LESSON 10 The Packaging Problem

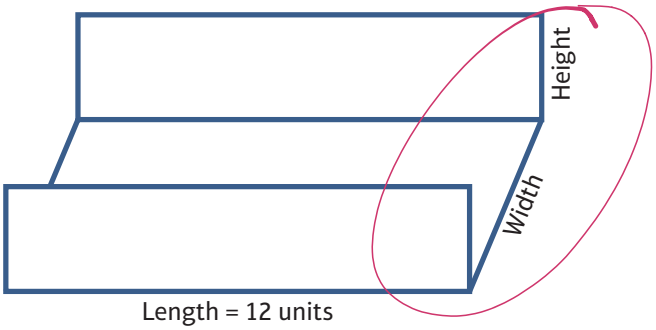
LEARNING OBJECTIVES

- Today I am: creating several "sleeves" for a licorice package.
- So that I can: determine which one will give me the greatest volume.
- I'll know I have it when I can: use a table, graph or equation to find the maximum volume.

Opening Exercise

Adapted from a lesson created by the UCLA Curtis Center, Department of Mathematics and Teaching

A candy company wants to package their licorice in an open-topped and open-sided cardboard "sleeve" and then place the filled sleeve in a plastic bag.



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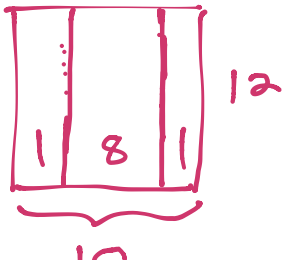
The licorice is 12 units long and they plan to use 12 unit by 10 unit pieces of cardboard to fold into sleeves. As the Package Design Engineer for this project, they've asked you to determine the dimensions of the sleeve that will result in the maximum amount of licorice.



You will need: Packaging handout, scissors

1. Use the Packaging handout to create at least 7 different-sized licorice sleeves. You'll record your data in the table below.
2. Record the height, width and volume of each sleeve you create. Notice that the length is 12 units for all of the sleeves since that is the length of the licorice.

$l \times w \times h$

h (height)	*w* (width)	*l* (length)	*V* (volume)
1	8	12	96
2	6	12	144
3	4	12	144
4	2	12	96
5	0	12	0
2.5	5	12	150
3.5	3	12	126
0	10		0

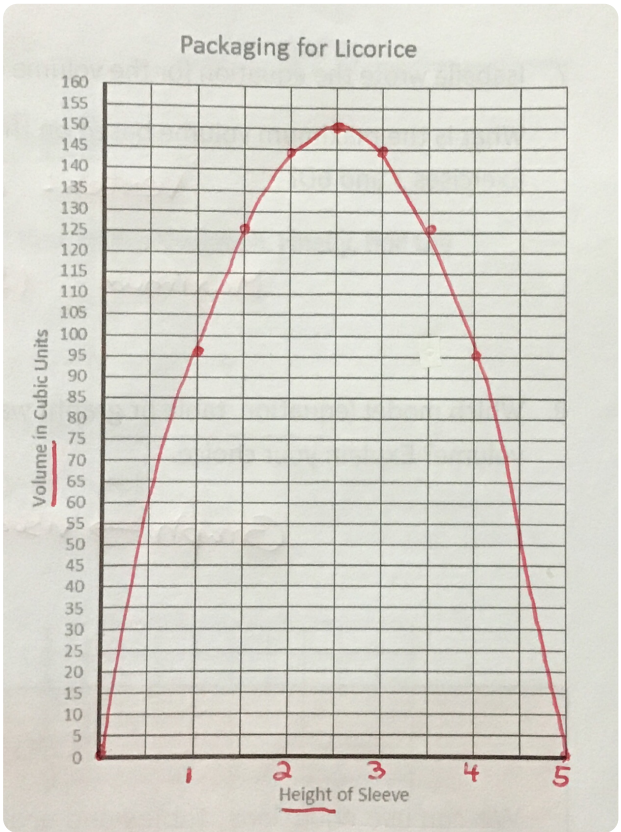
3. Based on the table, what is the maximum volume?

150 $unit^3$

4. When would the volume be 0? How can you tell?

When the width is 0.

5. On the grid at the right, plot the volume and height data. You'll need to write in values for the height. Try to use as much of the horizontal axis as possible.



6. A. Does it make sense to connect the data points? Explain your thinking.

Yes for continuous data.

B. What type of function is created from the data?

Quadratic

C. Use the graph to estimate where the horizontal intercepts would be. What does this mean in this context?

x-intercepts

0 and 5

D. Based on the graph, what is the maximum volume? Does this agree with your estimate in Exercise 3.

150 units^3

E. What is a reasonable domain for this problem? What does the domain mean in this context?

$[0, 5]$

F. What is a reasonable range for this problem? What does the range mean in this context?

$[0, 150]$

7. Isabella wrote the equation for the volume of the sleeve in vertex form as $V = -24\left(h - \frac{5}{2}\right)^2 + 150$.

   What is the maximum volume based on this equation? Does this agree with your estimates in Exercises 3 and 6D?

8. Which model (equation, table or graph) was the easiest for you to use to find the maximum volume? Explain your choice.

Lesson Summary

We can use equations, tables and graphs to model real-world situations. In the graph below, we see the height of the basketball in relation to its horizontal distance traveled.

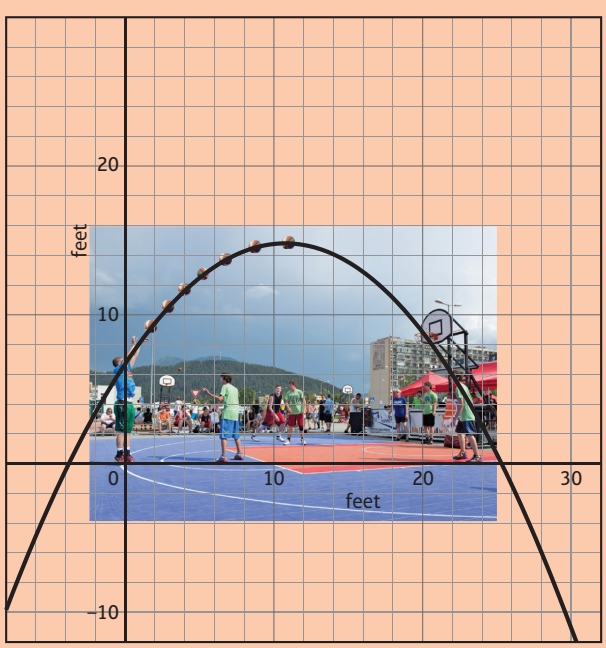


Photo source: © tempisch/Shutterstock.com

$$y = -0.07(x - 10.76)^2 + 14.8$$

x (horizontal distance)	y (height of the basketball)
0	6.70
6	13.21
12	14.69
18	11.13
24	2.53

NAME: _____ PERIOD: _____ DATE: _____

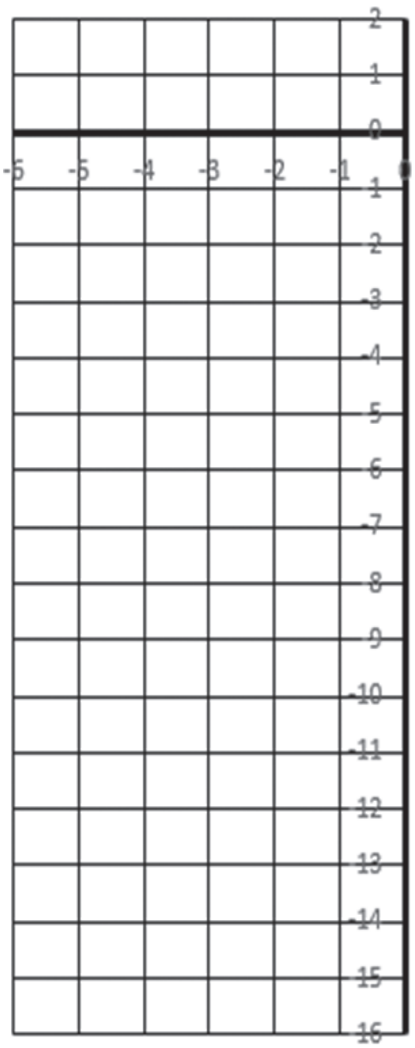
Homework Problem Set

For each problem, determine the vertex, the *y*-intercept, and then sketch the graph. Finally, find the *x*-intercepts for each graph.

1. $y = -2(x + 3)^2 + 2$

   vertex: (_____, _____)

   y-intercept: _____

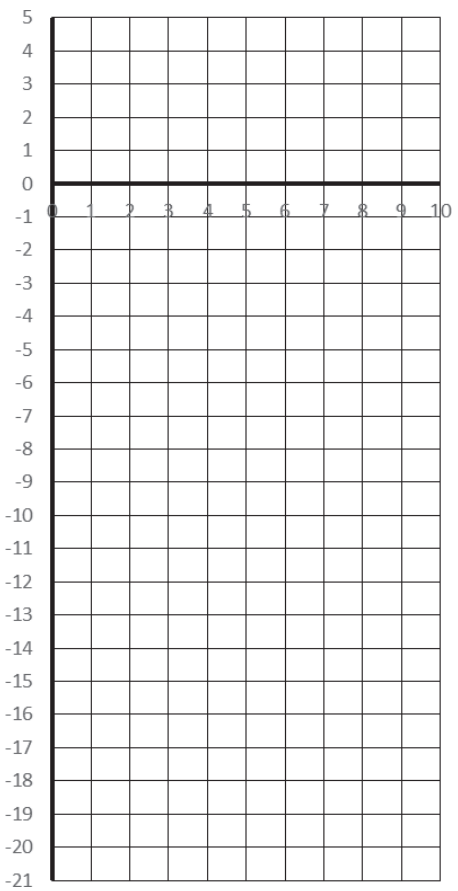


   x-intercepts: _____ and _____

2. $y = -(x - 5)^2 + 4$

   vertex: (_____, _____)

   y-intercept: _____

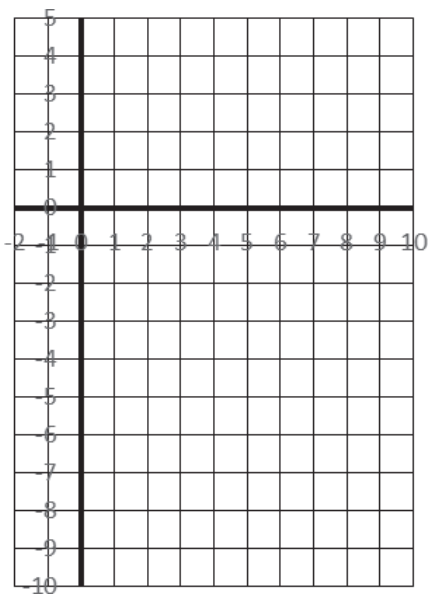


   x-intercepts: _____ and _____

3. $y = \frac{1}{2}(x - 4)^2 - 8$

vertex: (_____, _____)

y-intercept: _____

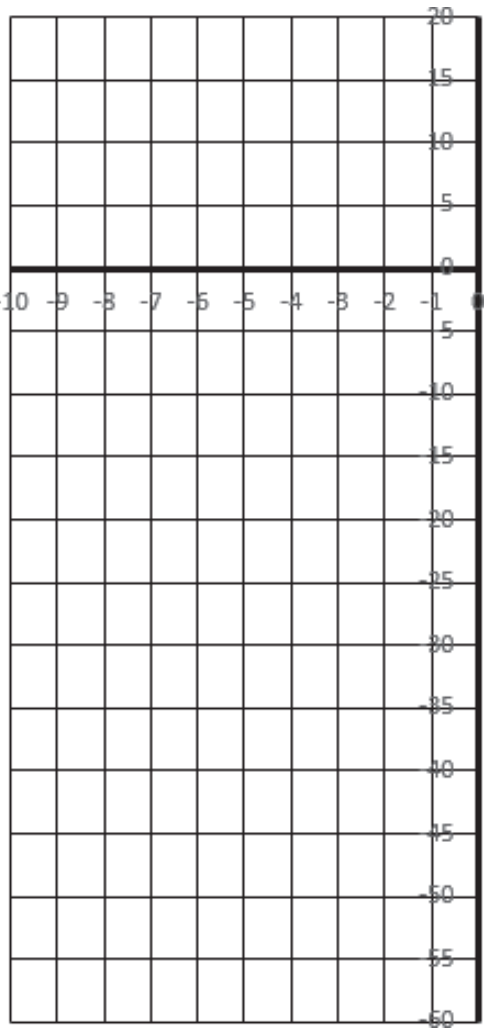


x-intercepts: _____ and _____

4. $y = -2(x + 6)^2 + 18$

vertex: (_____, _____)

y-intercept: _____



x-intercepts: _____ and _____

Spiral REVIEW—Simplifying Radical Expressions

Simplify each radical expression.

5. $\sqrt{36}$

6. $-\sqrt{49}$

7. $\sqrt{100} + \sqrt{100}$

8. $\sqrt{200}$

9. $\sqrt{16x^2}$

10. $3\sqrt{64}$

11. $-2\sqrt{9} + 3\sqrt{16}$

12. $\sqrt{81} + \sqrt{4}$

13. $\sqrt{1} - \sqrt{0}$

14. $\sqrt{28}$

15. $\sqrt{56}$

16. $\sqrt{4 + 9 + 3}$

Spiral REVIEW

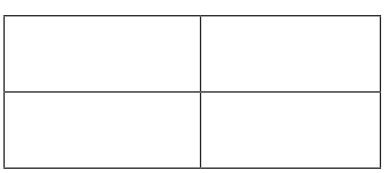
Multiply—Monomial with a Binomial

17. $6(y^2 + 3)$	18. $-9y(3y + 2)$	19. $3(7b - 5a)$
20. $-2c(7c - 1)$	21. $3(x^2 - 9)$	22. $-2(3x - 6)$

Spiral REVIEW

23. Below are examples of two methods for multiplying two binomials.

A. Use each method to multiply $(2x - 3)$ with $(x + 4)$.

Method 1: Table or Box	**Method 2: Double Distribution**
$x \quad - \quad 5$ x + 3: x^2 | $-5x$; $3x$ | -15 $x^2 - 2x - 15$	$(x + 3)(x - 5) = x(x - 5) + 3(x - 5)$ $= x^2 - 5x + 3x - 15$ $= x^2 - 2x - 15$
	$(2x - 3)(x + 4)$

B. Which method do you prefer? Explain your thinking.

Multiply—Binomial with a Binomial

24. $(x + 1)(x - 7)$	25. $(x + 9)(x + 2)$	26. $(x - 5)(x - 3)$
27. $(x - 1)(x + 7)$	28. $(x - 9)(x - 2)$	29. $(x + 5)(x + 3)$