LESSON 12 Roots of Quadratic Functions

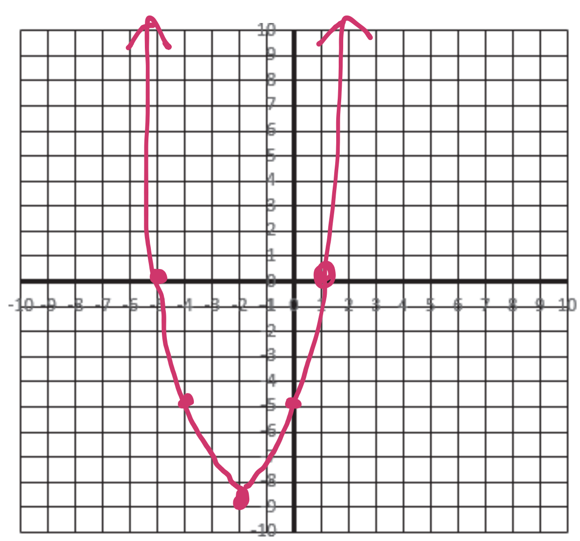
LEARNING OBJECTIVES

- Today I am: sketching parabolas with limited information.
- So that I can: identify the strengths of each form of a quadratic equation.
- I'll know I have it when I can: use a graph to determine the x-intercepts and an equation for the function.

Opening Activity

1. You are told to sketch a parabola with the following attributes:

   - The x-intercepts or *roots* are at $(-5, 0)$ and $(1, 0)$.
   - The vertex is at $(-2, -9)$.
   - The y-intercept is at $(0, -5)$.



Use the grid provided to sketch this parabola. Is it possible to sketch another parabola with the same attributes? Explain your thinking.

2. A. Write an equation of a parabola with a vertex at (−2, −9). $y = (x+2)^2 - 9$

$y = a(x-h)^2 + k$

B. Check if this equation fulfills the other attributes for the parabola on the previous page. Be sure to show your work.

$y = (x+2)(x+2) - 9$

$y = x^2 + 4x + 4 - 9$

3. Write the equation of your parabola from Exercise 2 in standard form. $y = x^2 + 4x - 5$

$(-5, 0) \quad (1, 0)$

4. A. Substitute each x-intercept into your standard form equation. What value do you get for y?

$y = (-5)^2 + 4(-5) - 5$

$y = 25 - 20 - 5$

B. Why does this make sense?

$y = 0$

5. Another form of the equation for this parabola is $y = (x + 5)(x - 1)$. This is called ***factored form*** and gives us information about the x-intercepts. How does this form of the equation help us find the x-intercepts of −5 and 1?

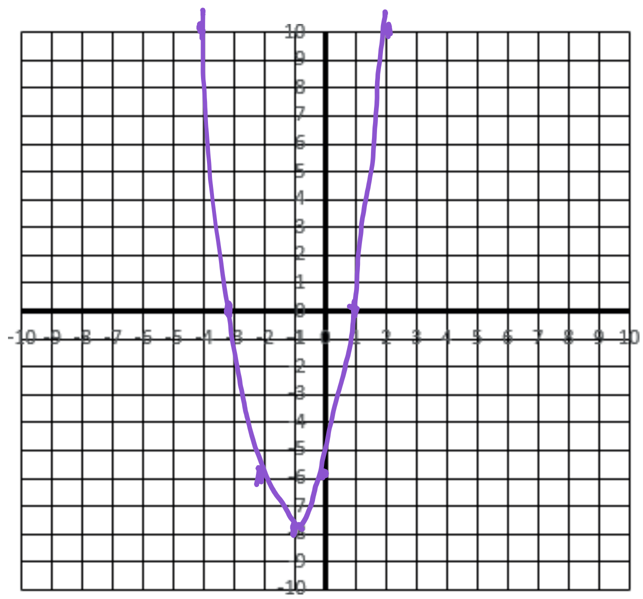
$y = (x+5)(x-1)$

$0 = (x+5)(x-1)$

$x + 5 = 0$ or $x - 1 = 0$

$x = -5 \qquad x = 1$

6. So far we've worked with vertex form and standard form of quadratic functions. The factored form can be very useful too. Let's explore one quadratic function written in three different ways and determine what is useful about each form. (Notice that **you** need to write the standard form.)

Form of the Equation	**What can you easily find? Find it!**	**Sketch of the Parabola**
Vertex: $y = 2(x + 1)^2 - 8$ $2(0+1)^2-8$	-vertex: $(-1,-8)$ -concave up -axis: $x=-1$	
Standard: $y = 2(x+1)(x+1)-8$ $2(x^2+2x+1)-8$ $2x^2+4x+2-8$ $y=2x^2+4x-6$	- y-inter: $(0,-6)$	
Factored: $y = 2(x - 1)(x + 3)$ $0=2(x-1)(x+3)$	- x-inter. $(1,0)$ $(-3,0)$	

$x-1=0$ $x+3=0$
$x=1$ $x=-3$

One example is just not enough to see the patterns in factored form. You'll now do a Desmos activity with this idea.

You will need: a Chromebook, class code for the Desmos activity *Roots of Quadratic Functions: Looking for Patterns in Special Cases*

7. Go to student.desmos.com and type in class code: ______________________. You will be rewriting equations in standard form given the factored form, and writing the factored form given the graph.

8. On Screen 9, you are given the standard form $y = -4x^2 - 24x - 36$. In factored form this will look like

$$y = -4(x + \_\_\_\_)(x + \_\_\_\_)$$

9. On Screen 15, you are given the standard form $y = -12x^2 - 12x$. In factored form this will look like

$$y = -12x(x + \_\_\_\_)$$

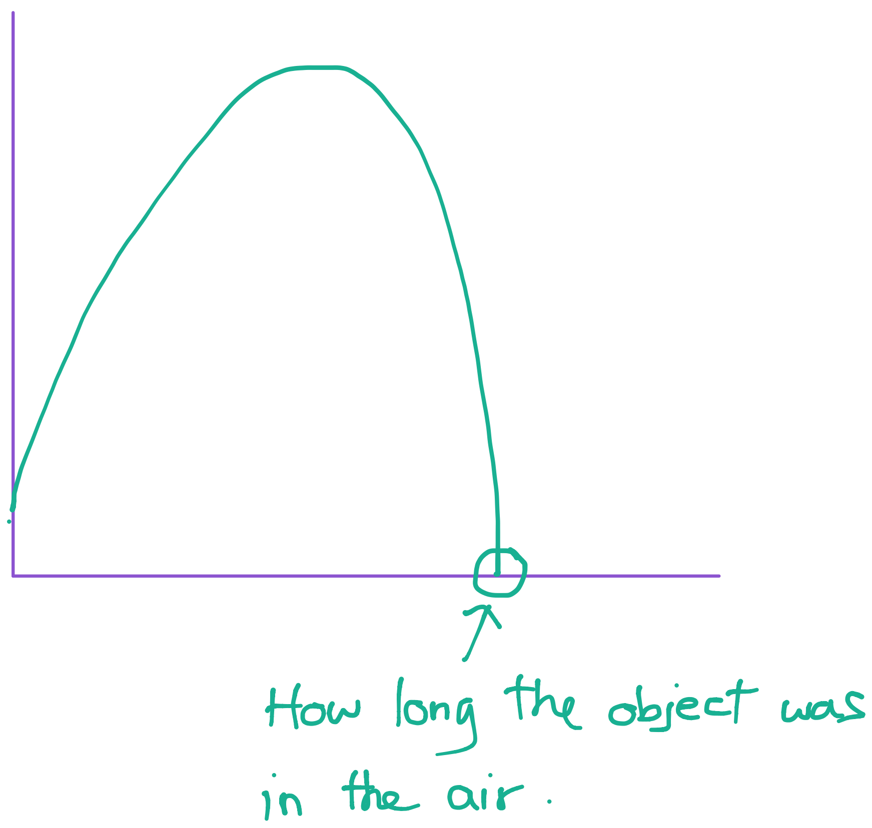
10. On Screens 22 and 23, the three functions given can be written in factored form as:

$f(x) = (x - 3)(x + 3)$

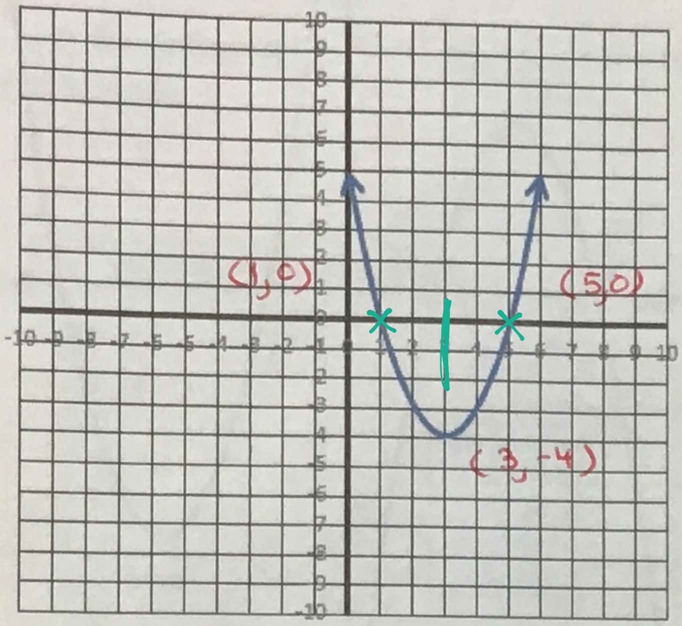
$g(x) = 2x(x - 9)$

$h(x) = (x + 3)(x + 3)$

11. Besides being useful when graphing parabolas, the x-intercepts also help us solve real-life problems. In the ball toss activity from Lesson 1, what information did the x-intercept provide?



Looking for Patterns



x-intercepts: 1 and 5

Vertex: (3 , -4)

Equations: $y=(x-1)(x-5)$ $y=(x-3)^2-4$

$y=x^2-5x-x+5$

$y=x^2-6x+5$

$y=(x-3)(x-3)-4$

$y=x^2-3x-3x+9-4$

$y=x^2-6x+5$

13. Prove that these equations are equivalent.

See above.

14. What is the axis of symmetry for this parabola? Can you find it with only the *x*-intercepts or the vertex? Explain your thinking.

vertex (3, -4)

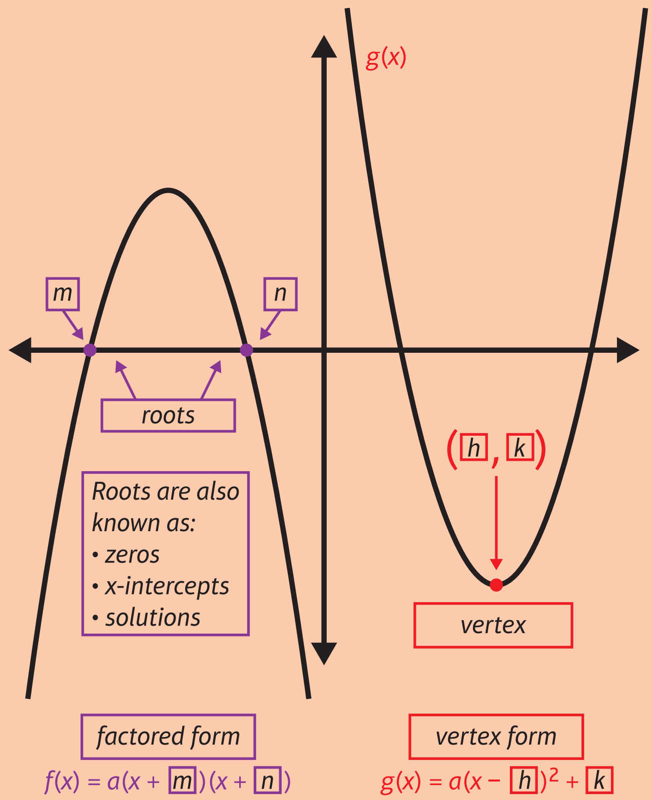
axis of sym: $x=3$

x-intercepts 1, 5

midway between the two

$\frac{1+5}{2}=3 \rightarrow x=3$

Lesson Summary



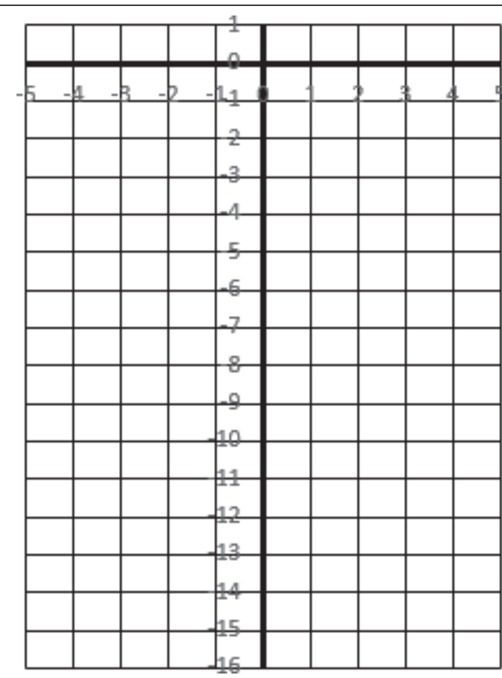
Source: Planetxmap.com

NAME: ______________________ PERIOD: __________ DATE: __________

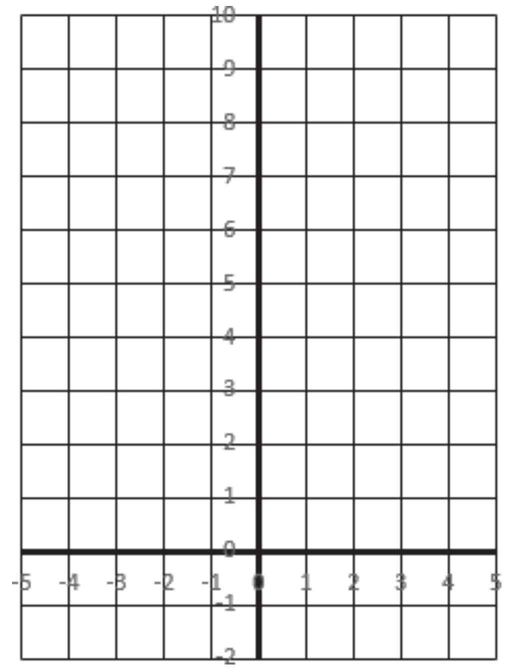
Homework Problem Set

For each problem below, tell what information you can get from the form of the equation and then sketch a graph of the parabola.

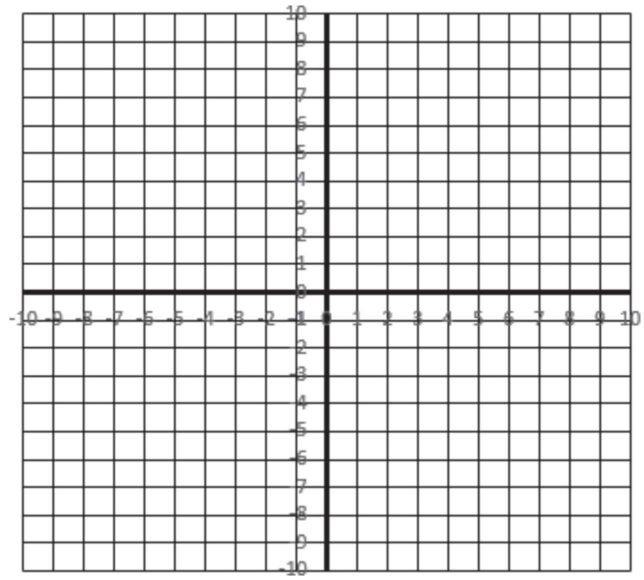
1.

Form of the Equation	What can you easily find? Find it!	Sketch of the Parabola
Vertex: $y = x^2 - 16$		
Standard: $y = x^2 - 16$		
Factored: $y = (x - 4)(x + 4)$		

2.

Form of the Equation	What can you easily find? Find it!	Sketch of the Parabola
Vertex: $y = -(x - 2)^2 + 9$		
Standard: $y = -x^2 + 4x + 5$		
Factored: $y = -(x - 5)(x + 1)$		

3.

<table>
<tr><th>Form of the Equation</th><th>What can you easily find? Find it!</th><th>Sketch of the Parabola</th></tr>
<tr><td>Vertex:
$y = -(x - 1)^2 + 9$</td><td></td><td rowspan="3"></td></tr>
<tr><td>Standard:
$y = -x^2 + 2x + 8$</td><td></td></tr>
<tr><td>Factored:
$y = -(x - 4)(x + 2)$</td><td></td></tr>
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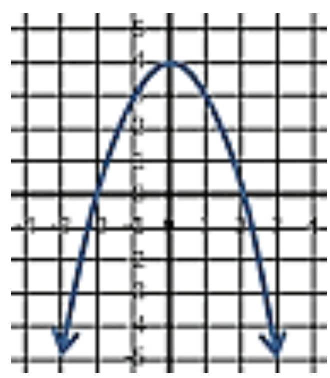
4. For the parabola graph at the right, determine the *x*-intercepts and vertex. Then write two equations using the information given.

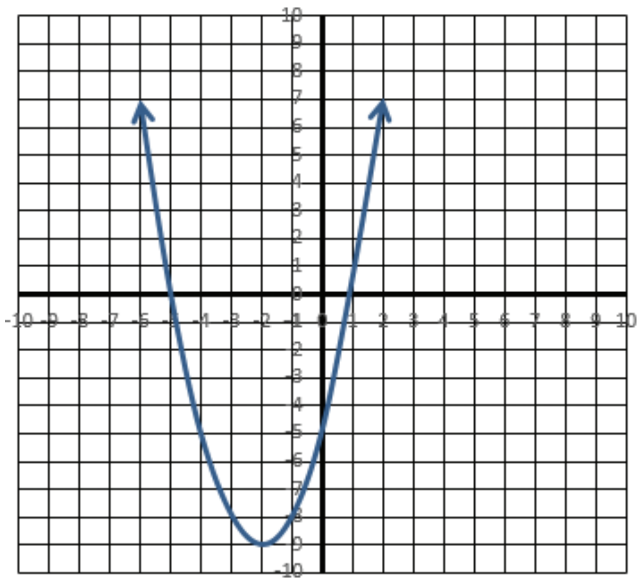


x-intercepts: ________ and ________ Vertex: (________, ________)

Equations: ___________________________ ___________________________

5. Prove that the equations in Problem 4 are equivalent.

6. For the parabola graph below, determine the *x*-intercepts and vertex. Then write two equations using the information given.



x-intercepts: ________ and ________ Vertex: (________, ________)

Equations: ____________________________________ ____

7. Prove that the equations in Problem 6 are equivalent.

Spiral REVIEW—Multiplying Binomials

Multiply the following binomials. Write the answers in standard form, which in this case takes the form $ax^2 + bx + c$, where a, b, and c are constants.

8. $(x + 1)(x - 7)$

9. $(x + 9)(x + 2)$

10. $(x - 5)(x - 3)$

11. $(x + 2)(x - 1)$

12. Describe any patterns you noticed in Problems 8–11.

13. The square parking lot at Gene Simon's Donut Palace is going to be enlarged so that there will be an additional 30 ft. of parking space in the front of the lot and an additional 30 ft. of parking space on the side of the lot, as shown in the figure below. Write an expression in terms of x that can be used to represent the area of the new parking lot. Explain how your solution is demonstrated in the area model.