LESSON 9

Geometric Sequences

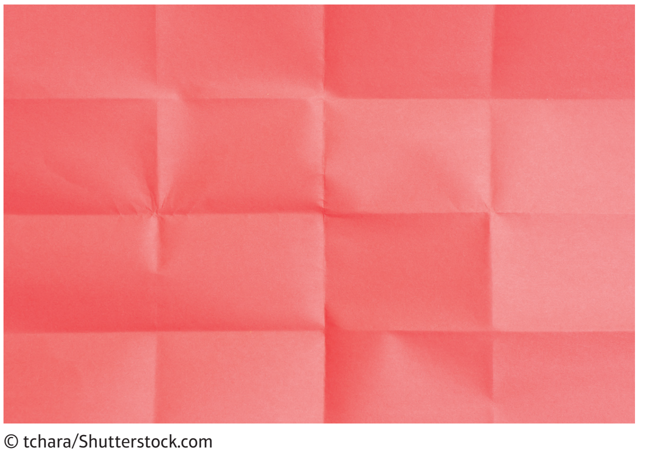
LEARNING OBJECTIVES

- Today I am: folding paper and counting the number of rectangles formed.
- So that I can: see how a geometric sequence is formed.
- I'll know I have it when I can: write an explicit formula for a geometric sequence.

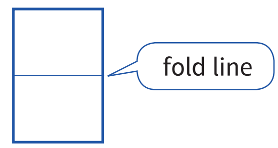
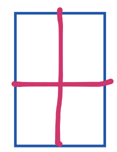
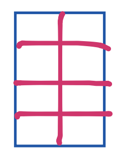
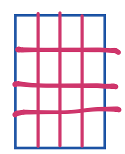
Opening Exercise

You will need: one sheet of paper

1. A. **Prediction** If we fold a rectangular piece of paper in half multiple times and count the number of rectangles created, what pattern do you expect in the number of rectangles?



B. **TRY IT!** Use the table below to record the number of rectangles in the paper folding experiment.

Term number (number of folds)	Sequence (number of rectangles)	Sketch of Unfolded Paper
1	2	
2	4 $2 \cdot 2$	
3	8 $2 \cdot 2 \cdot 2$	
4	16 $2 \cdot 2 \cdot 2 \cdot 2$	
5	32 $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	
6	64 $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot$	
n	2^n 2n	

C. Write an explicit formula for this sequence.

Function Notation

$f(n) = 2^n$

"*a* sub" Notation

$a_n = 2^n$

The sequence in the Opening Exercise is an example of a *geometric sequence*. Unlike arithmetic sequences, there is no common difference. Geometric sequences have a *common ratio*. = r

A geometric sequence is formed by **multiplying** each term, after the first term, by a non-zero constant.

The amount by which we multiply each time is called the **common ratio** of the sequence.

mult.

$\times \frac{1}{2}$

The sequence in the Opening Exercise was fairly simple and you saw something similar to it in Lesson 3 with Aunt Lucy's Plan 4. Let's look at more challenging sequences and determine the pattern for the explicit formula.

$\frac{6}{2} = 3$ $\frac{2^{nd}}{1^{st}} = r = \frac{\text{term after}}{\text{term before}}$

2. Consider the sequence: 2, 6, 18, …

$\times 3$ $\times 3$

$a_1 = 2$

$r = 3$

A. Write the next three terms.

2, 6, 18, 54, 162, 486

$\times 3$ $\times 3$ $\times 3$

B. Why isn't this an arithmetic sequence?

No adding, no common difference.

C. What is the pattern? What is the common ratio?

$r = 3$

D. Fill in the table below.

n	1	2	3	4	5	6
***f*(*n*)**	2	6	18	54	162	486
Pattern	2	2(3)	2(3)(3)	2(3)(3)(3)	2(3)(3)(3)(3)	
Exponential form	$2 \cdot 3^0$	$2 \cdot 3^1$	$2 \cdot 3^2$	$2 \cdot 3^3$	$2 \cdot 3^4$	$2 \cdot 3^5$

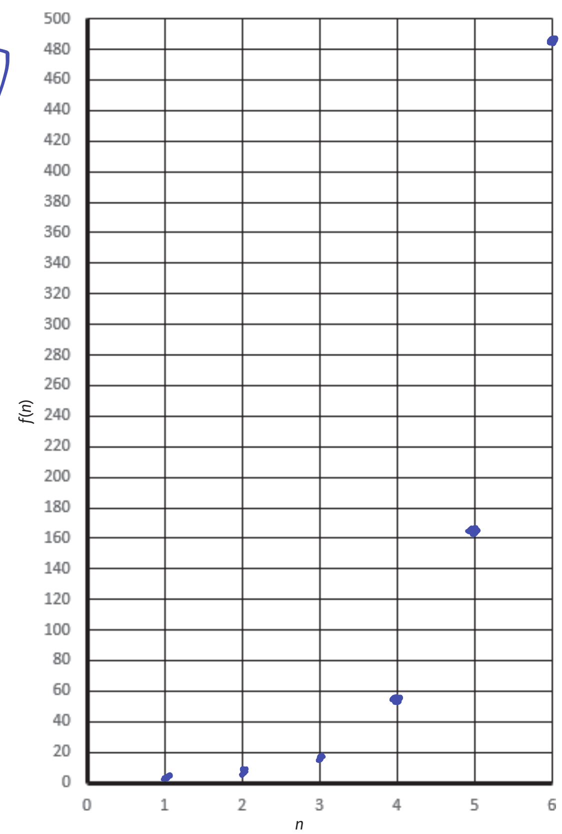
$f(n) = 2(3)^{n-1}$

E. Plot the points on the graph. What type of graph do the points make?

$f(n) = f(1)\, r^{n-1}$

$a_n = a_1 \cdot r^{n-1}$

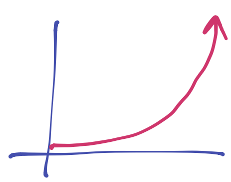
Geometric Sequence.



F. Write the explicit formula for this sequence based on the pattern in the table.

$f(n) = 2 \cdot 3^{n-1}$

3. We can now say that . . .



Arithmetic sequences are modeled by linear functions.
Geometric sequences are modeled by exponential functions.

4. To find the general term or explicit formula, $f(n)$ or a_n, of a geometric sequence you need the first term, $f(1)$, and the common ratio, r.

$f(n) = f(1) \cdot r^{n-1}$ **or** $a_n = a_1 \cdot r^{n-1}$

$f(1) = a_1 = 1^{st}$ term

r = common ratio = term after

r = common ratio term before

5. Find the common ratio for the following geometric sequences:

A. 1, 5, 25, 125, 625, …

$r = 5$

$f(n) = 1 \cdot (5)^{n-1}$

$a_n = 5^{n-1}$

B. 9, −3, 1, $\frac{-1}{3}$, $\frac{1}{9}$, …

$r = \frac{-3}{9} = -\frac{1}{3}$

$a_1 = 9$

$f(n) = 9\left(-\frac{1}{3}\right)^{n-1}$

6. Write the first six terms of the geometric sequence with first term 6 and common ratio −2.

6, −12, 24, −48, 96, −192

$\times(-2)$

$f(1) = 6$ $r = -2$

$f(n) = 6 \cdot (-2)^{n-1}$

7. Write a formula for the n^{th} term of each sequence below. Then find $f(8)$. ← 8^{th}

A. 12, 6, 3, $\frac{3}{2}$,……..

$f(1) = 12$

$r = \frac{6}{12} = \frac{1}{2}$

$f(n) = 12\left(\frac{1}{2}\right)^{n-1}$

$f(8) = 12\left(\frac{1}{2}\right)^{8-1}$

$= 12\left(\frac{1}{2}\right)^{7}$

$= 12\left(\frac{1}{128}\right)$

B. 3, 6, 12, 24, 48, ……

$a_1 = 3$

$r = 2$

$a_n = 3(2)^{n-1}$

$a_8 = 3(2)^{8-1}$

$= 3(2)^7$

$= 3(128)$

$= 384$

$= \frac{12}{128} = \frac{3}{32}$

8. Complete the example in the Lesson Summary.

Lesson Summary

A *geometric sequence* is a sequence in which each term after the first is obtained by multiplying the preceding term by a fixed non-zero constant. The amount by which we multiply each time is called the *common ratio* of the sequence.

Geometric sequences can be modeled by exponential functions.

The common ratio, r, is found by dividing any term after the first term by the term that directly precedes it.

General Term or Explicit Formula of a Geometric Sequence

The nth term of a geometric sequence with first term $f(1)$ or a_1 and common ratio r is

$$f(n) = f(1)r^{n-1} \text{ or } a_n = a_1 \cdot r^{n-1}$$

Both of these equations are explicit formulas. They are also known as the general term of the sequence.

Example: Find $f(8)$ of the geometric sequence when $f(1) = -4$ and the common ratio is -2.

$$f(n) = -4 \cdot (-2)^{n-1}$$
$$f(8) = -4 \cdot (-2)^{\_\_-1}$$
$$= -4 \cdot (-2)^{\_\_}$$
$$= -4 \cdot (\_\_\_\_\_)$$
$$= \_\_\_\_\_$$

NAME: ______________________ PERIOD: __________ DATE: __________

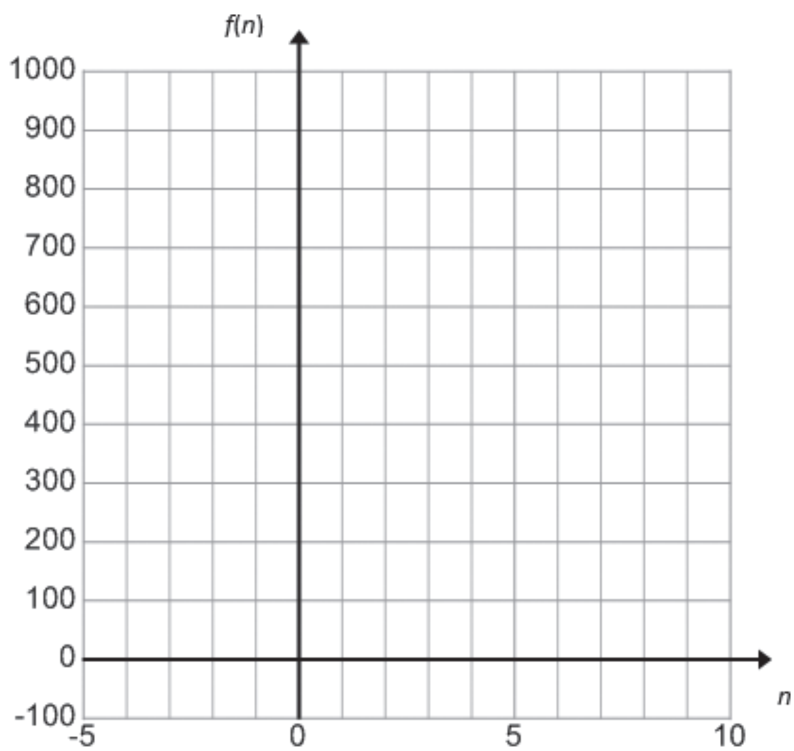
Homework Problem Set

1. Consider a sequence that follows a times 5 pattern: 1, 5, 25, 125, ….

   A. Write a formula for the n^{th} term of the sequence. Be sure to specify what value of n your formula starts with.

   B. Using the formula, find the 10^{th} term of the sequence.

   C. Graph the terms of the sequence as ordered pairs (n, f, (n)) on a coordinate plane.



2. A radioactive substance decreases in the amount of grams by one-third each year. If the starting amount of the substance in a rock is 1,452 g, write an explicit formula for a sequence that models the amount of the substance left after the end of each year.

3. Write the first five terms of each geometric sequence.

   A. $f(1) = 20, r = \frac{1}{2}$

   B. $a_1 = 4, r = 3$

Write the explicit formula for the general term (n^{th} term) of a geometric sequence described. Then use it to find the indicated term of each sequence. The first term is $f(1)$ or a_1, and the common ratio is r.

4. Find $f(8)$ when $f(1) = 6, r = 2$.

5. Find a_{12} when $a_1 = 5, r = -2$.

6. Find a_{25} when $a_1 = 1000, r = -\frac{1}{2}$.

7. Find $f(8)$ when $f(1) = 9000, r = -\frac{1}{3}$

Write a formula for the n^{th} term of each geometric sequence. Then use the formula to find $f(7)$.

8. 3, 12, 48, 192, ……

9. $18, 6, 2, \frac{2}{3}, \ldots\ldots$

Find the first 5 terms of the following functions.

10. $a_n = 1^n$

11. $f(n) = 3^{n-2}$

Write a formula for the general term (the n^{th} term) of each geometric sequence. Then use the formula for $f(n)$ to find $f(9)$.

12. $5, -1, \frac{1}{5}, -\frac{1}{25}, \ldots\ldots$

13. 0.07, 0.007, 0.0007, 0.00007,…

14. A mine worker discovers an ore sample containing 500 mg of radioactive material. It is discovered that the radioactive material has a half life of 1 day. (This means that each day, half of the material decays, and only half is left.) Find the amount of radioactive material in the sample at the beginning of the 7^{th} day.

15. A culture of bacteria doubles every 2 hours. If there are 500 bacteria at the beginning, how many bacteria will there be after 24 hours?

16. **Challenge** You complain that the hot tub in your hotel suite is not hot enough. The hotel tells you that they will increase the temperature by 10% each hour. If the current temperature of the hot tub is 75° F, what will be the temperature of the hot tub after 3 hours, to the nearest tenth of a degree?

Challenge Problems

17. Find the common ratio and an explicit form in each of the following geometric sequences.

    A. 4, 12, 36, 108, …

    B. 162, 108, 72, 48, …

    C. $\frac{4}{3}, \frac{2}{3}, \frac{1}{3}, \frac{1}{6}, \ldots$

    D. $xz,\ x^2z^3,\ x^3z^5,\ x^4z^7, \ldots$

18. The first term in a geometric sequence is 54, and the 5th term is $\frac{2}{3}$. Find an explicit form for the geometric sequence.

19. If 2, a, b, -54 forms a geometric sequence, find the values of a and b.

20. Find the explicit form $f(n)$ of a geometric sequence if $f(3) - f(1) = 48$ and $\frac{f(3)}{f(1)} = 9$.