LESSON 22

A Little Bit More

Completing the Square

LEARNING OBJECTIVES

- Today I am: completing the square when there is a value for "c".
- So that I can: learn to complete the square of any quadratic expression.
- I'll know I have it when I can: complete the square with the expression $x^2 - \frac{1}{2}x + \frac{3}{4}$.

Opening Exercise

1. Suppose you got the two expressions below and were asked to complete the square on each one. What trouble might you have? Why?

$x^2 + 18x + c$

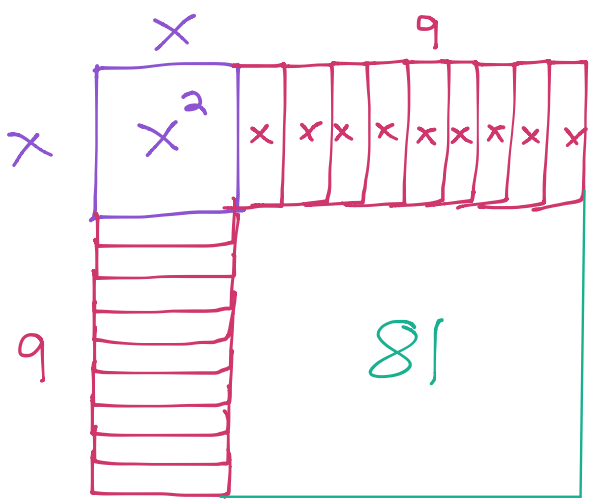
$\left(\frac{18}{2}\right)^2$

$x^2 + 18x + 81$

$(x + 9)^2$

$x^2 + 18x + 80$ → 81

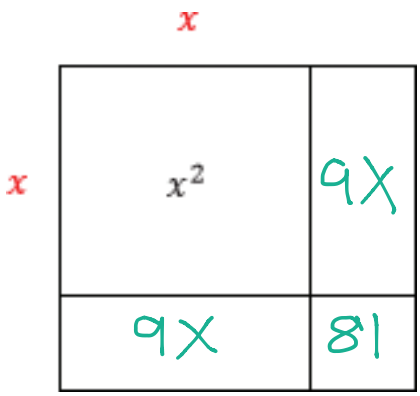
2. Draw a algebra tile model for $x^2 + 18x$. How many unit tiles are needed to complete the square?



Exploratory Challenge

When an expression is not a perfect square, you can use a tabular method to rewrite this expression as an equivalent perfect square binomial.

3. We are looking for a perfect square binomial that matches our quadratic expression as closely as possible. How do we know there must be an x-term in our binomial?



4. The quadratic expression in standard form has a linear term of $+18x$. What constant term must the perfect square binomial have if the linear term coefficient is positive 18? Fill in the missing cells both outside and inside the square.

5. Find an expression equivalent to $x^2 + 18x + 80$ that includes a perfect square binomial. How can you check that your expression is equivalent?

Perfect square binomial → $a^2+2ab+b^2$

$(a+b)^2$

$a^2-2ab+b^2$

$(a-b)^2$

Practice Exercises

Rewrite each expression by completing the square.

6. $a^2 - 4a + 15$

$(a^2-4a \underline{+4})+15 \underline{-4}$

$(-\frac{4}{2})^2$

$(a-2)^2+11$

7. $n^2 - 2n - 15$

$(n^2-2n \underline{+1})-15-1$

$(-\frac{2}{2})^2$

$(n-1)^2-16$

8. $c^2 + 20c - 40$

$(c^2+20c+100)-40-100$

$(c+10)^2-140$

9. $x^2 - 1000x + 60\,000$

$(x^2-1000x+250000)+60000-250000$

$(x-500)^2-190000$

10. $y^2 - 8y + 10$

$y^2-8y+16+10-16$

$(y-4)^2-6$

11. $k^2 + 6k + 6$

$k^2+6k+9+6-9$

$(k+3)^2-3$

12. $z^2 - 2z + 3$

$z^2-2z+1+3-1$

$(z-1)^2+2$

13. $p^2 + 10p + 12$

$p+10p+25+12-25$

$(p+5)^2-13$

14. $x^2 - \frac{1}{2}x + \frac{3}{4}$

$\left(x^2-\frac{1}{2}x+\frac{1}{16}\right)+\frac{3}{4}-\frac{1}{16}$

$\left(-\frac{1}{2}\cdot\frac{1}{2}\right)^2$ $\quad \frac{4\cdot3}{4\cdot4}-\frac{1}{16}=\frac{12-1}{16}$

$\left(x-\frac{1}{4}\right)^2+\frac{11}{16}$

15. How could you write $y = x^2 - \frac{1}{2}x + \frac{3}{4}$ in vertex form?

$y=\left(x-\frac{1}{4}\right)^2+\frac{11}{16}$

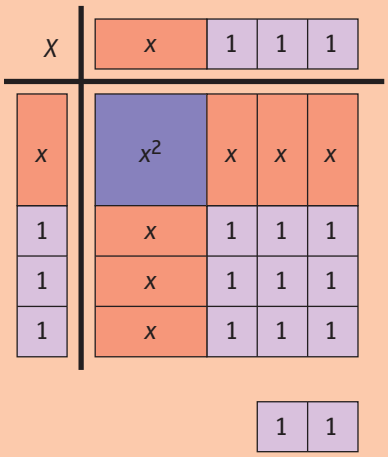
vertex: $\left(\frac{1}{4}, \frac{11}{16}\right)$

Lesson Summary

A visual model can help you understand the process of completing the square.

Completing the square

Write $x^2 + 6x + 11$ in the form $(x + a)^2 + b$



Both factors are the same so you need to share your x terms equally between columns and rows.

The remainder 1s left over is the b term.

$(x + 3)^2 + 2$

Source: http://www.greatmathsteachingideas.com/2015/04/04/algebra-tiles-from-counting-to-completing-the-square/

NAME: _____ PERIOD: _____ DATE: _____

Homework Problem Set

Rewrite each expression by completing the square.

1. $q^2 + 12q + 32$

2. $m^2 - 4m - 5$

3. $x^2 - 12x + 6$

4. $a^2 + 70a + 1225$

5. $z^2 - 30z + 10$

6. $y^2 - 6by + 20$

7. Which of these expressions would be most easily rewritten by factoring? Justify your answer.

Spiral REVIEW—Simplifying Radicals

Simplify each radical expression.

8. $3+\sqrt{4}$

9. $3-\sqrt{4}$

10. $4+\sqrt{8}$

11. $3\sqrt{4}$

12. $4\sqrt{8}$

13. $3+5\sqrt{4}$

14. $3-5\sqrt{4}$

15. $2+\sqrt{37-1}$

16. $\sqrt{36-9}$

$\sqrt{27}$

$\sqrt{9}\sqrt{3}$

$3\sqrt{3}$

17. $\sqrt{36}-\sqrt{9}$

$6-3$

3

18. $\sqrt{25-9}$

19. $\sqrt{25}-\sqrt{9}$

20. $\sqrt{16-4}$

21. $\sqrt{10-6}$

22. $2\sqrt{9}+3\sqrt{25-16}$

23. $5\sqrt{12-3}$

Spiral REVIEW—Multiplying Radical Binomials

24. $\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)$

25. $\left(x+\sqrt{6}\right)\left(x+\sqrt{8}\right)$

$\sqrt{4}\sqrt{12}$ → $2\sqrt{12}$ → $2\sqrt{4}\sqrt{3}$ → $2\cdot 2\sqrt{3}$

$x^2 + x\sqrt{8} + x\sqrt{6} + \sqrt{48}$ ($\sqrt{8} = 2\sqrt{2}$; $\sqrt{48} = \sqrt{16}\sqrt{3}$)

$x^2 + 2x\sqrt{2} + x\sqrt{6} + 4\sqrt{3}$

26. $\left(a-\sqrt{2}\right)\left(a-\sqrt{2}\right)$

27. $\left(b+\sqrt{3}\right)\left(b-2\sqrt{3}\right)$

28. $\left(2w-3\right)\left(2+\sqrt{2}\right)$

29. $\left(y+\sqrt{6}\right)\left(y-\sqrt{6}\right)$

30. $\left(4x+\sqrt{2}\right)\left(4x-\sqrt{2}\right)$

31. $\left(a-2\sqrt{2}\right)\left(a+2\sqrt{2}\right)$