LESSON 14

Factored Form When $a = 1$

LEARNING OBJECTIVES

- Today I am: using a quadratic graph to write the x-intercepts and related equations.
- So that I can: see the patterns between the factored form and standard form of a quadratic equation.
- I'll know I have it when I can: determine the factored form of the equation $y = x^2 + 5x - 24$ using number sense.

Opening Activity—Graph Exchange

Your group will need: one quadratic graph

1. A. For your given graph, circle the graph number on the table below and determine the x-intercepts. Then write the equation of the graph in factored form, standard form and vertex form. You can assume that for each graph there is no stretching or shrinking so $a = 1$ in all cases, where $ax^2 + bx + c = y$ is the equation of the quadratic function. Be prepared to share your results with the class.

Graph Number	x-intercepts	Factored Form	Standard Form	Vertex Form
1				
2				
3				
4				
5				

B. Once you have finished your graph, you'll be given a new graph from another group. Fill in the chart with the *x*-intercepts and equations. Continue getting new graphs from other groups until you have seen all 5 graphs.

Sharing Your Results—Math Practice 3

2. Each group will share their *x*-intercepts and equations with the class. If your group's equations don't match the presenting group's equations, be sure to point it out. It is important that all equations are correct for the next part of this lesson.

Looking for Patterns

3. Look back at the **factored** forms and **standard** forms of the graph. Discuss with your group what relationships you see in the numbers. List 3 patterns you see between the two forms of the equation.

-

-

-

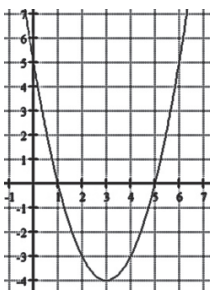
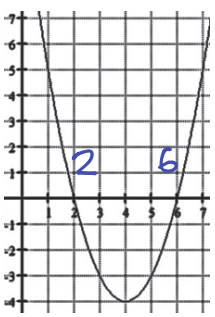
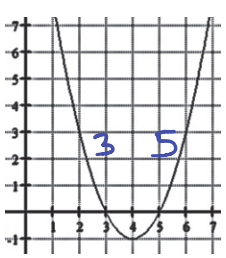
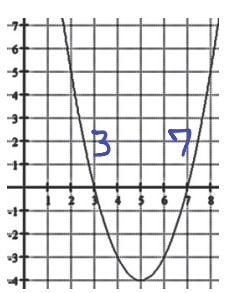
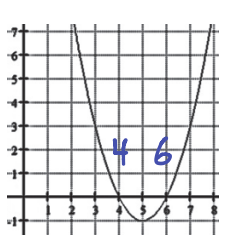
4. If we focus on the numbers, we may be able to see the patterns a little more clearly. We can write the equations abstractly as $y = (x + p)(x + q)$ (factored form) and $y = ax^2 + bx + c$ (standard form). Use your equations from Exercise 1 to fill in the chart. Then write the pattern you see between the p and q values and the a, b and c values. Graph 1 has been started for you.

Factored Form		**Standard Form**			**Pattern**
p	***q***	***a***	***b***	***c***	
1	5	1	6	5	
		1			
		1			
		1			
		1			

5. For the equation $y = x^2 + 7x + 12$, what would the factored form be?

$$y = (x + \_\_\_\_\_)(x + \_\_\_\_\_)$$

6. In Exercise 4, both x-intercepts were negative. Now we'll look at *x*-intercepts that are positive. For each graph below, write the factored form of the equation and the standard form of the equation. Then write the *p*, *q*, *b*, and *c* values. What pattern do you see? The first one has been started for you.

Graph	Factored Form		Standard Form			Patterns
	p	*q*	*a*	*b*	*c*	
	$y = (x - 1)(x - 5)$		$y = x^2 - 6x + 5$			add, −6, −1, −5, 5, mult.
	x-inter. +1	+5	1	−6	5	
	$y=(x-2)(x-6)$		$y=x^2-8x+12$			add, −8, −2, −6, 12, mult.
	2	6	1			
	$y=(x-3)(x-5)$		$y=x^2-8x+15$			+, −8, −5, −3, 15, x
	3	5	1			
	$y=(x-3)(x-7)$		$y=x^2-10x+21$			+, −10, −3, −7, 21, x
	3	7	1			
	$y=(x-4)(x-6)$		$y=x^2-10x+24$			
	4	6	1			

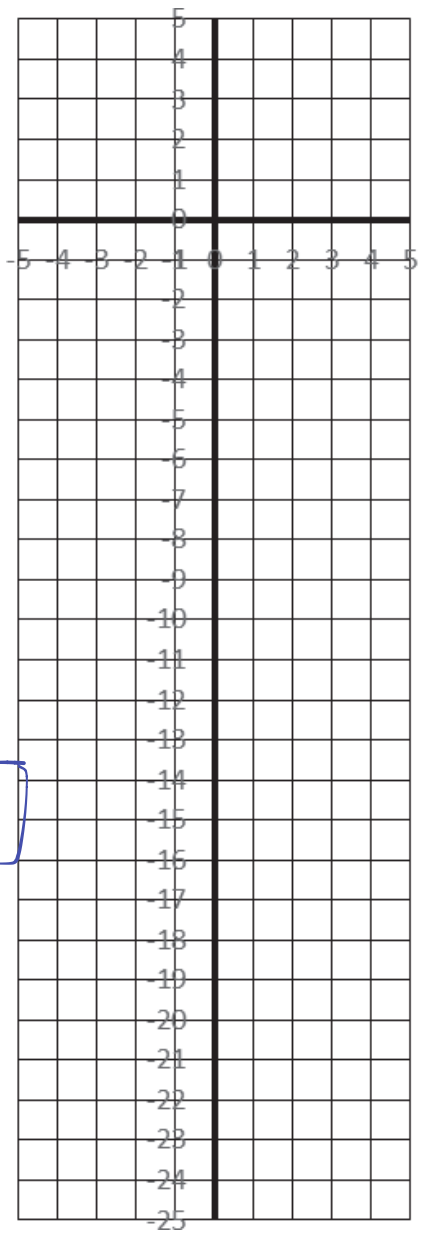
7. For the equation $y = x^2 - 9x + 14$, what would the factored form be? Why must both signs in the factored form be negative?

-9
-7 -2
14

$y = (x - \underline{7})(x - \underline{2})$

Exploration Negative and Positive *x*-Intercepts

8. In all of these cases, both *x*-intercepts were negative or both were positive. It is possible to have a factored form with one negative *x*-intercept and one positive *x*-intercept.



   A. Create an equation in factored form with one negative *x*-intercept and one positive *x*-intercept. So that you can graph your equation, keep your *x*-intercepts between −5 and +5. What are the *x*-intercepts?

   Equation: $y = (x-4)(x+2)$

   x-intercepts: 4 and −2

   B. Rewrite your equation in standard form. What is the *y*-intercept?

$y = x^2 - 2x - 8$

   C. What is the axis of symmetry of your quadratic?

$x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = 1$ $x=1$

   D. What is the vertex of your quadratic?

(1, −9)

$y = (1)^2 - 2(1) - 8$
$= 1 - 2 - 8 = -1 - 8 = -9$

   E. Graph your quadratic equation.

   F. What is the equation of your quadratic in vertex form?

$y = (x-1)^2 - 9$

9. For the equation $y = x^2 + 5x - 24$, what would the factored form be? Be sure to write in the signs.

5
8 -3
-24
x

$y = (x \underline{+8})(x \underline{-3})$

$x = -8, 3$

10. For the equation $y = x^2 - 5x - 24$, what would the factored form be? Be sure to write in the signs.

-5
-8 3
-24

$y = (x \underline{-8})(x \underline{+3})$

Practice Problems

For each equation below, write the factored form of the equation.

11. $y = x^2 + 4x + 4$

12. $y = x^2 + 5x + 4$

13. $y = x^2 - 4x + 4$

$y = (x-2)(x-2)$

$y = (x-2)^2$

14. $y = x^2 + 5x + 6$

$y = (x+2)(x+3)$

15. $y = x^2 - 7x + 6$

16. $y = x^2 + 7x + 6$

17. $y = x^2 - 1x - 12$

-1, -4, 3, -12

$y = (x-4)(x+3)$

18. $y = x^2 + 4x - 12$

19. $y = x^2 - 11x - 12$

+, -11, -12, 1, -12, x

$y = (x-12)(x+1)$

20. $y = x^2 - 2x - 8$

$y = (x-4)(x+2)$

21. $y = x^2 + 2x - 8$

$y = (x+4)(x-2)$

22. $y = x^2 - 7x - 8$

Lesson Summary

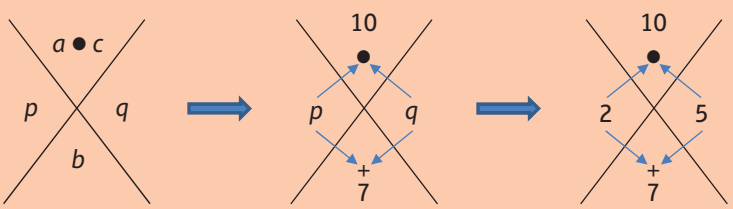
Factoring trinomials means finding two binomials that when multiplied together produce the given trinomial. You may also need to factor out the GCF of the trinomial or one of the binomials.

For trinomials of the form $y = ax^2 + bx + c$ and $a = 1$, _____ of c add up to _____.

In factored form, $y = (x + p)(x + q)$, $p \cdot q =$ _____ and $p + q =$ _____.

Some techniques to factoring include the X method as shown below.

Example: $y = x^2 + 7x + 10 = (x + 2)(x + 5)$



NAME: _____ PERIOD: _____ DATE: _____

Homework Problem Set

For each equation below, write the factored form of the equation.

1. $y = x^2 + 9x + 20$
2. $y = x^2 + 8x + 12$
3. $y = x^2 + 13x + 42$
4. $y = x^2 + 10x + 16$
5. $y = x^2 + 11x + 10$
6. $y = x^2 + 6x + 8$
7. $y = x^2 - 8x + 7$
8. $y = x^2 - 9x + 14$
9. $y = x^2 - 8x + 15$
10. $y = x^2 - 4x - 60$
11. $y = x^2 + x - 20$
12. $y = x^2 - 2x - 15$

13. The expression $x^2 + 3x + c$ can be factored and c is an integer.

    A. What is a value c can have? Show how it would be factored.

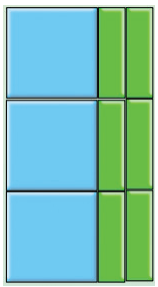
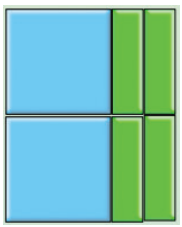
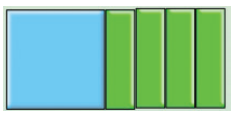
    B. Are there any other values of c that would make the expression factorable? Explain your reasoning.

14. The expression $x^2 + bx + 24$ can be factored and b is an integer.

    A. What is a value b can have? Show how it would be factored.

    B. Are there any other values of b that would make the expression factorable? Explain your reasoning.

For each algebra tile model below, write the expression it represents as a product and as a sum.

15.	16.	17.
		
Product: _____	Product: _____	Product: _____
Sum: _____	Sum: _____	Sum: _____

18. Factor each of the following expressions. You may draw a model.

A. $4x^2 - 8x$ B. $9x^2 + 27x$ C. $10x^2 - 25x$

19. Discuss how you could factor the expression $2x^2 + 10$. The algebra tile model is shown on the right.