LESSON 17 The Zero Product Property Revisited

LEARNING OBJECTIVES

$a \cdot b = 0$

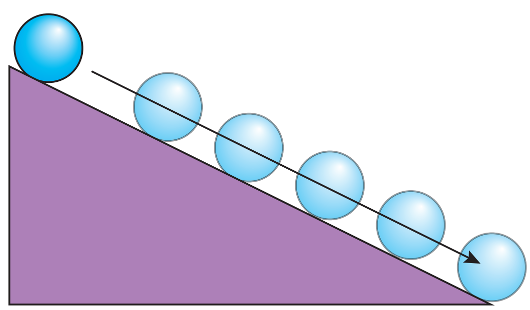
$a = 0$ or $b = 0$

- Today I am: looking at equations like $a \cdot b \cdot c \cdot d = 0$.
- So that I can: tell when a factor will equal 0.
- I'll know I have it when I can: solve a problem like $7r^2 - 14r = -7$.

Opening Exercise

1. A physics teacher put a ball at the top of a ramp and let it roll down toward the floor. The class determined that the height of the ball could be represented by the equation $h = -16t^2 + 4$, where the height, h, is measured in feet from the ground and time, t, is measured in seconds.

$4 - 16t^2$



A. What do you notice about the structure of the quadratic expression in this problem?

No linear term, concave down

B. In the equation, explain what the 4 represents.

Initial height 4 feet (y-intercept)

C. Explain how you would use the equation to determine the time it takes the ball to reach the floor.

$h = 0$ $\quad 0 = -16t^2 + 4$

D. Now consider the possible solutions for t. Which values are reasonable? What values would make no sense in this situation?

Throughout this unit, we've been putting equations into factored form so that we could find the x-intercepts.

2. What is the y-value of all x-intercepts? Are there any exceptions?

$y=0$

The x-intercepts give rise to the Zero Product Property. We've used this property since Lesson 11, but now we'll explore it more analytically.

3. Consider the equation $a \cdot b \cdot c \cdot d = 0$. What values of a, b, c, and d would make the equation true?

$a=0$, $b=0$, $c=0$, or $d=0$

Find values of c and d that satisfy each of the following equations. (There may be more than one correct answer.)

4. $cd = 0$

$c=0$ or $d=0$

5. $(c - 5)d = 2$

7 1
6 2

Many Solutions

6. $(c - 5)d = 0$

$c-5=0$ or $d=0$
$c=5$

7. $(c - 5)(d + 3) = 0$

$c-5=0$ or $d+3=0$
$c=5$ or $d=-3$

8. The equation in the Opening Exercise, $h = -16t^2 + 4$, describes the height (in feet) of the ball at different times (in seconds). If we want to find the time when the ball reaches the ground, we would set $h = 0$. There are two ways you could approach the problem $0 = -16t^2 + 4$ as shown below. Complete each method's example.

Factoring Method	Square Root Method
diff of two squares $0 = -16t^2 + 4 \rightarrow 4 - 16t^2$ $0 = (-4t + 2)(4t + 2)$ $(2+4t)(2-4t)$ $-4t + 2 = 0$ OR $4t + 2 = 0$ $-4t = -2$ $4t = -2$ $t = \frac{2}{4}$ $t = -\frac{2}{4}$ $t = \frac{1}{2}$ OR $t = \frac{-1}{2}$	$0 = -16t^2 + 4$ $16t^2 = 4$ $\frac{16t^2}{16} = \frac{4}{16}$ $t^2 = \frac{1}{4}$ important $t = \pm\sqrt{\frac{1}{4}}$ $t = \frac{1}{2}$ OR $t = \frac{-1}{2}$

Practice—Factoring Method and Square Root Method

For the next two exercises, you and your partner will switch off using the Factoring Method and the Square Root Method. When you are done, check your answers with your partner's.

Partner A's Work	**Partner B's Work**
Factoring Method	**Square Root Method** * only works when "x"-linear term is missing.
9. $0 = 3x^2 - 108$ $0 = 3(x^2 - 36)$ $0 = \cancel{3}(x+6)(x-6)$ $x+6=0$ or $x-6=0$ $x=-6$ $x=6$	9. $0 = 3x^2 - 108$ solve for x^2 $108 = 3x^2$ $\pm\sqrt{36} = \sqrt{x^2}$ $\pm 6 = x$ $x=6$ $x=-6$
Square Root Method	**Factoring Method**
10. $0 = 4x^2 - 100$ $\frac{100}{4} = \frac{4x^2}{4} = 25 = x^2$ $\pm\sqrt{25} = \sqrt{x^2}$ $\pm 5 = x$	10. $0 = 4x^2 - 100$ $0 = 4(x^2 - 25)$ $0 = 4(x-5)(x+5)$ $x-5=0$ $x+5=0$ $x=5$ $x=-5$

Back in Lessons 10 and 11, you explored the licorice-packaging problem. Recall, Isabella wrote the equation for the volume of the sleeve in vertex form as $V=-24\left(h-\frac{5}{2}\right)^2+150$.

11. Isabella's work to find the horizontal intercepts is shown below. Explain what Isabella did at each step.

$V=-24\left(h-\frac{5}{2}\right)^2+150$

$0=-24\left(h-\frac{5}{2}\right)^2+150$ ____

$-150=-24\left(h-\frac{5}{2}\right)^2$ ____

$6.25=\left(h-\frac{5}{2}\right)^2$ ____

$\pm\sqrt{\frac{25}{4}}=\sqrt{\left(h-\frac{5}{2}\right)^2}$ ____

$\pm\frac{5}{2}=h-\frac{5}{2}$ ____

$\frac{5}{2}\pm\frac{5}{2}=h$ ____

$\frac{5}{2}+\frac{5}{2}=h$ OR $\frac{5}{2}-\frac{5}{2}=h$

____

$\frac{10}{2}=5=h$ OR $0=h$

Additional Practice—Solving for Horizontal Intercepts

Isabella solved for *h* to find the horizontal intercepts. Use her method to find the *x*-intercepts for each of the exercises below. Remember, *y* is zero when the function hits the *x*-axis.

vertex form

12. $y = 3(x - 2)^2 - 27$ 0 $+27 \quad +27$ $\frac{27}{3} = \frac{3(x-2)^2}{3}$ $* \rightarrow \pm\sqrt{9} = \sqrt{(x-2)^2}$ $\pm 3 = x - 2$ $+2 \quad +2$ $2 \pm 3 = x$ $2+3$ or $2-3$ $x = 5$ or -1	13. $y = 5(x + 1)^2 - 20$ $0 = 5(x+1)^2 - 20$ $+20 \quad +20$ $\frac{20}{5} = \frac{5(x+1)^2}{5}$ $\pm\sqrt{4} = \sqrt{(x+1)^2}$ $\pm 2 = x + 1$ $-1 \quad -1$ $-1 \pm 2 = x$ $-1+2$ or $-1-2$ $1 \quad -3$	14. $y = (x - 7)^2 - 25$

15. What is the most difficult part of solving for the *x*-intercepts? Explain your thinking.

Practice Exercises 16–27

Solve. Show your work.

16. $(x - 2)^2 = 16$

$\sqrt{(x-2)^2} = \sqrt{16}$

$x - 2 = \pm 4$

$x - 2 = 4$, $+2 \quad +2$, $x = 6$

$x - 2 = -4$, $+2 \quad +2$, $x = -2$

17. $(2x + 4)^2 = 36$

$\sqrt{(2x+4)^2} = \sqrt{36}$

$2x + 4 = \pm 6$

$2x + 4 = 6$, $-4 \quad -4$, $\frac{2x}{2} = \frac{2}{2}$, $x = 1$

$2x + 4 = -6$, $-4 \quad -4$, $\frac{2x}{2} = \frac{-10}{2}$, $x = -5$

18. $3x^2 + 5x + 2 = 0$

Factor

$3x^2 + 3x + 2x + 2 = 0$

$3x(x+1) + 2(x+1) = 0$

$(3x+2)(x+1) = 0$

$3x+2=0$ $x=-1$

$x=-2/3$

19. $x^2 - 9x = -18$

20. $x^2 - 11x + 19 = -5$

Factoring

$+5$ $+5$

$1x^2 - 11x + 24 = 0$

$(x-8)(x-3) = 0$

$x-8=0$ or $x-3=0$

$x=8$ $x=3$

21. $7x^2 + x = 0$

$x(7x+1) = 0$

$x=0$ $7x+1=0$

$x = \frac{-1}{7}$

22. $7r^2 - 14r = -7$

23. $2d^2 + 5d = 12$ Long Factor

$2d^2 + 5d - 12 = 0$

$2d^2 + 8d - 3d - 12 = 0$

$2d(d+4) - 3(d+4) = 0$

$(2d-3)(d+4) = 0$

$d = \frac{3}{2}$ $d = -4$

24. $x^2 + 4 = 29$

25. $2x^2 = 72$

26. $3(x-3)^2 = 27$

$\frac{3(x-3)^2}{3} = \frac{27}{3}$

$\sqrt{(x-3)^2} = \sqrt{9}$

$x-3 = \pm 3$

$+3$ $+3$

$x=0$

$x=6$

27. $5(x-5)^2 + 2 = 22$

-2 -2

$\frac{5(x-5)^2}{5} = \frac{20}{5}$

$\sqrt{(x-5)^2} = \sqrt{4}$

$x-5 = 2$ $x-5=-2$

$+5$ $+5$ $+5$ $+5$

$x=7$ $x=3$

28. Write in the steps to solve for the x-intercepts.

Lesson Summary

When solving for the variable in a quadratic equation, rewrite the quadratic expression in factored form and set equal to zero. Using the zero product property, you know that if one factor is equal to zero, then the product of all factors is equal to zero.

Going one step further, when you have set each binomial factor equal to zero and have solved for the variable, all of the possible solutions for the equation have been found. Given the context, some solutions may not be viable, so be sure to determine if each possible solution is appropriate for the problem.

Zero Product Property

If $ab = 0$, then $a = 0$ or $b = 0$ or $a = b = 0$.

Solving for the x-intercepts	Steps
$y = (x + 1)^2 - 16$	
$0 = (x + 1)^2 - 16$	
$16 = (x + 1)^2$	
$\pm\sqrt{16} = \sqrt{(x + 1)^2}$	
$\pm 4 = x + 1$	
$4 = x + 1$ or $-4 = x + 1$	
$x = 3$ or $x = -5$	

NAME: ______________________ PERIOD: ____________ DATE: ____________

Homework Problem Set

Solve the following equations.

1. $(2x - 1)(x + 3) = 0$

2. $(t - 4)(3t + 1)(t + 2) = 0$

3. $x^2 - 9 = 0$

4. $(x^2 - 9)(x^2 - 100) = 0$

5. $x^2 - 9 = (x - 3)(x - 5)$

6. $x^2 + x - 30 = 0$

7. $p^2 - 7p = 0$

8. $p^2 - 7p = 8$

9. $3x^2 + 6x + 3 = 0$

10. $2x^2 - 9x + 10 = 0$

11. $x^2 + 15x + 40 = 4$

12. $7x^2 + 2x = 0$

13. $7x^2 + 2x - 5 = 0$

14. $b^2 + 5b - 35 = 3b$

15. $6r^2 - 12r = 18$

16. $2x^2 + 11x = x^2 - x - 32$

17. Write an equation (in factored form) that has solutions of $x = 2$ or $x = 3$.

18. Write an equation (in factored form) that has solutions of $a = 0$ or $a = -1$.

19. Quinn looks at the equation $(x - 5)(x - 6) = 2$ and says that since the equation is in factored form it can be solved as follows:

$$(x - 5)(x - 6) = 2$$

$$x - 5 = 2 \text{ or } x - 6 = 2$$

$$x = 7 \text{ or } x = 8.$$

Explain to Quinn why this is incorrect. Show her the correct way to solve the equation.

For each problem, determine the *x*-intercepts, the vertex and the *y*-intercept.

20. $y = -2(x + 5)^2 + 18$	21. $y = -(x - 5)^2$	22. $y = \frac{1}{3}(x - 1)^2 - 3$
x-intercepts: ______ and ______	*x*-intercepts: ______ and ______	*x*-intercepts: ______ and ______
vertex: (_______, _______)	vertex: (_______, _______)	vertex: (_______, _______)
y-intercept: ____________	*y*-intercept: ____________	*y*-intercept: ____________

23. $y = \frac{3}{4}(x+4)^2 - \frac{27}{16}$ *x*-intercepts: ______ and ______ vertex: (_______, _______) *y*-intercept: ___________	24. $y = -\frac{1}{3}(x+1)^2 + 3$ *x*-intercepts: ______ and ______ vertex: (_______, _______) *y*-intercept: ___________	25. $y = 4(x+2)^2 - 4$ *x*-intercepts: ______ and ______ vertex: (_______, _______) *y*-intercept: ___________
26. $y = -(x+2)^2 + 25$ *x*-intercepts: ______ and ______ vertex: (_______, _______) *y*-intercept: ___________	27. $y = \frac{1}{2}(x+3)^2 - 18$ *x*-intercepts: ______ and ______ vertex: (_______, _______) *y*-intercept: ___________	28. $y = -\frac{1}{3}(x-1)^2 + 27$ *x*-intercepts: ______ and ______ vertex: (_______, _______) *y*-intercept: ___________