LESSON 15 Factor by Grouping

LEARNING OBJECTIVES

- Today I am: examining Juan's method for factoring trinomials.
- So that I can: factor by grouping.
- I'll know I have it when I can: apply factoring to graphing quadratic functions.

Exploration 1—Factor Trinomials by Grouping

Juan found a pattern for factoring that works for all types of trinomials.

1. Here is the beginning of Juan's work. Discuss with your group, each step that Juan did. There are thought bubbles to help you identify each step.

A. $y = 1x^2 + 8x + 15$

Multiples of: 15

Adds to: 8

The 15 is from 3×5.

The 8 is from $3 + 5$.

$1 \cdot 15$

$1 \cdot 15$
$3 \cdot 5$

$(x + 3)(x + 5)$

B. $y = 3x^2 + 5x - 2$

Multiples of: -6

Adds to: 5

The -6 is from $3 \cdot (-2)$.

The 5 is from $-1 + 6$.

-6

$1 \cdot (-6)$
$-1 \cdot 6$
$2 \cdot (-3)$
$-2 \cdot 3$

What is the purpose of these boxes?

2. Juan's next steps are shown below. Determine what he did at each stage.

A. $y = x^2 + 8x + 15$

$1 \cdot 15$

$3 \cdot 5$

Multiples of: 15

Adds to: 8

The $8x$ became $3x + 5x$.

$y = x^2 + 3x + 5x + 15$

$y = x(x + 3) + 5(x + 3)$

What did Juan do at this step?
Factor GCF

B. $y = 3x^2 + 5x - 2$

$1 \cdot (-6)$

$-1 \cdot 6$

$2 \cdot (-3)$

$-2 \cdot 3$

Multiples of: -6

Adds to: 5

The $5x$ became $-1x + 6x$.

$y = 3x^2 - 1x + 6x - 2$

$y = x(3x - 1) + 2(3x - 1)$

What did Juan do at this step?
Factor GCF

3. What is Juan's final equation for each problem?

A. $y = x(x + 3) + 5(x + 3)$

$y = (x + 5)(x + 3)$

B. $y = x(3x - 1) + 2(3x - 1)$

$y = (x + 2)(3x - 1)$

4. Use a generic rectangle to factor each of Juan's equations.

A.

x^2	
	15

B.

$3x^2$	
	-2

Practice Problems—Factor Trinomials by Grouping

a c

5. $y = 1x^2 + 7x + 10$

$ac =$ 10

factors of ac:

5 · 2

$y=(x+5)(x+2)$

$y = x^2 + 5x + 2x + 10$

$y = x(x+5) + 2(x+5)$

$y = (x+2)(x+5)$

6. $y = 4x^2 + 5x + 1$

$ac =$ ______

factors of ac:

× 4 4 1 5 +

$y = 4x^2 + 4x + x + 1$

$y = 4x(x+1) + 1(x+1)$

$y = (4x+1)(x+1)$

7. $y = 3x^2 + 5x + 2$

$ac =$ ______

factors of ac:

× 6 2 3 5 +

$y = 3x^2 + 2x + 3x + 2$

$y = x(3x+2) + 1(3x+2)$

$y = (x+1)(3x+2)$

8. $y = 5x^2 + 12x + 4$

$ac =$ 20

factors of ac:

1 · 20

4 · 5

2 · 10

$y = 5x^2 + 10x + 2x + 4$

$y = 5x\boxed{(x+2)} + 2\boxed{(x+2)}$

$y = (5x+2)(x+2)$

Exploration 2—Looking for the GCF

In Lesson 13, we saw that many trinomials have a GCF. We can still use Juan's method for these trinomials, but we'll factor out the GCF first.

9. Factor $y = 2x^2 + 8x + 8$.

   First, we'll factor out the GCF of $2x^2 + 8x + 8$ to get $y = 2(x^2 + 4x + 4)$.

   Then use factor by grouping on the trinomial in the parenthesis.

Focus on $x^2 + 4x + 4$

$y = 2(x^2 + 4x + 4)$

$ac = \underline{\ 4\ }$

factors of ac:

Reflection

10. Melissa said you don't have to factor out the GCF right away. Melissa's work is shown below.

$y = 2x^2 + 8x + 8$

$y = 2x^2 + 4x + 4x + 8$

$y = 2x(x + 2) + 4(x + 2)$

$y = (x + 2)(2x + 4)$

$y = 2(x + 2)(x + 2)$

$ac = \underline{16}$

factors of ac:

$1 \cdot 16$

$2 \cdot 8$

$4 \cdot 4$

What are the advantages to factoring out the GCF right away? What are the advantages to waiting to factor out the GCF?

Practice Problems—Looking for the GCF

11. $y = 3x^2 + 12x + 9$ $y = 3(x^2 + 4x + 3)$ $y = 3(x+3)(x+1)$ $ac =$ ______ factors of ac:	12. $y = -4x^2 + 2x + 2$ -2, 2, 1, -1 $y = -2(2x^2 - x + 1)$ $2x^2 - 2x + x + 1$ $y = -2$ $ac =$ ______ factors of ac:
13. $y = 4x^2 + 4x - 48$ $y = 4(x^2 + x - 12)$ $y = 4(x-3)(x+4)$ -12, -3, 4, 1 $ac =$ ______ factors of ac:	14. $y = 4x^2 + 6x - 18$ $y = 2(2x^2 + 3x - 9)$ -18, 6, -3, 3 $y = 2(2x^2 + 6x - 3x - 9)$ $2x(x+3) - 3(x+3)$ $y = 2(2x-3)(x+3)$ $ac =$ ______ factors of ac:

15. **Discussion** What are the advantages to factoring by grouping (Juan's method)? What are the advantages to the generic rectangle method?

$a^2 - b^2 = (a-b)(a+b)$

Exploration 3—Graphing Quadratic Equations Revisited

Determine the key features of each quadratic and then graph the parabola.

$a=1 \quad b=8 \quad c=15$

16. $y = x^2 + 8x + 15$ Standard Form

$y = (x+3)(x+5)$ Factored Form

Key features:

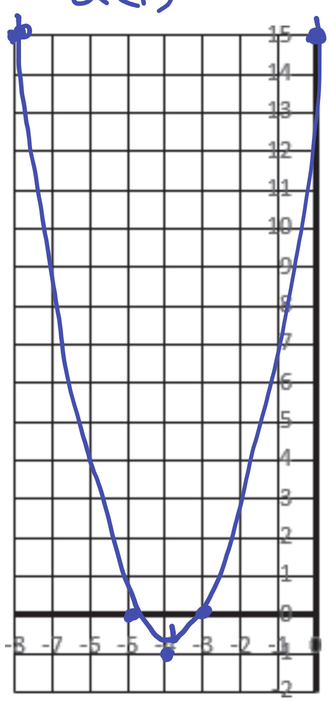
3×5: product 15, sum 8

x-intercepts: -3, -5

y-intercept: 15

vertex: $(-4, -1)$

$$x = \frac{-b}{2a} = \frac{-8}{2(1)} = -4$$



17. $y = 3x^2 - 12$ Standard Form

$3(x-0)^2 - 12$

$3(x^2-4)$

$y = 3(x+2)(x-2)$ Factored Form

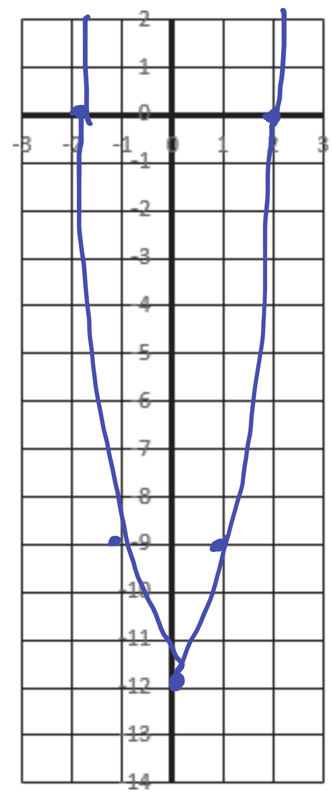
diff of two squares

Key features:

x-intercepts: -2, 2

y-intercept: -12

vertex: $(0, -12)$



NAME: ______________ PERIOD: ______________ DATE: ______________

Homework Problem Set

Use Juan's method to factor each equation.

1. $y = x^2 + 14x + 24$

   ac = ________

   factors of *ac*:

2. $y = 3x^2 + 11x + 6$

   ac = ________

   factors of *ac*:

3. $y = 2x^2 + 11x + 14$

   ac = ________

   factors of *ac*:

4. $y = 5x^2 + 17x + 6$

   ac = ________

   factors of *ac*:

Factor each equation in Problems 5–10. Use any method.

5. $y = x^2 + 8x + 7$	6. $y = x^2 - 11x + 10$
7. $y = x^2 + 3x - 54$	8. $y = 10x^2 + 13x - 30$
9. $y = 12x^2 - 43x + 35$	10. $y = 14x^2 + 19x - 3$

Factor each equation in Problems 11–16.

11. $y = 3x^2 + 16x - 35$	12. $y = 10x^2 + 71x - 72$
13. $y = 4x^2 + 11x - 20$	14. $y = 3x^2 - 28x + 9$
15. $y = 7x^2 + 6x - 1$	16. $y = 3x^2 - 10x + 7$