LESSON

11

Using the Zero Product Property to Find Horizontal Intercepts

LEARNING OBJECTIVES

- Today I am: exploring what happens when two factors have a product of 0.
- So that I can: solve equations that are in factored form.
- I'll know I have it when I can: write an equation that only has 53 and 22 as solutions.

Opening Discussion

1. A. Jenna said the product of two numbers is 20. Would the factors have to be 4 and 5? Why?


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   B. Julie said the product of two numbers is 20. Would both factors have to be less than 20? Why?

   C. Justin said the product of two numbers is 20. Would both factors have to be positive? Why?

2. Jeremy said the product of two numbers is 0. What do you know must be true about the factors? Why?

3. A. Demanding Dwight insists that you give him two solutions to the following equation.

let y=0 → (x-inter.)

$$\overset{y}{0} = (x-10)(x+20)$$

Equations written in this way are in ***factored form***.

What are the two solutions?

$a \cdot b = 0$

$a = 0$ or $b = 0$

$x - 10 = 0$ or $x + 20 = 0$

$x = 10$ $\quad x = -20$

B. Demanding Dwight now wants FIVE solutions to the following equation:

$$(x-10)(2x+6)(x^2-36)(x^2+10)(x+20) = 0$$

What are the five solutions?

$x^2 - 36 = 0$

$\sqrt{x^2} = \sqrt{36} \rightarrow x = \pm 6$

Do you think there might be a sixth solution? Explain.

$x - 10 = 0$ $\quad 2x + 6 = 0$ $\quad \cancel{x^2 + 10 = 0}$ $\quad x + 20 = 0$

$x = 10$ $\quad x = -3$ $\quad \cancel{x^2 = -10}$ $\quad x = -20$

4. Let's summarize what we've observed.

If a • b = 0, then either a = 0 *or b =* 0 *or a = b =* 0.

This is known as the **zero product property**.

5. Consider the equation $(x - 4)(x + 3) = 0$.

A. Rewrite the equation as a compound statement.

$x - 4 = 0$ or $x + 3 = 0$

$x = 4$ $\quad x = -3$

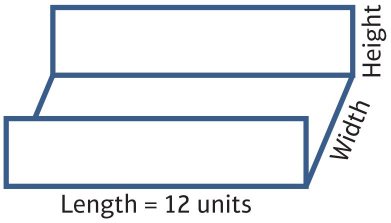
B. Find the two solutions to the equation.

see above.

$\{-3, 4\}$

set notation.

Let's think back on our work with the licorice packaging from the last lesson. We examined the volume of each "sleeve" in both table and graph form and we were given the equation for the volume. We'll now come up with our own equation for the volume.

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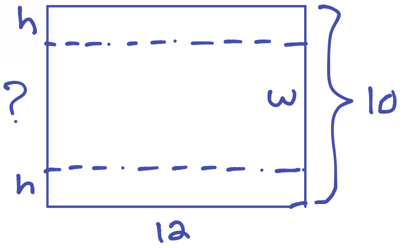
$w=10-2h$?



Here are the details we know:

- The length is always 12 units. (This is the length of the licorice.)
- The pieces of cardboard to make the sleeve are 12 units by 10 units.
- The cardboard is folded to create a "sleeve" for the licorice to be placed in.

6. A. How can we determine the width of the sleeve, if we know the height?

$w = 10 - 2h$

B. Write an expression for the width based on the height. Fill in the appropriate cell in the table below.

h (height)	*w* (width)	*l* (length)	*V* (volume)
h	$10-2h$	12	$V =$ $12h(10-2h)$

C. Complete the equation for the general formula for the volume of the sleeves. Fill in the appropriate cell in the table above.

7. Which of the equations below could be used to accurately represent the volume of the sleeve? Circle all that would work.

$V=12h(10-2h)$

[✓ **$V = h \cdot (10 - 2h) \cdot 12$**]

[✓ **$V = 12h \cdot (10 - 2h)$**]

[✓ **$V = (10h - 2h^2) \cdot 12$**]

[✓ **$V = -24h \cdot (h - 5)$**]

[✓ **$V = -24h^2 + 120h$**]

x-intercept, let y=0

8. The horizontal intercept is where the curve meets the horizontal axis. In this case, the horizontal axis is the height. What does the horizontal intercept mean in our licorice packaging problem?

$V = 12h(10-2h)$

$0 = 12h(10-2h)$

$12h=0$ or $10-2h=0$

$h=0$ or $h=5$

9. Use one of the circled equations to find the horizontal intercepts algebraically.

See above

Reflection

10. Which form of the equation made it easiest to find the horizontal intercepts? Which would be the most difficult to use?

11. A. Can you easily tell what the vertex is from these equations?

B. What would the vertex tell us in this situation?

12. Isabella wrote the equation for the volume of the sleeve in vertex form as $V = -24\left(h - \frac{5}{2}\right)^2 + 150$. Prove that Isabella's equation $V = -24\left(h - \frac{5}{2}\right)^2 + 150$, is equivalent to any one of the equations you circled in Exercise 7.

13. Could you use Isabella's equation to find the horizontal intercepts? Explain your reasoning.

x-intercepts, zeros, roots

Practice Problems—Find the solutions for each equation.

14. $(x + 1)(x + 2) = 0$ $x+1=0$ or $x+2=0$ $x=-1$ $x=-2$	15. $(3x - 2)(x + 12) = 0$ $3x-2=0$ or $x+12=0$ $x=\frac{2}{3}$ $x=-12$	16. $(x - 3)(x - 3) = 0$ $x-3=0$ $x=3$
17. $(x + 4)(x - 6)(x - 10) = 0$ $x=-4$ or $x=6$ or $x=10$	18. $x(x - 6) = 0$ $x=0$ or $x-6=0$ $x=6$	19. $(x + 4)(x - 5) = 0$ $x=-4$ or $x=5$

Practice Problems—Multiply each set of binomials.

20. $(x + 1)(x - 1)$ $x^2-x+x-1$ x^2-1	21. $(x + 9)(x - 9)$ $x^2-9x+9x-81$ x^2-81	22. $(x - 5)(x + 5)$ x^2-25
23. $(x - 7)(x + 7)$ x^2-49	24. $(x + 2)(x - 2)$ x^2-4	25. $(x - 3)(x + 3)$ x^2-9

26. What pattern do you notice with Exercises 20–25?

The middle terms cancel.

27. State this as a rule with variables.

$(a - b)(a + b) = a^2 \cancel{+ab} \cancel{-ab} - b^2$

$= a^2 - b^2$

difference of two squares.

Lesson Summary

The *zero-product property* says that if $a \cdot b = 0$,

then either $a =$ ____ or $b =$ ____ or $a = b =$ ____.

We can use the factored form of the equation to find the x-intercepts (horizontal intercepts):

Factored Form of the Quadratic
$y = (x - 3)(x + 5)$
$0 = (x - 3)(x + 5)$
$x - 3 = 0$ or $x + 5 = 0$
$x = 3$ or $x = -5$

$y = 0$ when the parabola hits the x-axis.

Binomials expressions in the form $(x - y)(x + y)$ give a binomial product of the form $x^2 - y^2$. We can use this as a short cut when finding the factored form of a quadratic function.

NAME: ______________________ PERIOD: __________ DATE: __________

Homework Problem Set

Find the solution set of each equation:

1. $(x - 1)(x - 2)(x - 3) = 0$

2. $(x - 16.5)(x - 109) = 0$

3. $x(x - 4)(x + 5) = 0$

4. $3(x + 5)(2x - 3) = 0$

5. $\left(\frac{1}{2}x + 6\right)(x - 3) = 0$

6. $(5x - 10)(x + 2) = 0$

7. $(\mathrm{x} - 2)^2 = 0$

8. $(x + 8)^2 = 0$

9. $x^2 \cdot (x - 7) = 0$

10. Using what you learned in this lesson, create an equation that has 53 and 22 as its only solutions.

Multiply each set of binomials.

11. $(x - 10)(x + 10)$

12. $(x - 1)(x + 1)$

13. $(x - 5)(x + 5)$

14. $(y - 3)(y + 3)$

15. $(a - 7)(a + 7)$

16. $(b - 6)(b + 6)$

17. $(x + 10)(x + 10)$

18. $(y + 4)(y - 4)$

19. $(c + 2)(c - 2)$

20. $(x + \frac{1}{2})(x - \frac{1}{2})$

21. $(w + 2)(w + 3)$

22. $(x - 0.2)(x + 0.2)$

23. $(b - 1.5)(b + 1.5)$

24. $(x - 2y)(x + 2y)$

25. $(2x - 3)(2x + 3)$

Spiral REVIEW—Solving Equations

Solve each equation.

26. $3x + 4 = 2x - 5$

27. $\frac{1}{4}x = 5$

28. $\frac{-m}{8} = -5$

29. $\frac{-5}{6}x = \frac{3}{4}$

30. $\frac{1}{4} + \frac{1}{2}t = 4$

31. $\frac{2}{5}(x - 2) = -3$

32. $3.5x + 0.8 = -50.9 - 5.9x$

33. $0.3x - 0.24 = 0.36 + 0.52x$

34. $\frac{3}{4}(2x + 1) = 2$

35. $5x = -7x + 6$

36. $7 - 3x = x - 4(2 + x)$

37. $6(4x - 5) = 24x - 30$

38. $2x + 5 = 2x - 3$

39. $3(x + 1) - 5 = 3x - 2$

40. $4(2x - 8) = 3(2 - 3x)$

41. $-2x = -3x + 12 - 2x$

42. $8(b + 1) + 4 = 3(2b - 8) - 16$

43. $4x - 6 = x + 9$