LESSON 13

Exploring Factored Form

LEARNING OBJECTIVES

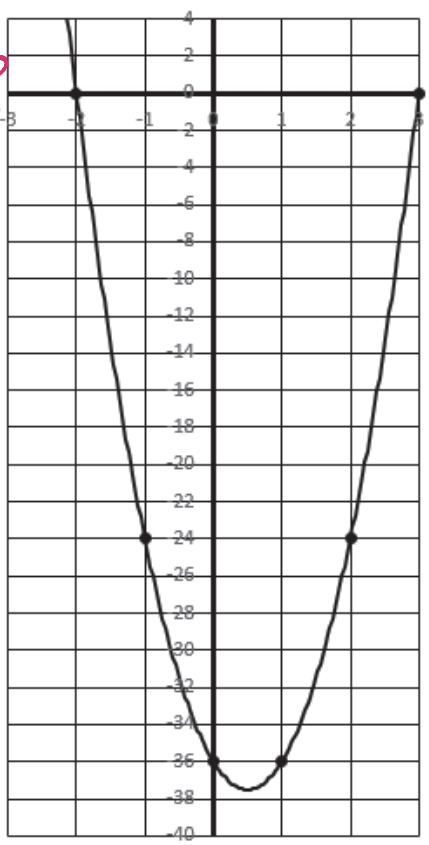
- Today I am: using Algebra Tiles to model factoring quadratic expressions.
- So that I can: find patterns to make factoring easier.
- I'll know I have it when I can: draw the generic rectangle to factor $y = 4x^2 - 12x + 9$.

Opening Activity

On the right is a graph of the equation $y = 6(x - 3)(x + 2)$.

$x - 3 = 0$ $\quad x + 2 = 0$

$x = 3$ $\quad x = -2$



It is also the graph of:

$y = 3(2x - 6)(x + 2)$ $\qquad$ $y = 2(3x - 9)(x + 2)$

$y = 2(x - 3)(3x + 6)$ $\qquad$ $y = 3(x - 3)(2x + 4)$

$y = (3x - 9)(2x + 4)$ $\qquad$ $y = (2x - 6)(3x + 6)$

$3x - 9 = 0$ $\quad 2x + 4 = 0$

$3x = 9$ $\quad 2x = -4$

$x = 3$ $\quad x = -2$

1. **Discussion** How is this possible? What other equations can you write that are equivalent to these?

2. We saw from the Opening Exercise that there are many ways to write equivalent equations or expressions. For the five expressions below, match each to the equivalent expressions.

A. $4x^2 - 16$ Matches:	B. $3x^2 - 9x$ Matches:	C. $10x + 10$ Matches:	D. $5x^2 + 25x$ Matches:	E. $8x - 24$ Matches:
#6, 12	2 #7, 11	#1, 4, 13	#3, 8, 14	#5, 9, 10

1. $5(2x + 2)$	2. $3x(x - 3)$	3. $5x(x + 5)$
4. $10(x + 1)$	5. $4(2x - 6)$	6. $4(x^2 - 4)$ $4(x+2)(x-2)$
7. $3(x^2 - 3x)$	8. $x(5x + 25)$	9. $2(4x - 12)$
10. $8(x - 3)$	11. $x(3x - 9)$	12. $2(2x^2 - 8)$
13. $2(5x + 5)$	14. $5(x^2 + 5x)$	

3. **Discussion** Which of the matched expressions do you think is the simplest? Circle your choices. Explain your thinking.

In expressions 2, 3, 4, 6 and 10 the *greatest common factor* or *GCF* was factored out. For each exercise below, find the GCF of the expression.

Example: $15x^2 - 25x \rightarrow \underline{5x(3x - 5)}$

GCF: $\underline{5x}$

4. $12x^2 - 36 \rightarrow$ $12(x^2-3)$

   GCF: 12

   x^2 | -3

   12 | $12x^2$ | -36

5. $9x^2 + 15x \rightarrow$ $3x(3x+5)$

   GCF: $3x$

6. $22x^2 - 11 \rightarrow$ _____

   GCF: _____

7. $5x^2 - 15x \rightarrow$ _____

   GCF: _____

8. $10x - 2 \rightarrow$ _____

   GCF: _____

9. $17x - 3 \rightarrow$ _____

   GCF: 1

10. $-3x^2 + 9 \rightarrow$ $-3(x^2-3)$

    GCF: -3

11. $-4x^2 - 8x \rightarrow$ $-4x(x+2)$

    GCF: $-4x$

12. $3x^2 - 6x + 12 \rightarrow$ _____

    GCF: _____

13. $-2x^2 - 6x + 4 \rightarrow$ _____

    GCF: -2

    $-2(x^2+3x-2)$

14. $10x^2 + 5x - 25 \rightarrow$ _____

    GCF: 5

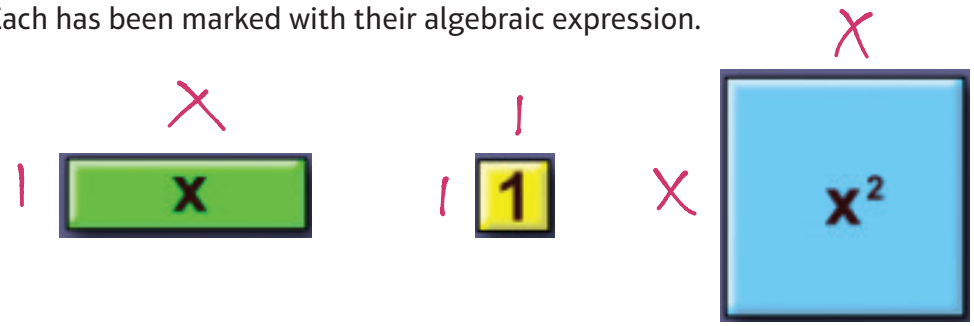
    $5(2x^2+x-5)$

15. $7x^2 - 14x + 14 \rightarrow$ _____

    GCF: 7

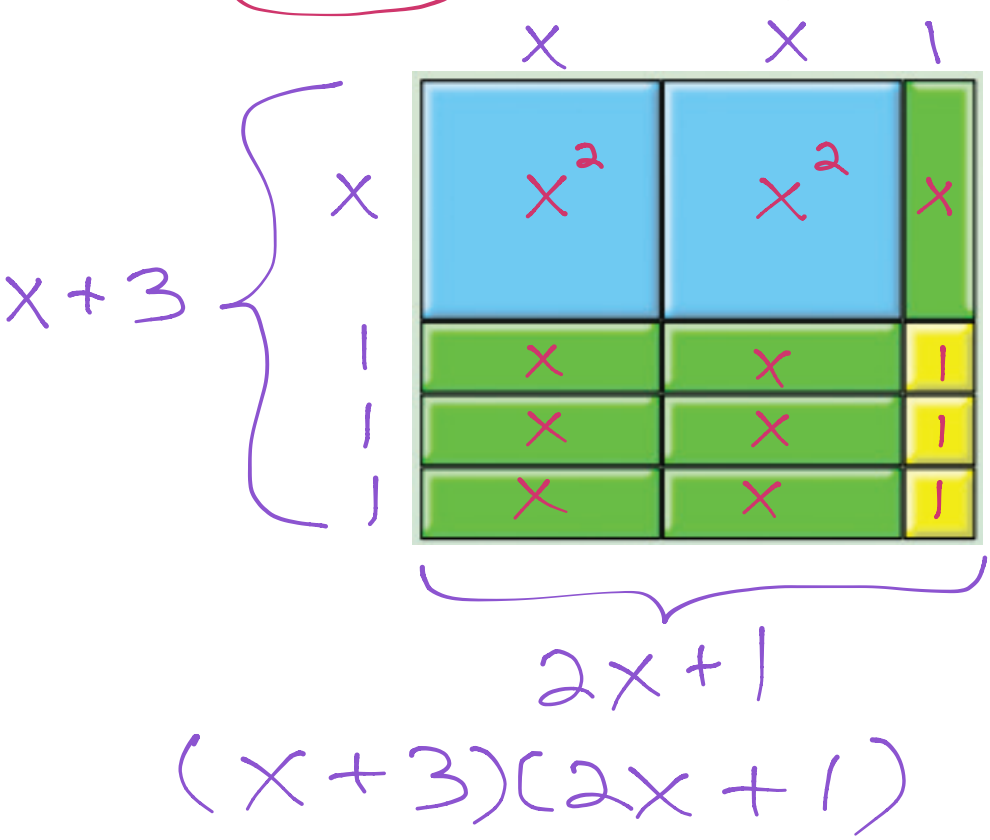
    $7(x^2-2x+2)$

Algebra tiles are often used to show how expressions can be factored. Below are the three tiles we'll be using. Each has been marked with their algebraic expression.

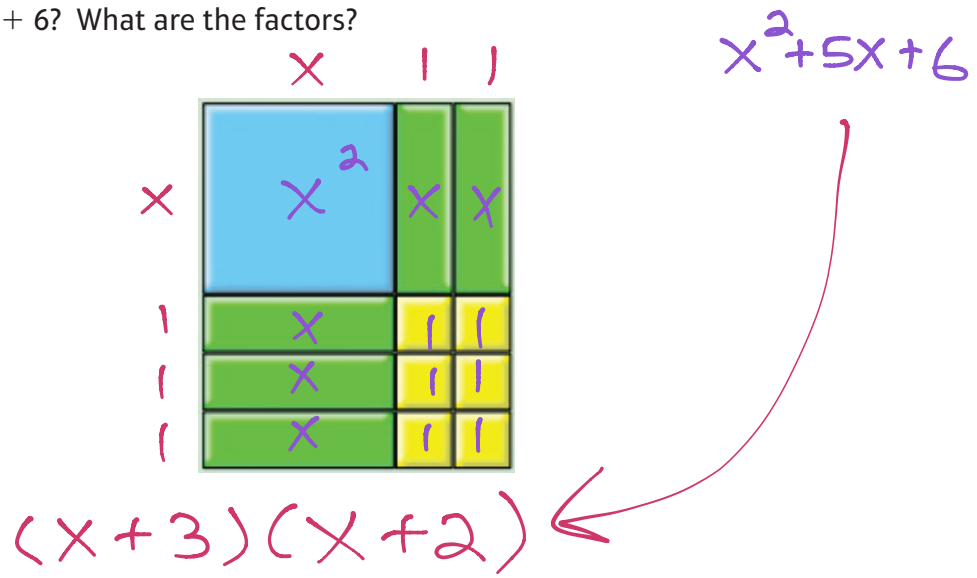


16. The 1-tile is 1 unit by 1 unit as shown below. Discuss how the dimensions of the other two tiles are represented by their algebraic expressions.

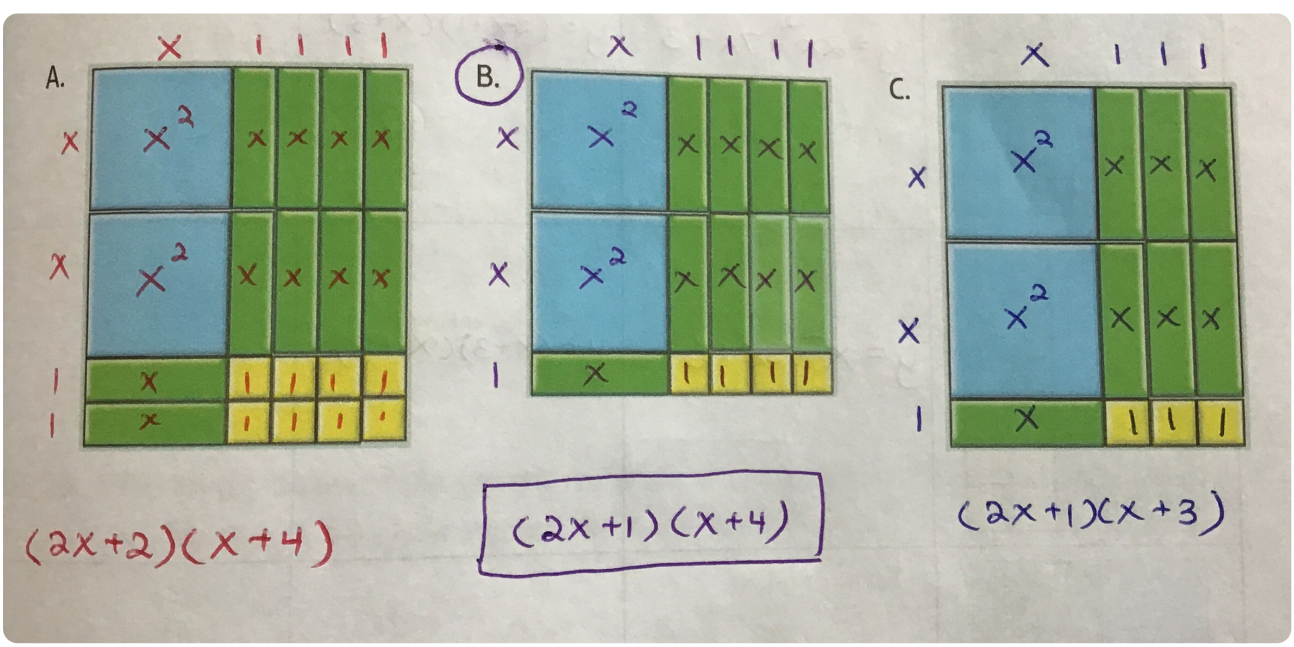
17. The tiles for $2x^2 + 7x + 3$ are shown below. Where do you see the factors of $(2x + 1)$ and $(x + 3)$?



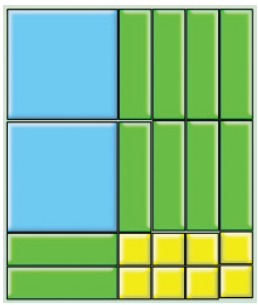
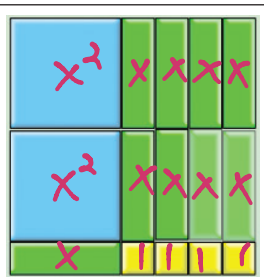
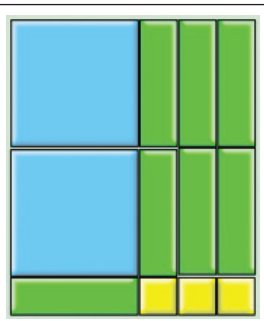
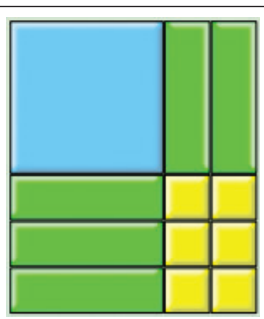
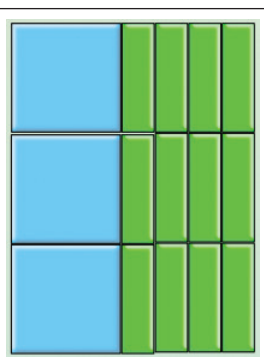
18. The model below is illustrating how $x^2 + 5x + 6$ can be factored. How is the model showing the factors of $x^2 + 5x + 6$? What are the factors?



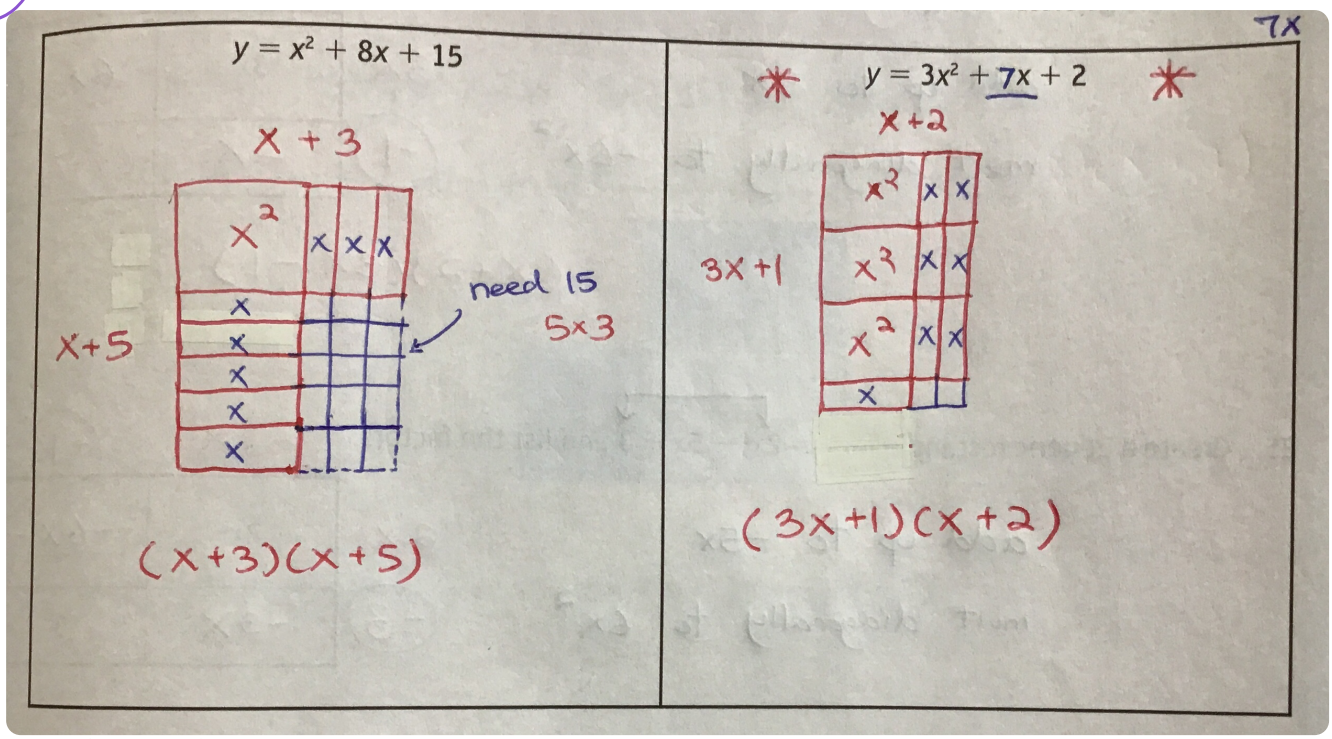
19. Which of the following is the correct model for $2x^2 + 9x + 4$? What are the factors of $2x^2 + 9x + 4$?



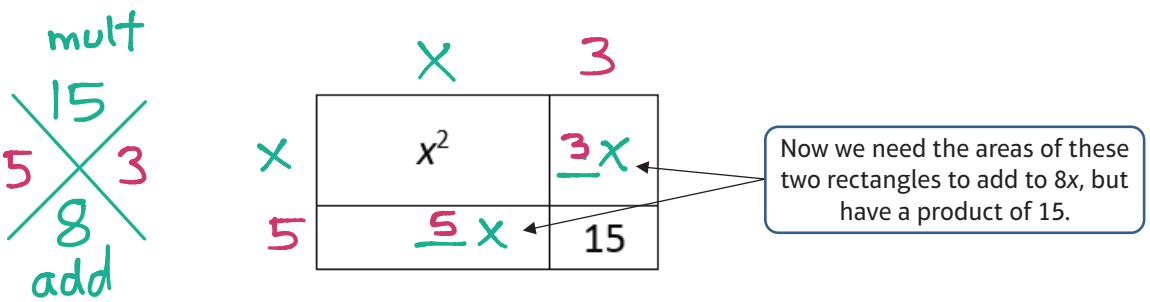
20. Let's look for some patterns between the standard form and the factored form. The first one has been done for you.

Model	Standard Form (sum of terms)	Factored Form (product of terms)	Patterns I Noticed
	$y = 2x^2 + 10x + 8$	$y = (2x + 2)(x + 4)$	
	$y = 2x^2 + 9x + 4$	$y = (2x+1)(x+4)$	
	$y = 2x^2 + 7x + 3$	$y = (2x+1)(x+3)$	
	$y = x^2 + 5x + 6$	$y = (x+3)(x+2)$	
	$y = 3x^2 + 12x$	$y = 3x(x+4)$	

21. Draw algebra tile models for each expression below. Then determine the factors of each one.



We can find the factors without having to draw out the algebra tiles each time by using a *generic rectangle*. For example, we can take the expression $x^2 + 8x + 15$ and place the first and last terms in a rectangle as shown below.



22. Fill in the empty boxes of the generic rectangle above. What are the factors of $x^2 + 8x + 15$? How did this compare to your algebra tile model in Exercise 21?

23. Go back to Exercise 20 and draw in a generic rectangle for each algebra tile model. What additional patterns do you see?

24. Create a generic rectangle for $y = 3x^2 + 5x - 2$ and list the factors.

add up to 5x
mult. to $-6x^2$

$3x^2$	$6x$
$-x$	-2

-6
$2 \quad 3$
$-1 \quad 6$

$(x+2)(3x-1)$

25. Create a generic rectangle for $y = 2x^2 - 5x + 3$ and list the factors.

add to -5
mult to 6

$2x^2$	$-2x$
$-3x$	3

$-2 \cdot -3 = 6$
$-2 + -3 = -5$

$(2x-3)(x-1)$

26. Create a generic rectangle for $y = 6x^2 - x - 1$ and list the factors.

add to -1
mult to -6

$6x^2$	$2x$
$-3x$	-1

$-3 \cdot 2 = -6$
$-3 + 2 = -1$

$(2x-1)(3x+1)$

27. Create a generic rectangle for $y = 4x^2 - 12x + 9$ and list the factors.

add to -12
mult to 36 (4×9)

$4x^2$	$-6x$
$-6x$	9

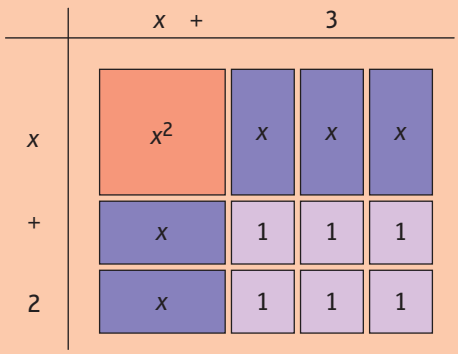
$-6 \cdot -6 = 36$
$-6 + -6 = -12$

$(2x-3)(2x-3)$

$(2x-3)^2$

Lesson Summary

Algebra Tile Model Example: $y = x^2 + 5x + 6 = (x + 2)(x + 3)$



Generic Rectangle Example: $y = x^2 + 5x + 6 = (x + 2)(x + 3)$

	x	3
x	x^2	$3x$
2	$2x$	6

NAME: _____ PERIOD: _____ DATE: _____

Homework Problem Set

For each problem below, find the GCF of the expression, if one exists. Be careful, some expressions have no GCF.

1. $9x^2 - 81 \rightarrow$ _____
   GCF: _____

2. $-4x^2 + x \rightarrow$ _____
   GCF: _____

3. $45x^2 - 9 \rightarrow$ _____
   GCF: _____

4. $-31x^2 - 3x \rightarrow$ _____
   GCF: _____

5. $27x - 9 \rightarrow$ _____
   GCF: _____

6. $9x - 4 \rightarrow$ _____
   GCF: _____

7. $-14x^2 + 35 \rightarrow$ _____
   GCF: _____

8. $-2x^2 - 3x \rightarrow$ _____
   GCF: _____

9. $x^2 - 6x + 12 \rightarrow$ _____
   GCF: _____

10. $-6x^2 - 6x + 6 \rightarrow$ _____
    GCF: _____

11. $16x^2 + 8x - 24 \rightarrow$ _____
    GCF: _____

12. $60x^2 - 10x + 6 \rightarrow$ _____
    GCF: _____

13. $8x^2 - 4xy + 2xw \rightarrow$ _____
    GCF: _____

14. $ax^2 + ax - a \rightarrow$ _____
    GCF: _____

15. $6bx^2 - 2bx + 8b \rightarrow$ _____
    GCF: _____

Factor each equation in Problems 16–21. You may use either the algebra tile model or the generic rectangle.

16. $y = x^2 + 2x + 1$

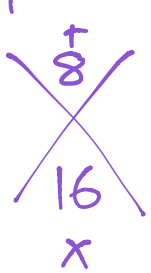
17. $y = x^2 + 4x + 4$

18. $y = x^2 + 6x + 5$

19. $y = x^2 + 7x + 10$

20. $y = x^2 + 6x + 9$

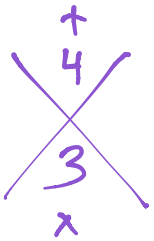
21. $y = x^2 + 8x + 16$



Factor each equation in Problems 22–27.

22. $y = 2x^2 + 7x + 3$

23. $y = 3x^2 + 4x + 1$



24. $y = 4x^2 + 13x + 10$

25. $y = 5x^2 + 14x + 8$

26. $y = 3x^2 + 11x + 6$

27. $y = 2x^2 + 11x + 15$

Factor each equation in Problems 28–33.

28. $y = 2x^2 - 7x + 3$	29. $y = 3x^2 - 14x - 5$
30. $y = 4x^2 + x - 3$	31. $y = 4x^2 - 4x - 3$
32. $y = 6x^2 + x - 1$	33. $y = 8x^2 - 10x - 3$

Challenge Problems

34. Fill in the boxes with any numbers that make the equation true.

$$(\square x - 3)(\square x + \square) = 12x^2 - \square x - 15$$

35. Fill in the blanks by finding the largest and smallest integers that will make the quadratic expression factorable.

$$2x^2 + 3x + \square$$

36. An algebra tile model was started but not completed and it is missing some pieces. What trinomial could this be representing? What is it in factored form?