}f^{-1}} = d^3 \cdot d^3 \cdot e^2 \cdot e^2 \cdot f \cdot f$

$= d^6e^4f^2$

11. $d^3 \times d^2 \times d^{-4} = d$

12. $\dfrac{e^3 \cdot e^2}{e^7e^{-4}} = \dfrac{e^3 \cdot e^2 \cdot e^4}{e^7}$

$= \dfrac{e^9}{e^7} = e^2$

13. $\dfrac{a^3b^4}{a^{-2}b^{-1}}$

14. $\left(a^{-3}b^{-3}\right)^0$

15. $\dfrac{2m^3 \cdot m^2}{m^{-3}}$

16. $\dfrac{\left(x^{-3}\right)^4 x^4}{2x^{-3}}$

17. $\left(x^{-2}x^{-3}\right)^4$

18. $\dfrac{2x^4y^{-4}z^{-3}}{3x^2y^{-3}z^4}$

19. $\dfrac{3m^{-4}}{m^3}$

20. $\left(2x^2\right)^{-4}$

21. $\dfrac{3n^4}{3n^3}$

Determine what integer can be placed in the blank to make the statement true.

22. $\left(\dfrac{a^3b^2}{b^{-3}}\right)^4 = \dfrac{a^{12}}{b^{\_\_}}$

23. $\left(a^4b^{-2}\right)^{-3} = \left(\dfrac{b^2}{a^4}\right)^{\_\_}$

24. $a^{-2}b^3 = \dfrac{a^2}{a^{\_\_}b^{-3}}$

25. $\left(a^3b^5\right)^0 = \dfrac{a^3}{a^{\_\_}b^{\_\_}}$

Spiral REVIEW—Order of Operations

26. **Four Number Game**: Use the numbers 1, 2, 3 and 4 no more than once for each problem. You may use any operations including powers and parentheses. You may NOT create a 2-digit number ($1 \,\&\, 2 \neq 12$). For example, if the number was 10, then we could do any of the following: $1 + 2 + 3 + 4 = 10$ or $2 \cdot 3 + 4 = 10$ or $3^2 + 1 = 10$.

   A. The number is 12: ______

   B. The number is 15: ______

   C. The number is 25: ______

   D. The number is 32: ______

27. Think about order of operations to insert parentheses to make each statement true. If you are having trouble remembering the Order of Operations, watch the YouTube video by Math Antics https://www.youtube.com/watch?v=dAgfnK528RA.

   A. $2 + 3 \times 4^2 + 1 = 81$

   B. $2 + 3 \times 4^2 + 1 = 85$

   C. $2 + 3 \times 4^2 + 1 = 51$

   D. $2 + 3 \times 4^2 + 1 = 53$