LESSON 24 Finding x-Intercepts Again?

LEARNING OBJECTIVES

- Today I am: comparing two methods for finding the x-intercepts.
- So that I can: determine when to use each method.
- I'll know I have it when I can: identify the advantages and disadvantages of each method.

Opening Discussion

The quadratic function, $y = x^2 - 6x + 8$, can be written as $y = (x - 2)(x - 4)$ and as $y = (x - 3)^2 - 1$. Deshi and Ame wanted to find the x-intercepts of this function. Their work is shown below.

1. Read over each method and then discuss which method you think is the easiest to use.

Deshi's Method: Using Factored Form	Ame's Method: Using Vertex Form
y=0 $y = (x - 2)(x - 4)$ $0 = (x - 2)(x - 4)$ $(x - 2) = 0$ or $(x - 4) = 0$ $x = 2$ or $x = 4$	$y = (x - 3)^2 - 1$ $+1\ 0 = (x - 3)^2 - 1\ +1$ $\sqrt{1} = \sqrt{(x - 3)^2}$ $\pm 1 = x - 3$ $x = \pm 1 + 3$ $x = 1 + 3 = 4$ or $x = -1 + 3 = 2$ square root.

Practice Exercises

For each quadratic function, use the form given to determine the x-intercepts. Your partner will use the other form. Then check that you are getting the same x-intercepts. Be sure to switch methods for each problem.

$y=0$

2. A. Using Factored Form to Find x-Intercepts	B. Using Vertex Form to Find x-Intercepts
$y = (x + 1)(x - 3)$ $0 = (x+1)(x-3)$ $x+1=0 \quad x-3=0$ $x=-1 \quad x=3$	$y = (x - 1)^2 - 4$ $0 = (x-1)^2 - 4$ $+4 \quad +4$ $\pm\sqrt{4} = \sqrt{(x-1)^2}$ $\pm 2 = x-1$ $+1 \quad +1$ $1 \pm 2 = x$ $1+2=3 \quad 1-2=-1$
3. A. Using Factored Form to Find x-Intercepts	**B. Using Vertex Form to Find x-Intercepts**
$y = 2(x - 4)(x + 2)$ $0 = 2(x-4)(x+2)$ $x = 4, -2$	$y = 2(x - 1)^2 - 18$ $0 = 2(x-1)^2 - 18$ $+18 \quad +18$ $\frac{18}{2} = \frac{2(x-1)^2}{2}$ $\sqrt{9} = \sqrt{(x-1)^2}$ $\pm 3 = x-1$ $1 \pm 3 = x$ $x=4, -2$

4. A. Using Factored Form to Find x-Intercepts

$$y = \frac{1}{3}(x-7)(x+1)$$

$$0 = \frac{1}{3}(x-7)(x+1)$$

$$x = 7, -1$$

B. Using Vertex Form to Find x-Intercepts

$$y = \frac{1}{3}(x-3)^2 - \frac{16}{3}$$

$$0 = \frac{1}{3}(x-3)^2 - \frac{16}{3}$$

$$\cancel{3} \cdot \frac{16}{\cancel{3}} = \cancel{3} \cdot \frac{1}{\cancel{3}}(x-3)^2$$

$$\sqrt{16} = \sqrt{(x-3)^2}$$

$$\pm 4 = x - 3$$

$$3 \pm 4 = x$$

$$x = 7, -1$$

5. A. Using Factored Form to Find x-Intercepts

$$y = (2x-1)(2x+1)$$

$$0 = (2x-1)(2x+1)$$

$$2x - 1 = 0 \qquad 2x + 1 = 0$$

$$x = \frac{1}{2} \qquad x = \frac{-1}{2}$$

B. Using Vertex Form to Find x-Intercepts

$$y = 4x^2 - 1$$

$$+1 \qquad +1$$

$$1 = 4x^2$$

$$\frac{\sqrt{1}}{\sqrt{4}}$$

$$\pm\sqrt{\frac{1}{4}} = \sqrt{x^2}$$

$$\pm\frac{1}{2} = x$$

Discussion

6. Avery prefers the vertex form of quadratic functions. He found the x-intercepts for $y = (x - 5)^2 - 12$ in the following way.

   A. Write in each step Avery took.

   $y = (x - 5)^2 - 12$

   $0 = (x - 5)^2 - 12$ — Let y = 0

   $12 = (x - 5)^2$ — Add 12

   $\pm\sqrt{12} = x - 5$ — Square root both sides

   $x = 5 \pm \sqrt{12}$ — Add 5

   $x = 5 + 2\sqrt{3}$ or $x = 5 - 2\sqrt{3}$ — Simplify $\sqrt{12}$

   B. What is different about the equation Avery worked with as compared to the ones you used in Exercises 1–5?

7. Kenja really likes the vertex form but she ran into a problem with $y - 10 = (x - 1)^2$. What problem did Kenja face? What are the x-intercepts? What does this mean?

   0

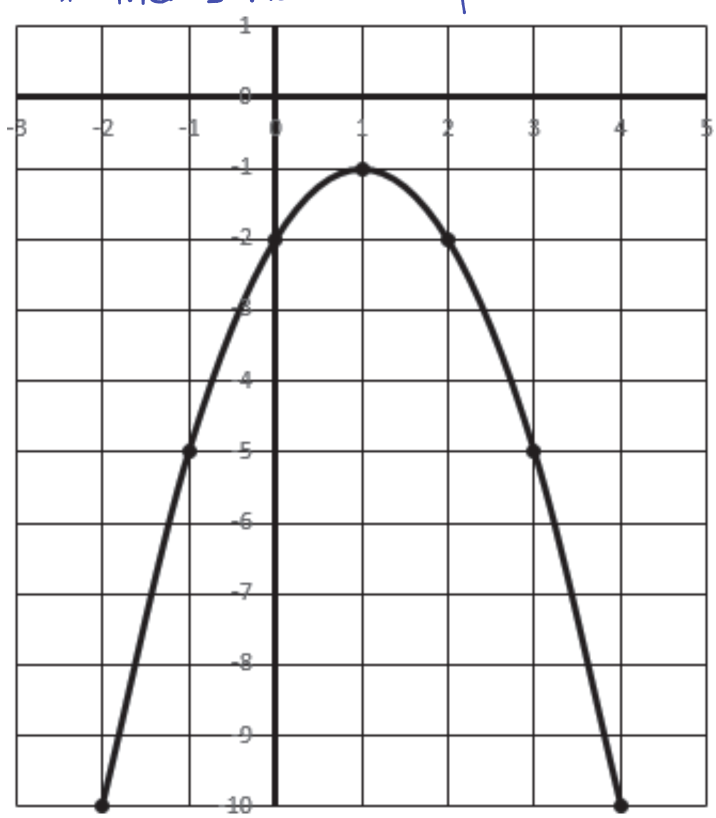
   The y should be zero

   $0 - 10 = (x - 1)^2$

   $\sqrt{-10} = \sqrt{(x-1)^2}$

   * There's no x-intercept.

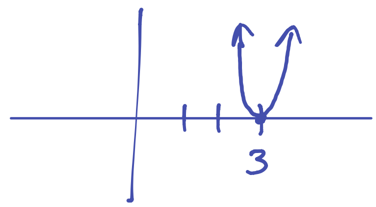
8. Neema wondered how to write the equation of the graph at the right in factored form. What should Neema do to get an equation of this function? Is there a factored form?



9. Write an equation that has no x-intercepts. Exchange equations with your partner and prove there are no x-intercepts for your partner's equation.

10. Marine thought she made a mistake when finding the x-intercepts of the equation $y = 2(x - 3)(x - 3)$ and got only one x-intercept of 3. Explain what Marine's graph would look like and why this isn't a mistake.

$x = 3$



11. Write the advantages and disadvantages to each method.

	Advantages	**Disadvantages**
Using Factored Form to Find x-Intercepts	- Fast/easy to find x-intercept(s) - No square root (±)	- Factor (short & long) (special cases)
Using Vertex Form to Find x-Intercepts	- Good if the answer is irrational. $\sqrt{11}$, $\sqrt{12}$	- Have to complete the square - longer. - remember to put ±

12. Are there any advantages to using the standard form to get the x-intercepts? Explain your thinking.

Practice Exercises—Finding *x*-Intercepts

For each equation below, determine the *x*-intercepts, if there are any. You may use any method.

13. $y = (x - 5)(3x + 2)$

$0 = (x-5)(3x-2)$

$x - 5 = 0$

$x = 5$

$3x + 2 = 0$

$3x = -2$

$x = \frac{-2}{3}$

14. $y = (4x + 1)(2x - 1)$

15. $y = -3(x + 4)(2x + 3)$

16. $y = 2(x - 3)^2 - 50$

17. $y = -3(x + 4)^2 + 12$

$0 = -3(x+4)^2 + 12$

$\frac{-12}{-3} = \frac{-3(x+4)^2}{-3}$

$\sqrt{4} = \sqrt{(x+4)^2}$

$\pm 2 = x + 4$

$-4 \qquad -4$

$-4 \pm 2 = x$

$-6, -2$

18. $y + 10 = \frac{1}{4}(x - 1)^2$

$0 + 10 = \frac{1}{4}(x-1)^2$

$10 = \frac{1}{4}(x-1)^2$

$\pm\sqrt{40} = \sqrt{(x-1)^2}$

$\sqrt{4}\sqrt{10}$

$\pm 2\sqrt{10} = x - 1$

$+1 \qquad +1$

$1 \pm 2\sqrt{10} = x$

$1+2\sqrt{10}, 1-2\sqrt{10}$

19. $y = 2x^2 + 4x + 4$

$0 = 2x^2 + 4x + 4$

* complete the square (can not factor)

$0 = 2(x^2 + 2x + 1) + 4 - 2$

$\left(\frac{2}{2}\right)^2$

$0 = 2(x+1)^2 + 2$

$-2 \qquad -2$

$-2 = 2(x+1)^2$

$\sqrt{-1} = \sqrt{(x+1)^2}$

No real x-intercepts

20. $y = x^2 + 10x + 9$

21. $y - 11 = x^2 - 6x$

Practice Exercises—Finding the Vertex

For each equation below, determine the vertex. You may use any method.

22. $y = (x - 5)(3x + 2)$	23. $y = (4x + 1)(2x - 1)$	24. $y = -3(x + 4)(2x + 3)$
25. $y = 2(x - 3)^2 - 50$	26. $y = -3(x + 4)^2 + 12$	27. $y + 10 = \frac{1}{4}(x - 1)^2$
28. $y = 2x^2 + 4x + 4$	29. $y = x^2 + 10x + 9$	30. $y - 11 = x^2 - 6x$

Lesson Summary

When a quadratic equation is not conducive to factoring, we can solve by completing the square.

Completing the square can be used to find solutions that are irrational, something very difficult to do by factoring.

$$0 = x^2 + 6x - 12$$

$$0 = (x^2 + 6x + 9) - 12 - 9$$

$$0 = (x+3)^2 - 21$$

$$\pm\sqrt{21} = \sqrt{(x+3)^2}$$

$$\pm\sqrt{21} = x + 3$$

$$-3 \qquad\qquad -3$$

$$-3 \pm \sqrt{21} = x \rightarrow \pm\sqrt{21} - 3$$

$$-3 \pm \sqrt{21}$$

$$-3 + \sqrt{21} \text{ or } -3 - \sqrt{21}$$

NAME: ______________________ PERIOD: __________ DATE: __________

Homework Problem Set

1. Solve the equation for b: $2b^2 - 9b = 3b^2 - 4b - 14$.

2. Solve for x. $12 = x^2 + 6x$

3. Solve for x. $4x^2 - 40x + 93 = 0$

Solve each equation by completing the square.

4. $x^2 - 2x = 12$

5. $\frac{1}{2}r^2 - 6r = 2$ (Hint: Consider multiplying every term by 2.)

6. $2p^2 + 8p = -6$

7. **Challenge** $2y^2 + 3y - 5 = 4$

Solve each equation. Use any method.

8. $p^2 - 2p = 8$

9. $2q^2 + 8q = 4$

10. $\frac{1}{3}m^2 + 2m + 8 = 5$

11. $-4x^2 = 24x + 11$

12. **Challenge** Rewrite the expression by completing the square: $\frac{1}{2}b^2 - 4b + 13$.

Determine the *x*-intercepts of each quadratic function, if there are any.

13. $y = (2x - 1)(x + 2)$	14. $y = x(4x + 1)$	15. $y = (x - 7)(2x - 5)$
16. $y = (x - 4)^2 - 1$	17. $y = 2(x + 3)^2 - 2$	18. $y + 16 = (x - 2)^2$
19. $y = 3x^2 - 7x$	20. $y = x^2 - 5x - 24$	21. $y = 25x^2 - 1$