NAME: _____ PERIOD: _____ DATE: _____

Homework Problem Set

1. Consider a sequence that follows a times 5 pattern: 1, 5, 25, 125, ….

   A. Write a formula for the n^{th} term of the sequence. Be sure to specify what value of n your formula starts with. $a_1 = 1 \quad r = 5$

   $f(n) = 1(5)^{n-1} \longrightarrow f(n) = 5^{n-1}$

   B. Using the formula, find the 10^{th} term of the sequence.

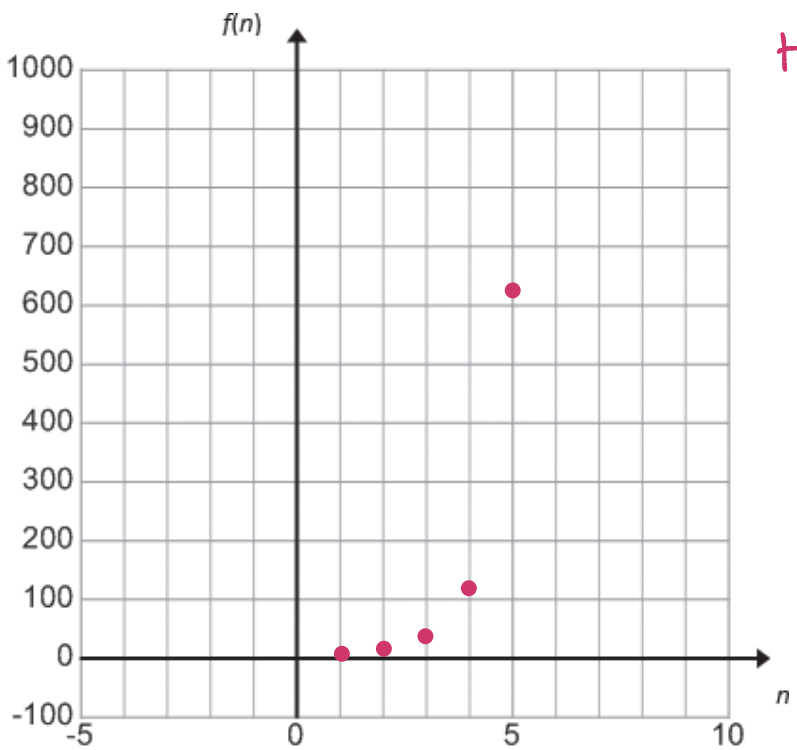
   $f(10) = 5^{10-1}$

   $f(10) = 5^9$

   $f(10) = 1{,}953{,}125$

   $a_1 = 5^{1-1} = 5^0 = 1$
   $a_2 = 5^{2-1} = 5^1 = 5$
   $a_3 = 5^{3-1} = 5^2 = 25$

   C. Graph the terms of the sequence as ordered pairs $(n, f(n))$ on a coordinate plane.



term # → term value

(1, 1)
(2, 5)
(3, 25)
(4, 125)
(5, 625)

decreases by $\frac{1}{3}$ → $\frac{2}{3}$ remaining

2. A radioactive substance decreases in the amount of grams by one-third each year. If the starting amount of the substance in a rock is 1,452 g, write an explicit formula for a sequence that models the amount of the substance left after the end of each year.

$r = \frac{2}{3}$

$f(n) = 1452\left(\frac{2}{3}\right)^{n-1}$ $\quad a_1 = f(1) = 1452$

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3. Write the first five terms of each geometric sequence.

A. $f(1) = 20, r = \frac{1}{2}$

$20, 10, 5, \frac{5}{2}, \frac{5}{4}$

B. $a_1 = 4, r = 3$

$4, 12, 36, 108, 324$

Write the explicit formula for the general term (n^{th} term) of a geometric sequence described. Then use it to find the indicated term of each sequence. The first term is $f(1)$ or a_1, and the common ratio is r.

4. Find $f(8)$ when $f(1) = 6, r = 2$.

$f(n) = 6(2)^{n-1}$

$f(8) = 6(2)^{8-1}$
$= 6(2)^7$
$= 6(128)$

$f(8) = 768$

* * * Don't forget order of operations

5. Find a_{12} when $a_1 = 5, r = -2$.

$a_n = 5(-2)^{n-1}$

$a_{12} = 5(-2)^{12-1}$

$a_{12} = 5(-2)^{11}$

$a_{12} = -10{,}240$

6. Find a_{25} when $a_1 = 1000, r = -\frac{1}{2}$.

$a_n = 1000\left(-\frac{1}{2}\right)^{n-1}$

$a_{25} = 1000\left(-\frac{1}{2}\right)^{25-1}$

$a_{25} = 1000\left(-\frac{1}{2}\right)^{24}$

$a_{25} = \frac{125}{2097152}$

7. Find $f(8)$ when $f(1) = 9000, r = -\frac{1}{3}$

$f(n) = 9000\left(-\frac{1}{3}\right)^{n-1}$

$f(8) = 9000\left(-\frac{1}{3}\right)^{8-1}$

$f(8) = -\frac{1000}{243}$

Write a formula for the n^{th} term of each geometric sequence. Then use the formula to find $f(7)$.

8. 3, 12, 48, 192, ...

$f(1) = 3 \quad r = 4$

$f(n) = 3(4)^{n-1}$

$f(7) = 3(4)^{7-1}$

$f(7) = 3(4)^6$

$f(7) = 12{,}288$

9. $18, 6, 2, \frac{2}{3}, \ldots..$

$f(7) = \frac{2}{81}$

Find the first 5 terms of the following functions.

10. $a_n = 1^n$

1, 1, 1, 1, 1, ...

$a_1 = 1^1 = 1$

$a_2 = 1^2 = 1$

$a_3 = 1^3 = 1$

$a_4 = 1^4 = 1$

$a_5 = 1^5 = 1$

11. $f(n) = 3^{n-2}$

$\frac{1}{3}, 1, 3, 9, 27$

$f(1) = 3^{1-2} = 3^{-1} = \frac{1}{3}$

$f(2) = 3^{2-2} = 3^0 = 1$

$f(3) = 3^{3-2} = 3^1 = 3$

$f(4) = 3^{4-2} = 3^2 = 9$

$f(5) = 3^{5-2} = 3^3 = 27$

Write a formula for the general term (the n^{th} term) of each geometric sequence. Then use the formula for $f(n)$ to find $f(9)$.

12. $5, -1, \frac{1}{5}, -\frac{1}{25}, \ldots.$ $r = -\frac{1}{5}$

$f(n) = 5\left(-\frac{1}{5}\right)^{n-1}$

$f(9) = 5\left(-\frac{1}{5}\right)^{9-1}$

$= 5\left(-\frac{1}{5}\right)^8$

$f(9) = \frac{1}{78{,}125}$

13. 0.07, 0.007, 0.0007, 0.00007,...

$r = 0.1$

$f(n) = 0.07(0.1)^{n-1}$

$f(9) = 0.07(0.1)^{9-1}$

$f(9) = 0.07(0.1)^8$

$f(9) = 0.0000000007$

14. A mine worker discovers an ore sample containing 500 mg of radioactive material. It is discovered that the radioactive material has a half life of 1 day. (This means that each day, half of the material decays, and only half is left.) Find the amount of radioactive material in the sample at the beginning of the 7th day.

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$a_1 = 500 \quad r = \frac{1}{2}$

$a_n = 500\left(\frac{1}{2}\right)^{n-1}$

$a_7 = 500\left(\frac{1}{2}\right)^{7-1}$

$a_7 = 500\left(\frac{1}{2}\right)^{6}$

$a_7 = \frac{500}{64} = 7.8125 \text{ mg}$

500, 250, 125 . . .
1 2 3

15. A culture of bacteria doubles every 2 hours. If there are 500 bacteria at the beginning, how many bacteria will there be after 24 hours?

f(12)

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$f(n) = 500(2)^{n-1}$

$f(13) = 500(2)^{13-1}$

$f(13) = 500(2)^{12}$

$f(13) = 20{,}480{,}000$

500, 1000, 2000, . . . 13
1 2 3
2 hours 4 hours

16. **Challenge** You complain that the hot tub in your hotel suite is not hot enough. The hotel tells you that they will increase the temperature by 10% each hour. If the current temperature of the hot tub is 75° F, what will be the temperature of the hot tub after 3 hours, to the nearest tenth of a degree?

r = 110% → 1.1

3 hours → 4th term in sequence.

75° 82.5° 90.75° 99.8125°

×1.1 ×1.1 ×1.1

$f(n) = 75(1.1)^{n-1}$

$f(4) = 7.5(1.1)^{4-1} = 99.825°F$

Challenge Problems

17. Find the common ratio and an explicit form in each of the following geometric sequences.

A. 4, 12, 36, 108, … $r = 3$ $a_1 = 4$

$a_n = 4(3)^{n-1}$

B. 162, 108, 72, 48, …

$r = \frac{108}{162} = \frac{2}{3}$ $a_1 = 162$

$a_n = 162\left(\frac{2}{3}\right)^{n-1}$

C. $\frac{4}{3}, \frac{2}{3}, \frac{1}{3}, \frac{1}{6}, \ldots$

$r = \frac{1}{2}$ $a_1 = \frac{4}{3}$

$a_n = \frac{4}{3}\left(\frac{1}{2}\right)^{n-1}$

D. $xz, x^2z^3, x^3z^5, x^4z^7, \ldots$

$r = \frac{x^2z^3}{xz} = xz^2$ $f(1) = xz$

$f(n) = xz(xz^2)^{n-1}$

18. The first term in a geometric sequence is 54, and the 5th term is $\frac{2}{3}$. Find an explicit form for the geometric sequence.

$f(1) = 54$ $r = ?$

$\frac{2}{3} = 54(r)^{5-1}$

$\frac{2}{3} = 54(r)^4$

$\frac{2}{3} = 54r^4$

$\frac{\frac{2}{3}}{54} = \frac{54r^4}{54}$

$\frac{1}{81} = r^4$

$\sqrt[4]{\frac{1}{81}} = \sqrt[4]{r^4}$

$\frac{1}{3} = r$

So…

$f(n) = 54\left(\frac{1}{3}\right)^{n-1}$

Explicit Formula

19. If 2, a, b, -54 forms a geometric sequence, find the values of a and b.

$a_1 = 2 \quad r = ?$

$-54 = 2(r)^{4-1}$
$-54 = 2(r)^3$
$-54 = 2r^3$
$\frac{-54}{2} = \frac{2r^3}{2}$

$-27 = r^3$
$\sqrt[3]{-27} = \sqrt[3]{r^3}$
$r = -3$

$f(n) = 2(-3)^{n-1}$

2, −6, 18, −54
×−3 ×−3 ×−3

$a = -6$
$b = 18$

20. Find the explicit form $f(n)$ of a geometric sequence if $f(3) - f(1) = 48$ and $\frac{f(3)}{f(1)} = 9$.

$x = f(3)$
$y = f(1)$

* use systems of equations to help solve.

sys. of eq.

$\begin{cases} x - y = 48 \\ \frac{x}{y} = 9 \end{cases}$

$x - y = 48$
$x = 9y$

$9y - y = 48$
$8y = 48$
$y = 6$ ← first term

Back Sub
$x - 6 = 48$
$x = 54$ ← third term

$f(3) = 6(r)^{3-1}$
$\frac{54}{6} = \frac{6r^2}{6}$
$9 = r^2$
$r = 3$

Explicit Formula

$f(n) = 6(3)^{n-1}$