LESSON 18

Solving Quadratic Equations

LEARNING OBJECTIVES

- Today I am: looking at quadratic equations in real-world situations.
- So that I can: solve for the one-variable quadratic equations.
- I'll know I have it when I can: determine the amount of time for a paint bucket to hit the ground.

Opening Exercise

1. The area of a rectangle can be represented by the expression $x^2 + 2x - 3$. = Area

   $l \times w$

   A. If the dimensions of the rectangle are known to be the linear factors of the expression, write each dimension of this rectangle as a binomial. Write the area in terms of the product of the two binomials.

   4 x^2+2x-3 4

   $(x-1)(x+3)$

   l w

   x: -3, -1, 3, 2, $+$

   B. Draw and label a diagram that represents the rectangle's area.

   $x+3$

   $x-1$ | x^2+2x-3

C. Suppose the rectangle's area is 21 square units. Find the dimensions of the rectangle.

$(x-1)(x+3)=21$

$x^2+2x-3=21$

$-21 \quad -21$

$x^2+2x-24=0$ ←

D. What are the actual dimensions of the rectangle?

$(x+6)(x-4)=0$

$x=-6, \boxed{4}$

$\ell = 4-1 = 3$

$w = 4+3 = 7$

-24
6 -4
+2

2. A smaller rectangle can fit inside the rectangle from the Opening Exercise, and it has an area that can be represented by the expression $x^2 - 4x - 5$.

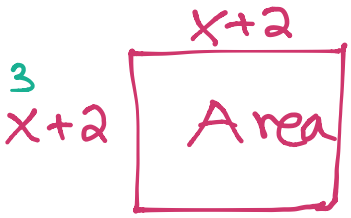
A. If the dimensions of the rectangle are known to be the linear factors of the expression, what are the dimensions of the smaller rectangle in terms of x?

B. What value for x would make the smaller rectangle have an area of $\frac{1}{3}$ that of the larger?

3. Lord Byron is designing a set of square garden plots so some peasant families in his kingdom can grow vegetables. The minimum size for a plot recommended for vegetable gardening is at least 2 m on each side. Lord Byron has enough space around the castle to make bigger plots. He decides that each side should be the minimum (2 m) plus an additional x m.

A. What expression can represent the area of one individual garden based on the undecided additional length x?

5×5



$(x+2)(x+2) = A$

$(x+2)^2 = A$

B. There are 12 families in the kingdom who are interested in growing vegetables in the gardens. What equation can represent the total area, A, of the 12 gardens? Assume all of the gardens are the same size.

$A = 12(x+2)^2$

C. If the total area available for the gardens is 300 sq m, what are the dimensions of each garden?

$\frac{300}{12} = \frac{12(x+2)^2}{12}$

$\pm\sqrt{25} = \sqrt{(x+2)^2}$

D. Find both values for x that make the equation in part C true (the solution set). What value of x does Lord Byron need to add to the 2 m?

$\pm 5 = x+2$

$-2 \quad -2$

$-2 \pm 5 = x$

$-2+5 \qquad -2-5$

Practice Exercises

Solve each equation. Some of them may have radicals in their solutions.

4. $3x^2 - 9 = 0$

$+9 \quad +9$

$\frac{3x^2}{3} = \frac{9}{3}$

$\sqrt{x^2} = \pm\sqrt{3}$

$x = \pm\sqrt{3}$

5. $(x - 3)^2 = 1$

6. $4(x - 3)^2 = 1$

7. $2(x - 3)^2 = 12$

$\frac{2}{2} \quad \frac{12}{2}$

$\sqrt{(x-3)^2} = \pm\sqrt{6}$

$x - 3 = \pm\sqrt{6}$

$+3 \quad +3$

$x = 3 \pm \sqrt{6}$

$3+\sqrt{6} \quad 3-\sqrt{6}$

8. Analyze the solutions for Exercises 4–7. Notice how the questions all had $(x - 3)^2$ as a factor, but each solution was different (radical, mixed number, whole number). Explain how the structure of each expression affected each problem-solution pair.

- Short factoring $a = 1$
- Long factoring $a \neq 1, 0$
- Square root method

9. Peter is a painter, and he wonders if he would have time to catch a paint bucket dropped from his ladder before it hits the ground. He drops a bucket from the top of his 9-foot ladder. The height, h, of the bucket during its fall can be represented by the equation, $h = -16t^2 + 9$, where the height is measured in feet from the ground, and the time since the bucket was dropped, t, is measured in seconds. After how many seconds does the bucket hit the ground? Do you think he could catch the bucket before it hits the ground? Be sure to justify your answer.

Lesson Summary

By looking at the structure of a quadratic equation (missing linear terms, perfect squares, factored expressions), you can find clues for the best method to solve it. Some strategies include setting the equation equal to zero, factoring out the GCF or common factors, and using the zero product property.

Be aware of the domain and range for a function presented in context, and consider whether answers make sense in that context.

NAME: _____ PERIOD: _____ DATE: _____

Homework Problem Set

1. Factor completely: $15x^2 - 40x - 15$.

Solve each equation.

2. $4x^2 = 9$

3. $3y^2 - 8 = 13$

4. $(d + 4)^2 = 5$

5. $4(g - 1)^2 + 6 = 13$

6. $12 = -2(5 - k)^2 + 20$

7. $-5x^2 = -500$

8. $7n^2 + 448 = 0$

9. $m^2 + 7 = 88$

10. $\frac{x^2}{25} - 6 = -2$

11. $4\left(x^2 - 15\right) = 84$

12. $2(x - 1)^2 = 8$

13. $(x + 2)^2 - 6 = 30$

14. $(3x + 6)^2 - 81 = 0$

15. $(4x - 5)^2 = 64$

16. Mischief is an Alaskan malamute dog that competes with her trainer in the agility course. Within the course, Mischief must leap through a hoop. Mischief's jump can be modeled by the equation $h = -16t^2 + 12t$, where h is the height of the leap in feet and t is the time since the leap, in seconds. At what values of t does Mischief start and end the jump?

17. A string 60 inches long is to be laid out on a tabletop to make a rectangle of perimeter 60 inches. Write the width of the rectangle as $15 + x$ inches. What is an expression for its length? What is an expression for its area? What value for x gives an area of the largest possible value? Describe the shape of the rectangle for this special value of x.