LESSON 25 Using the Quadratic Formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

discriminant

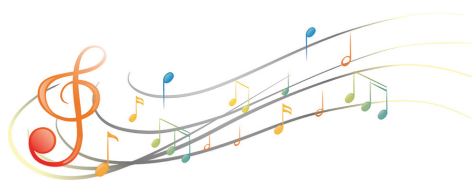
LEARNING OBJECTIVES

- Today I am: reading the lyrics to a quadratic formula song.
- So that I can: learn how to use the quadratic formula.
- I'll know I have it when I can: identify and correct an error made when using the quadratic formula.

Opening Exercise

Over the years, many students and teachers have thought of ways to help us all remember the quadratic formula. Below is the YouTube link to a video created by two teachers at Sequoyah High School near Canton, Georgia. They used Adele's "Rolling in the Deep" with modified lyrics.

https://www.youtube.com/watch?v=z6hCu0EPs-o



You will need: lyrics to the Quadratic Formula Song

The *a*, *b* and *c* in the quadratic formula refers to the coefficients when the quadratic equation is in the form, $0 = ax^2 + bx + c$.

1. Read through the lyrics and highlight the quadratic formula.
2. According to their song, what does the discriminant tell you?
   b^2-4ac
3. What do you have to make sure you have before you can use the quadratic formula?
   $y=ax^2+bx+c$

In Lesson 24 we reexamined two ways to find *x*-intercepts—factoring and completing the square. In this lesson, we'll use the quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, to find the *x*-intercepts. In Lesson 26, we'll look at how the discriminant can help us determine the type of solution we'll have. Finally in Lesson 27, we'll see how the quadratic formula came about.

Let's practice using the quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Follow the steps below to solve for *x*.

4. $2x^2 = x + 3$
$-x-3 \quad -x \quad -3$

$2x^2-x-3=0$ **Steps**	**Work**
A. Write the equation in standard form, $ax^2 + bx + c = 0$	
B. Identify the *a*, *b* and *c*.	$a =$ 2, $b =$ −1, $c =$ −3
C. Substitute the values for *a*, *b* and *c* into the quadratic equation.	$x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 2 \cdot (-3)}}{2 \cdot 2}$ $x = \dfrac{1 \pm \sqrt{1+24}}{4}$
D. Simplify.	$x = \dfrac{1 \pm \sqrt{25}}{4}$
E. Can you simplify the radical?	$x = \dfrac{1 \pm 5}{4}$ $\dfrac{1+5}{4} = \dfrac{6}{4} = \dfrac{3}{2}$ $\dfrac{1-5}{4} = \dfrac{-4}{4} = -1$
F. State both answers.	$x = \dfrac{3}{2}$; $x = -1$

Practice Problems

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve each quadratic equation. Some will have radical answers.

5. $6x^2 = 7x + 3$

6. $x^2 - 1 = x$

$-x \quad -x$

$x^2 - x - 1 = 0$ $a = 1$ $b = -1$ $c = -1$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{1 + 4}}{2}$$

$$x = \frac{1 \pm \sqrt{5}}{2} \nearrow \frac{1 + \sqrt{5}}{2} \quad \searrow \frac{1 - \sqrt{5}}{2}$$

7. $5x^2 + 6x = -1$

$5x^2 + 6x + 1 = 0$

$a = 5$ $b = 6$ $c = 1$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(5)(1)}}{2(5)}$$

$$x = \frac{-6 \pm \sqrt{36 - 20}}{10}$$

$$x = \frac{-6 \pm \sqrt{16}}{10}$$

$$x = \frac{-6 \pm 4}{10} \nearrow \frac{-6 + 4}{10} = \frac{-2}{10} = \frac{-1}{5} \quad \searrow \frac{-6 - 4}{10} = \frac{-10}{10} = -1$$

8. $x^2 = x$ $\rightarrow$ $x(x - 1) = 0$

$x^2 - x = 0$

$a = 1$
$b = -1$
$c = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(0)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{1}}{2}$$

$$x = \frac{1 \pm 1}{2}$$

$$x = \frac{1 + 1}{2} = 1 \rightarrow (1, 0)$$

$$x = \frac{1 - 1}{2} = 0 \rightarrow (0, 0)$$

Discussion

9. Which of the quadratic equations in Exercises 5–8 could be factored to find the solutions? Which method do you think would be easier (factoring or using the quadratic formula)?

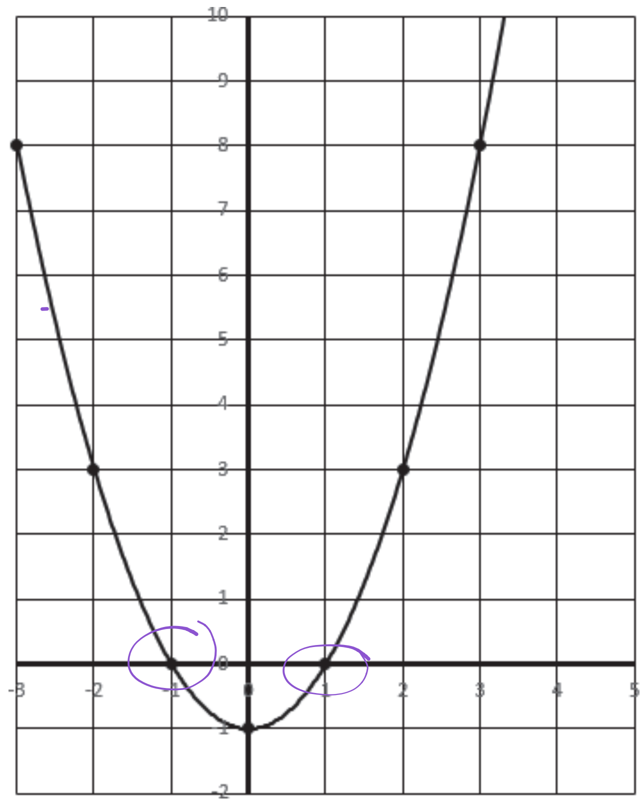
Exploration 1: Different Types of Solutions

10. A. Solve $x^2 - 1 = 0$.

$(x+1)(x-1)=0$ or $x^2-1=0$

$x=-1, x=1$ $\sqrt{x^2}=\sqrt{1}$

$x=\pm 1$



B. Which method is the most efficient for solving this problem? Why?

11. A. Solve $x^2 - 2x + 2 = 0$.

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

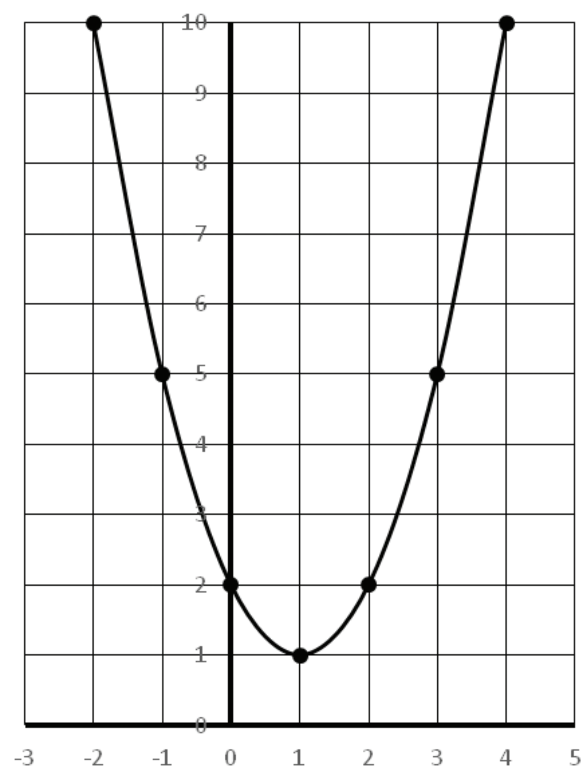
$a=1$
$b=-2$
$c=2$

$x = \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(2)}}{2(1)}$

$x = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm \sqrt{-4}}{2}$

B. What problem did you run into with this equation?

(radical)
Negative square root



C. At the right is a graph of the equation $x^2 - 2x + 2 = y$. How does the graph support your findings in Parts A and B?

No x-intercepts

12. A. Solve $x^2 - 2x + 1 = 0$.

$(x-1)(x-1)=0$

$x=1$

$a=1,\ b=-2,\ c=1$

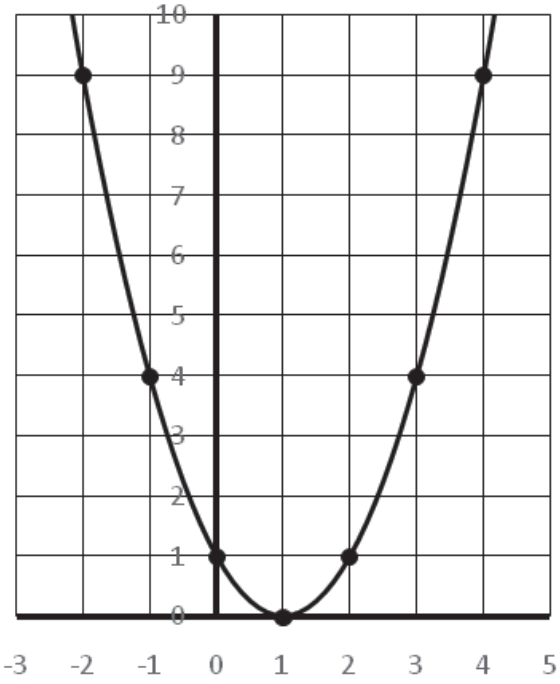
$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(1)}}{2(1)}$

$x=\frac{2\pm\sqrt{0}}{2}$

$x=\frac{2}{2}=1$

B. At the right is a graph of the equation $x^2 - 2x + 1 = y$. How does the graph support your findings in Part A?

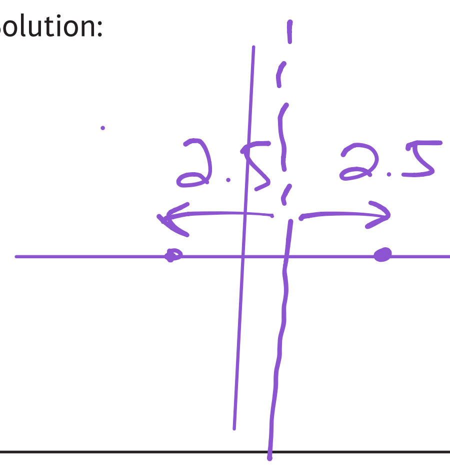
There's only one x-intercept.



13. Describe the solutions of these quadratic equations in your own words. What makes these equations different?

Exploration 2: Error Analysis

Below are some common mistakes made when using the Quadratic Formula. Identify each mistake and then write the correct solution.

14. $m^2 = 5m + 14$	15. $b^2 + 4 = 4b$	16. $2x^2 + 2x - 12 = 0$
$a = \underline{1}, b = \underline{5}, c = \underline{14}$	$a = \underline{1}, b = \underline{-4}, c = \underline{4}$	$a = \underline{2}, b = \underline{2}, c = \underline{-12}$
$m = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot 14}}{2 \cdot 1}$ $m = \frac{-5 \pm \sqrt{25 - 56}}{2}$ $m = \frac{-5 \pm \sqrt{-31}}{2}$	$b = \frac{-4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1}$ $b = \frac{-4 \pm \sqrt{16 - 16}}{2}$ $b = \frac{-4 \pm \sqrt{0}}{2} = \frac{-4}{2} = -2$	$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 2 \cdot (-12)}}{2 \cdot 2}$ $x = \frac{2 \pm \sqrt{4 + 96}}{4}$ $x = \frac{2 \pm \sqrt{100}}{4}$ $x = \frac{\cancel{2} \pm 10}{\cancel{4}_2} = \frac{\pm 10}{2} = \pm 5$
No real solutions.	1 real solution at $b = -2$.	2 real solutions at $x = 5$ and $x = -5$.
Mistake made:	Mistake made:	Mistake made:
Solution: $m^2 - 5m - 14 = 0$ $a = 1, b = -5, c = -14$ $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-14)}}{2(1)}$ $x = \frac{5 \pm \sqrt{25 + 56}}{2}$ $x = \frac{5 \pm \sqrt{81}}{2} = \frac{5 \pm 9}{2}$ $= \frac{14}{2} = 7$	Solution: $\frac{5 \pm 9}{2}$ $\frac{5}{2} \pm \frac{9}{2}$	Solution: 

$= \frac{-4}{2} = -2$

4.5

$2.5 + 4.5 = 7$

$2.5 - 4.5 = -2$

17. $m^2 = -4m - 3$

$a = \underline{1}$, $b = \underline{4}$, $c = \underline{3}$

$$m = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$$

$$m = \frac{-4 \pm \sqrt{16 - 12}}{2}$$

$$m = \frac{-4 \pm \sqrt{4}}{2}$$

$$m = \frac{-4 \pm \cancel{2}}{\cancel{2}} = -4$$

1 real solution at $m = -4$.

Mistake made:

Solution:

18. $9b^2 - 11 = 6b$

$a = \underline{9}$, $b = \underline{-6}$, $c = \underline{-11}$

$$b = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 9 \cdot (-11)}}{2 \cdot 9}$$

$$b = \frac{6 \pm \sqrt{36 + 396}}{18}$$

$$b = \frac{6 \pm \sqrt{432}}{18}$$

$$b = \frac{6 \pm \sqrt{12 \cdot 12 \cdot 3}}{18}$$

$$b = \frac{\overset{1}{\cancel{6}} \pm \overset{2}{\cancel{12}}\sqrt{\cancel{3}}}{\underset{3}{\cancel{18}_6}} = \frac{1 \pm 2\sqrt{3}}{3}$$

$$b = \frac{\cancel{6} \pm 12}{\cancel{6}} = \pm 12$$

2 real solutions at $b = 12$ and $b = -12$.

Mistake made:

Solution:

19. $x^2 = 9x - 20$

$a = \underline{1}$, $b = \underline{-9}$, $c = \underline{20}$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \cdot 1 \cdot 20}}{2 \cdot 1}$$

$$x = \frac{9 \pm \sqrt{81 - 80}}{2}$$

$$x = \frac{9 \pm \sqrt{1}}{2}$$

$$x = \frac{9 \pm 1}{2} = \pm\frac{10}{2} = \pm 5$$

2 real solutions at $x = 5$ and $x = -5$.

Mistake made:

Solution:

Lesson Summary

The quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, is one way to solve a quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$.

The formula can be used to solve any quadratic equation, and is especially useful for those that are not easily solved using any other method (i.e., by factoring or completing the square).

Because the quadratic formula has so many operations and places where numbers are substituted for variables, there are many ways to make mistakes when using it. It is important to make sure the quadratic equation is in standard form first and then to double-check that you've substituted the correct values into the formula. After that, be sure that you follow the order of operations.

When solving quadratic equations you have three possible ways to approach them:

- Factor—This is usually the easiest method. Don't forget about the GCF (greatest common factor).
- Complete the Square—This method is very helpful if you also need the vertex of the quadratic function.
- Quadratic Formula—Sometimes this is the only method that will work.

Graphing a quadratic function can also help you find the roots (solutions), but often you will only be able to get an approximate answer.

NAME: ______________________ PERIOD: __________ DATE: __________

Homework Problem Set

Examine the two equations below and *decide* what is the most efficient way to solve each one.

1. $4x^2 + 5x + 3 = 2x^2 - 3x$

2. $x^2 - 14 = 5x$

3. Solve each equation with the most efficient method.

4. What are the differences between these two quadratic equations? Is one easier to solve than the other? Explain your thinking.

5. Is one pathway to the solution *more correct* than another?

Solve each quadratic equation using the quadratic formula.

6. $x^2 + 2x - 8 = 0$

7. $d^2 + 5d - 6 = 0$

8. $2k^2 - 5k + 3 = 0$

9. $2a^2 - a - 13 = 2$

10. $8x^2 - 4x - 5 = 0$

11. $8m^2 + 6m = -5$

12. $10n^2 - n + 9 = 0$

13. $x^2 = -3x + 40$

14. $3f^2 = 6f - 3$

15. $3p^2 - 18 = 0$

16. $w^2 + 7w + 8 = 0$

17. $q^2 = 25$