LESSON 16 More Factoring Strategies

LEARNING OBJECTIVES

- Today I am: looking at special cases when factoring.
- So that I can: factor a difference of squares.
- I'll know I have it when I can: graph these special cases.

Opening Exploration—A Special Case

1. Consuela ran across the quadratic equation $y = 4x^2 - 16$ and wondered how it could be factored. She rewrote it as $y = 4x^2 + 0x - 16$.

-64

$-8 \quad +8$

0

A. Use one of the methods you've learned to factor this quadratic function.

$y = 4x^2 - 16$

$y = 4(x^2 - 4)$

$4(x-2)(x+2)$

$4x^2 - 8x + 8x - 16$

$4x(x-2) + 8(x-2)$

$(4x+8)(x-2)$

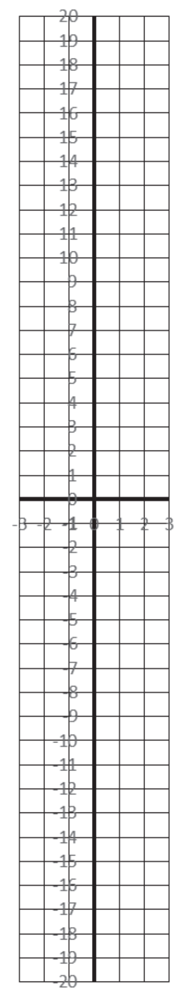
$4(x+2)(x-2)$

B. What are the key features of the parabola's graph?

x-intercepts: ______, ______ *y*-intercept: ______

vertex: (______, ______)

C. Graph the quadratic in the grid at the right.



Factor the following quadratic functions. Use Consuela's idea of a 0 middle term if necessary. Look for patterns as you go.

2. $y = x^2 - 9$

$y = x^2 + 0x - 9$

$y = (x+3)(x-3)$

3. $y = x^2 - 25$

$y = (x+5)(x-5)$

4. What is the generic rule for factoring a quadratic in the form $a^2 - b^2$?

$$\boxed{a^2 - b^2 = (a+b)(a-b)}$$

5. The expression $a^2 - b^2$ is called *The Difference of Squares.* Discuss with your partner where that name comes from.

$x^2 - 36 = (x+6)(x-6)$

$x^2 - 81 = (x+9)(x-9)$

$x^2 - 1 = (x+1)(x-1)$

$4x^2 - 9 = (2x+3)(2x-3)$

$49x^2 - 36$

$64x^2 - 25$

One of the other special cases is *Perfect Square Trinomials*. You've already encountered a few of these but we'll focus on them now and see why they are special.

6. Factor each of the following. Use any method.

A. $y = x^2 + 6x + 9$

$y = (x+3)(x+3)$

$y = (x+3)^2$

B. $y = x^2 - 4x + 4$

$y = (x-2)(x-2)$

$y = (x-2)^2$

7. A. What are the x-intercepts for these two equations? Remember the x-intercept is where $y = 0$.

B. What is different about the x-intercepts for these two equations?

Determine the key features of each quadratic and then graph the parabola.

8. $y = x^2 + 6x + 9$ Standard Form

$y = (x + \underline{3})(x + \underline{3})$ Factored Form

$y = \underline{(x+3)^2}$ Vertex Form

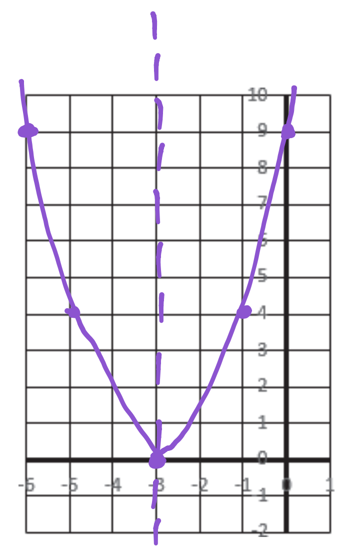
Key features:

x-intercepts: $\underline{-3}$, $\underline{-3}$

y-intercept: $\underline{9}$

vertex: $(\underline{-3}, \underline{0})$

x	y
-1	4



9. $y = x^2 - 4x + 4$ Standard Form

$y = (x - \underline{2})(x - \underline{2})$ Factored Form

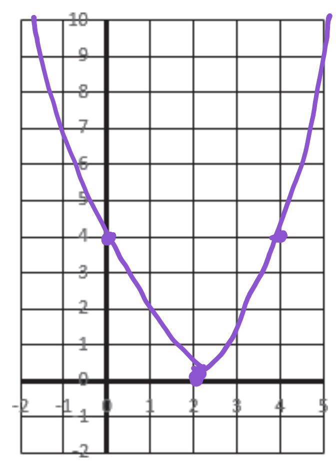
$y = \underline{(x-2)^2 + 0}$ Vertex Form

Key features:

x-intercepts: $\underline{2}$, $\underline{2}$

y-intercept: $\underline{4}$

vertex: $(\underline{2}, \underline{0})$



10. Both of the quadratic equations in Exercises 8 and 9 are perfect square trinomials. What is special about their graphs?

Let's look at a couple of perfect square trinomials that have $a \neq 1$. Factor each one.

11. $y = 4x^2 + 4x + 1$

12. $y = 9x^2 - 12x + 4$

36

−6 −6

−12

$y = 9x^2 - 6x - 6x + 4$

$y = 3x(3x-2) - 2(3x-2)$

$y = (3x-2)(3x-2)$

Determine the key features of each quadratic and then graph the parabola.

13. $y = 4x^2 + 4x + 1$ Standard Form

$y = (\_\_\_x + \_\_\_)(\_\_\_x + \_\_\_)$ Factored Form

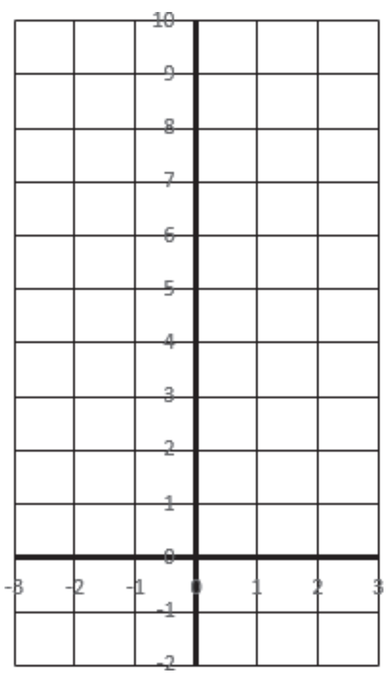
$y =$ ____________ Vertex Form

Key features:

x-intercepts: _____, _____

y-intercept: _____

vertex: (_____, _____)



14. $y = 9x^2 - 12x + 4$ Standard Form

$y = (3x-2)(3x-2)$ Factored Form

$y = (3x-2)^2$ Vertex Form

$3x - 2 = 0$

$3x = 2$

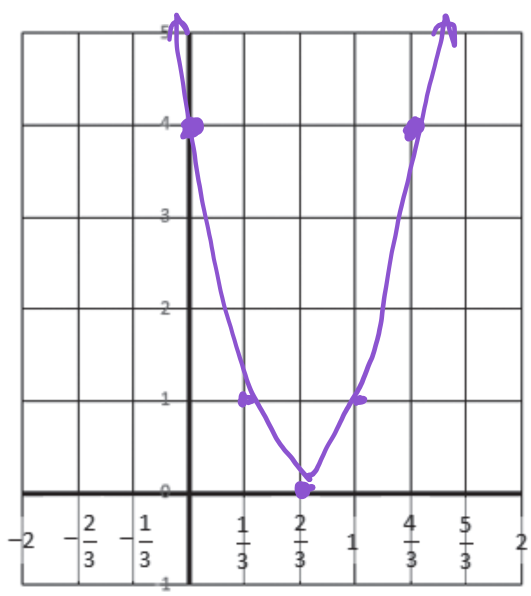
$x = 2/3$

Key features:

x-intercepts: $2/3$, $2/3$

y-intercept: 4

vertex: ($2/3$, 0)



Practice Problems—Factor each perfect square trinomial or difference of squares. Notice that these are not written as quadratic functions. They are simply expressions—we could not graph them or tell their key features.

15. $x^2 + 4x + 4$	16. $x^2 - 20x + 100$	17. $x^2 + 2x + 1$
18. $x^2 + 14x + 49$	19. $64x^2 + 48x + 9$	20. $25x^2 - 20x + 4$
21. $x^2 - 8x + 16$	22. $x^2 + 24x + 144$	23. $x^2 - 4$ $(x-0)^2 - 4$ $(x+2)(x-2)$
24. $4x^2 - 9$ $(2x-3)(2x+3)$	25. $25x^2 - 36$ $(5x-6)(5x+6)$	26. $x^2 - 49$ $(x+7)(x-7)$

Lesson Summary

Difference of Squares

$$(ax)^2 - b^2$$

$$(ax - b)(ax + b)$$

	ax	$-b$
ax	$(ax)^2$	$-abx$
b	abx	b^2

Perfect Square Trinomials

$$(ax)^2 + 2abx + b^2$$

$$(ax + b)^2$$

$$(ax)^2 - 2abx + b^2$$

$$(ax - b)^2$$

	ax	b
ax	$(ax)^2$	abx
b	abx	b^2

	ax	$-b$
ax	$(ax)^2$	$-abx$
$-b$	$-abx$	b^2

NAME: ____________________ PERIOD: __________ DATE: __________

Homework Problem Set

Factor the following examples of the difference of perfect squares. Notice that these are not written as quadratic functions. They are simply expressions—we could not graph them or tell their key features.

1. $t^2 - 25$

2. $4x^2 - 9$

3. $16h^2 - 36k^2$

4. $4 - b^2$

5. $x^4 - 4$

6. $x^6 - 25$

7. $9y^2 - 100z^2$

8. $a^4 - b^6$

9. **Challenge** $r^4 - 16s^4$ (Hint: This one factors twice.)

For each of the following, factor out the greatest common factor (GCF).

10. $6y^2 + 18$

11. $27y^2 + 18y$

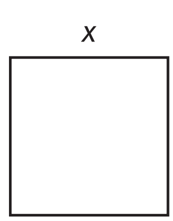
12. $21b - 15a$

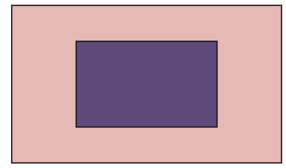
13. $14c^2 + 2c$

14. $3x^2 - 27$

15. The measure of a side of a square is x units. A new square is formed with each side 6 units longer than the original square's side. Write an expression to represent the area of the new square. (Hint: Draw the new square and count the squares and rectangles.)

Original Square





16. In the accompanying diagram, the width of the inner rectangle is represented by $x - 3$ and the length by $x + 3$. The width of the outer rectangle is represented by $3x - 4$ and the length by $3x + 4$.

    A. Write an expression to represent the area of the larger rectangle.

    B. Write an expression to represent the area of the smaller rectangle.

Mixed REVIEW

Factor completely.

17. $9x^2 - 25x$

18. $9x^2 - 25$

19. $9x^2 - 30x + 25$

20. $2x^2 + 7x + 6$

21. $6x^2 + 7x + 2$

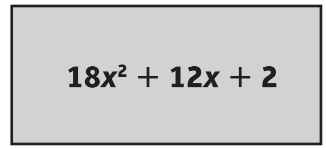
22. $8x^2 + 20x + 8$

23. $3x^2 + 10x + 7$

24. $4x^2 + 4x + 1$

Challenge Problems

25. The area of the rectangle at the right is represented by the expression $\mathbf{18x^2 + 12x + 2}$ square units. Write two expressions to represent the dimensions, if the length is known to be twice the width.



26. Two mathematicians are neighbors. Each owns a separate rectangular plot of land that shares a boundary and has the same dimensions. They agree that each has an area of $2x^2 + 3x + 1$ square units. One mathematician sells his plot to the other. The other wants to put a fence around the perimeter of his new combined plot of land. How many linear units of fencing does he need? Write your answer as an expression in x.

Note: This question has two correct approaches and two different correct solutions. Can you find them both?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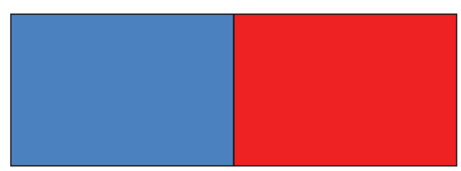

Spiral REVIEW—Factoring

Factor the following quadratic expressions.

27. $2x^2 + 10x + 12$

28. $6x^2 + 5x - 6$

29. $x^2 - 12x + 20$

30. $x^2 - 21x - 22$

31. $2x^2 - x - 10$

32. $6x^2 + 7x - 20$

33. $x^2 - 2x - 15$

34. $x^2 + 2x - 15$

35. $4x^2 + 12x + 9$

36. $49x^2 + 28x + 4$