

LESSON

19

Writing and Solving Quadratic Equations

LEARNING OBJECTIVES

- Today I am: writing a variety of quadratic equations.
- So that I can: model various real-life problems.
- I'll know I have it when I can: determine the width of a path that is needed to increase the area of a garden to 285 m².

Opening Exercise

1. The length of a rectangle is 5 in. more than twice a number. The width is 4 in. less than the same number. The perimeter of the rectangle is 44 in. Sketch a diagram of this situation, and find the unknown number.

$$l \times w = A$$

x

2. The length of a rectangle is 5 in. more than twice a number. The width is 4 in. less than the same number. If the area of the rectangle is 15 in², find the unknown number.

$$\textcircled{15} \quad l = 2x + 5$$

$$\textcircled{1} \quad w = x - 4$$

$$(2x + 5)(x - 4) = 15$$

$$2x^2 - 8x + 5x - 20$$

$$2x^2 - 3x - 20 = 15$$

$$2x^2 - 3x - 35 = 0$$

$$2x^2 - 10x + 7x - 35 = 0$$

$$2x + 7 = 0 \quad x - 5 = 0$$

$$x = -\frac{7}{2}$$

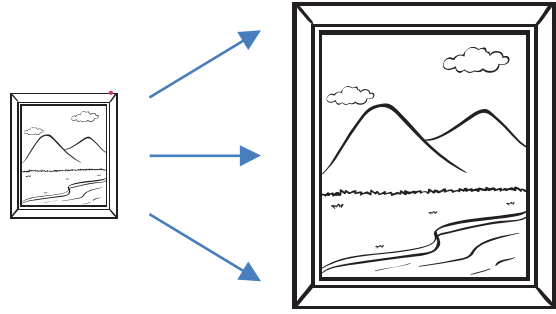
$$\boxed{x = 5}$$

$$\begin{array}{r} -70 \\ -10 \quad 7 \\ -3 \end{array}$$

$$2x(x-5)+7(x-5)=0$$

$$(2x+7)(x-5)=0$$

3. A picture has a height that is $\frac{4}{3}$ its width. It is to be enlarged so that the ratio of height to width remains the same, but the area is 192 in^2 . What are the dimensions of the enlargement?



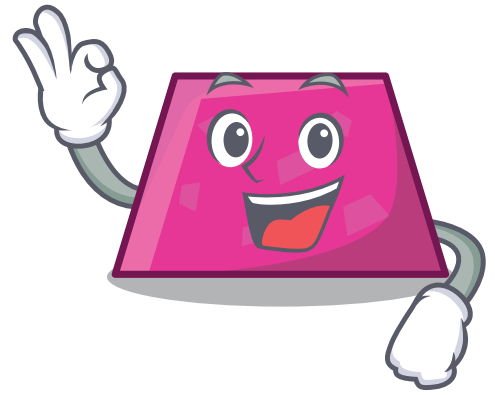
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Solve the following problems. Be sure to indicate if a solution is to be rejected based on the contextual situation.

4. The length of a rectangle is 4 cm more than 3 times its width. If the area of the rectangle is 15 cm^2 , find the width.
5. The ratio of length to width in a rectangle is 2 : 3. Find the length of the rectangle when the area is 150 in^2 .

6. One base of a trapezoid is 2 in. more than three times the length of the second base. The height of the trapezoid is the same as the second base. If the area of the trapezoid is 55 in^2 , find the dimensions of the trapezoid.

(Note: The area of a trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$.)



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7. A garden measuring 12 m by 16 m is to have a pedestrian pathway that is w meters wide installed all the way around it, increasing the total area to 285 m^2 . What is the width, w , of the pathway?



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8. **Challenge** Karen wants to plant a garden and surround it with decorative stones. She has enough stones to enclose a rectangular garden with a perimeter of 68 ft., and she wants the garden to cover 240 ft^2 . What is the length and width of her garden?



9. Find two consecutive odd integers whose product is 99. (Note: There are two different pairs of consecutive odd integers and only an algebraic solution will be accepted.)

$$n = 1^{\text{st}} \text{ odd number}$$

$$n+2 = 2^{\text{nd}} \text{ odd number}$$

$$n(n+2) = 99$$

$$n^2 + 2n = 99$$

$$n^2 + 2n - 99 = 0$$

$$(n+11)(n-9) = 0$$

$$n+11=0$$

$$n = -11, -9$$

$$n-9=0$$

$$n = 9, 11$$

10. **Challenge** You have a 500-foot roll of chain link fencing and a large field. You want to fence in a rectangular playground area. What are the dimensions of the largest such playground area you can enclose? What is the area of the playground?



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Lesson Summary

When provided with a verbal description of a problem, represent the scenario algebraically. Start by identifying the unknown quantities in the problem and assigning variables. For example, write expressions that represent the length and width of an object.

Solve the equation using techniques previously learned, such as factoring and using the zero product property. The final answer should be clearly stated and should be reasonable in terms of the context of the problem.

3. One base of a trapezoid is three times the length of the second base. The height of the trapezoid is 2 in. smaller than the second base. If the area of the trapezoid is 30 in^2 , find the lengths of the bases and the height of the trapezoid.

(Note: The area of a trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$.)

4. A student is painting an accent wall in his room where the length of the wall is 3 ft. more than its width. The wall has an area of 130 ft^2 . What are the length and the width, in feet?



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5. Find two consecutive even integers whose product is 80. (There are two pairs.) Be sure to show your work using algebraic methods.