

# LESSON

# 23

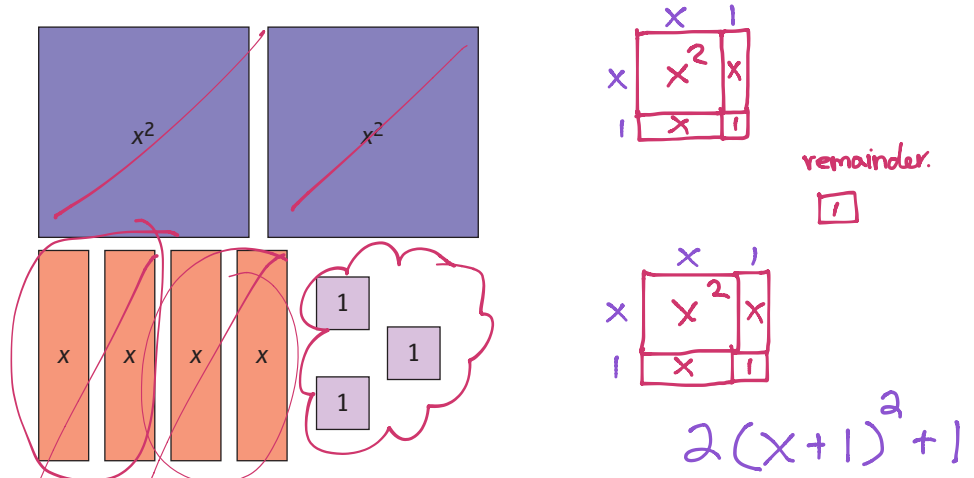
# Complicated Quadratics

## LEARNING OBJECTIVES

- Today I am: using algebra tile models of quadratic expression.
- So that I can: determine how to complete the square with complicated quadratics.
- I'll know I have it when I can: determine the correct order to complete the square with complicated quadratics.

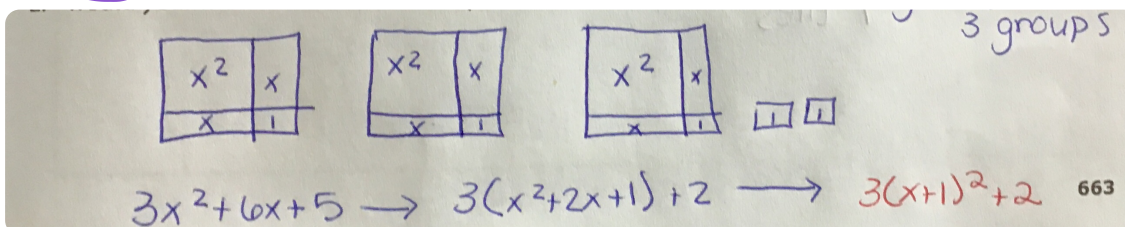
## Opening Discussion

1. The quadratic expression  $2x^2 + 4x + 3$  can be modeled with the algebra tiles as shown below. Discuss with your group a method to complete the square with this expression.



Source: [http://www.glencoe.com/sites/common\\_assets/mathematics/ebook\\_assets/vmf/VMF-Interface.html](http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html)

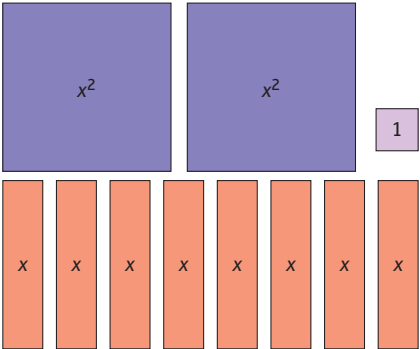
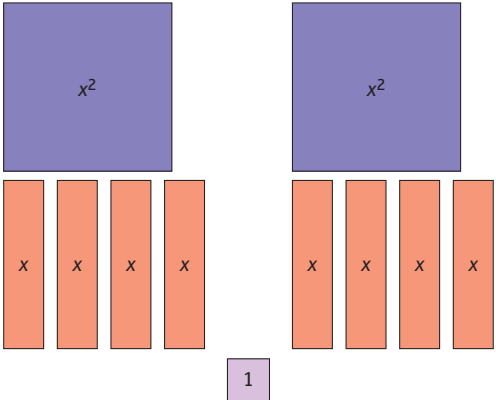
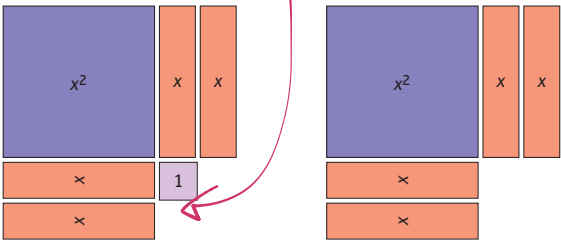
2. Would your method work for the expression  $3x^2 + 6x + 5$ ? Explain.



3. All of the quadratic functions we've rewritten from standard form to vertex form have had a **leading coefficient** of 1. What are the leading coefficients in Exercises 1 and 2?

Polynomial  
 monomial - 1-term  
 binomial - 2-term  
 trinomial - 3-term

4. One way to complete the square when the leading coefficient is not 1 is to factor out the leading coefficient. Fill in the missing parts of the example shown below and then answer the questions.

Algebra Tiles Model	Explanation in Words	Explanation with Algebra
	<p>This diagram is modeling the expression:  <math>2x^2 + 8x + 1</math></p>	<p>Trinomial  <math>2x^2 + 8x + 1</math></p>
	<p>At this step the <math>x^2</math> and <math>x</math> tiles are split into equal groups.</p>	<p>What happened here?</p> <p><math>2(x^2 + 4x) + 1</math></p> <p><math>2(x^2 + 4x + \underline{4}) + 1 - \underline{8}</math></p> <p><math>(\frac{4}{2})^2</math></p>
	<p>The algebra tiles are rearranged to create "squares". Draw in the missing tiles. Since they are missing we use a <b>negative</b> symbol.</p>	<p><math>2(x + 2)^2 - \underline{7}</math></p>

**Messy Mix Up Activity:** You and your partner will need two sets of Messy Mix Up cards, scissors and a glue stick.

$$3x^2 - 12x + 4$$

5. Cut out the steps of one of the sets of Messy Mix Up cards. Then place the cards in the correct order. Help your partner with theirs. Be sure your steps are in the correct order before gluing them down into the space below.

Step 1:	$3x^2 - 12x + 4$ <p>* Factor out the leading coefficient</p> $3(x^2 - 4x \underline{\quad}) + 4 \underline{\quad}$
Step 2:	<p>* Find the missing <u>C</u></p> $3(x^2 - 4x + \underline{4}) + 4 - \underline{12}$ <p><math>(\frac{-4}{2})^2</math></p>
Step 3:	<p>* Factor the perfect square trinomial</p> $3(x - \underline{2})^2 - 8$
Step 4:	<p>* Check your work.</p> $3(x - 2)(x - 2) - 8$ $3(x^2 - 4x + 4) - 8$
Step 5:	$3x^2 - 12x + 12 - 8$ $3x^2 - 12x + 4$

## Practice Exercises with Expressions

For each expression below, complete the square.

<p>6. <math>2x^2 - 8x - 7</math></p> $2(x^2 - 4x + \underline{4}) - \underline{7-8}$ $2(x-2)^2 - 15$	<p>7. <math>3x^2 + 12x + 4</math></p>	<p>8. <math>4x^2 - 24x + 3</math></p>
<p>9. <math>7x^2 + 14x - 8</math></p>	<p>10. <math>5x^2 - 20x - 2</math></p> $5(x^2 - 4x + \underline{4}) - \underline{2-20}$ $5(x-2)^2 - 22$	<p>11. <math>2x^2 + 12x + 5</math></p> $2(x^2 + 6x + \underline{9}) + \underline{5-18}$ $2(x+3)^2 - 13$

12. You may have noticed that the expressions that we get when we complete the square look a lot like a quadratic in vertex form. What part is missing?

$$y = a(x-h)^2 + k$$

$$f(x) = a(x-h)^2 + k$$

Each expression from Exercises 6–11, have been rewritten as an equation. Find the vertex of each one. Use your work from Exercises 6–11 to help you.

For each equation below, complete the square and then identify the vertex of the quadratic function.

<p>13. <math>y = 2x^2 - 8x - 7</math>  <math>y = 2(x^2 - 4x + 4) - 7 - 8</math>  <math>y = 2(x - 2)^2 - 15</math></p> <p>Vertex: ( <u>2</u> , <u>-15</u> )</p>	<p>14. <math>y = 3x^2 + 12x + 4</math>  <math>y = 3(x^2 + 4x + 4) + 4 - 12</math>  <math>y = 3(x + 2)^2 - 8</math></p> <p>Vertex: ( <u>-2</u> , <u>-8</u> )</p>	<p>15. <math>y = 4x^2 - 24x + 3</math>  <math>y = 4(x^2 - 6x + 9) + 3 - 36</math>  <math>y = 4(x - 3)^2 - 33</math></p> <p>Vertex: ( <u>3</u> , <u>-33</u> )</p>
<p>16. <math>y = 7x^2 + 14x - 8</math>  <math>y = 7(x^2 + 2x + 1) - 8 - 7</math>  <math>y = 7(x + 1)^2 - 15</math></p> <p>Vertex: ( <u>-1</u> , <u>-15</u> )</p>	<p>17. <math>y = 5x^2 - 20x - 2</math></p> <p>Vertex: ( _____ , _____ )</p>	<p>18. <math>y = 2x^2 + 12x + 5</math></p> <p>Vertex: ( _____ , _____ )</p>

## Lesson Summary

Here is an example of completing the square of a quadratic expression of the form  $ax^2 + bx + c$ .

$x^2 + 6x$

$x^2 + 6x$

$x^2 + 6x$

$+2$

$$3x^2 + 18x + 2$$

$$3(x^2 + 6x) + 2$$

$$3(x^2 + 6x + 9) - 3(9) + 2$$

$$3(x + 3)^2 - 3(9) + 2$$

$$3(x - 3)^2 - 27 + 2$$

$$3(x - 3)^2 - 25$$

19. What other steps could we show in the Lesson Summary to complete it?  
Show those steps.

NAME: \_\_\_\_\_ PERIOD: \_\_\_\_\_ DATE: \_\_\_\_\_

# Homework Problem Set

For each equation below, complete the square and then identify the vertex of the quadratic function.

1. $y = 3x^2 - 24x - 1$          Vertex: ( _____, _____ )	2. $y = 5x^2 + 20x + 7$          Vertex: ( _____, _____ )	3. $y = 2x^2 + 8x + 5$          Vertex: ( _____, _____ )
4. $y = 7x^2 - 14x + 10$          Vertex: ( _____, _____ )	5. $y = 3x^2 - 6x - 4$          Vertex: ( _____, _____ )	6. $y = 2x^2 + 8x + 1$          Vertex: ( _____, _____ )
7. $y = 5x^2 - 10x + 15$          Vertex: ( _____, _____ )	8. $y = 2x^2 - 12x - 13$          Vertex: ( _____, _____ )	9. $y = x^2 + 8x + 19$          Vertex: ( _____, _____ )

### Challenge Problems

Rewrite each expression by completing the square.

10.  $4x^2 - 12x + 9$

11.  $25k^2 - 75k + 10$

12.  $3b^2 + 21b - 5$

13.  $100c^2 - 500c + 625$

14.  $8n^2 + 4n + 5$