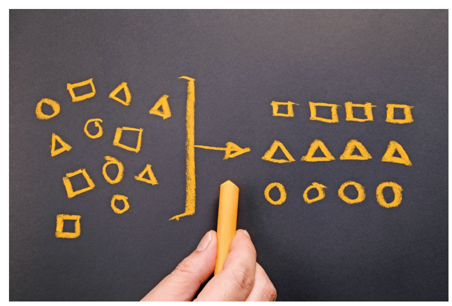
LESSON 26

The Power of the Discriminant

LEARNING OBJECTIVES

- Today I am: looking at the definition of “discriminant”.
- So that I can: understand how the mathematical definition is related to the everyday definition.
- I’ll know I have it when I can: tell when a quadratic equation will give one, two or no real solutions.

Opening Exercise



© patpitchaya/Shutterstock.com

1. Read the Google definition of the word “discriminant”. Circle the two words that you think are the most important in the everyday definition. Identify the corresponding words in the mathematical definition.

dis·crim·i·nant

/də'skrimənənt/

noun

an agent or characteristic that enables things, people, or classes to be distinguished from one another.

“anemia is commonly present in patients with both conditions, and is therefore not a helpful discriminant”

- MATHEMATICS

  a function of the coefficients of a polynomial equation whose gives information about the roots of the polynomial.

1 answer

2 answers

none

The mathematical definition can be a little tricky to decipher. Let's take it apart and see how it relates to the everyday definition.

2. A. Where have we seen the word "coefficients"?

   B. How are *coefficients* used in the quadratic formula?

   C. What are other words we've used for the word "root"?

3. Read the boxed explanation of the discriminant below. What do you think it means by "number and nature" of the solutions?

$b^2 - 4ac$

In the Quadratic Formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression under the radical is called the *discriminant*: $b^2 - 4ac$.

The value of the discriminant determines the number and nature of the solutions for a quadratic equation.

$\sqrt{\text{positive}}$ $\sqrt{\text{negative}}$

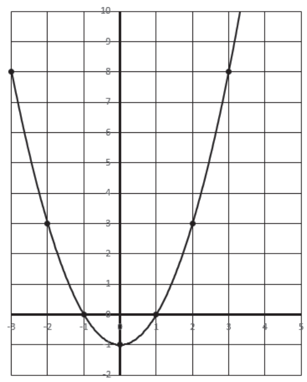
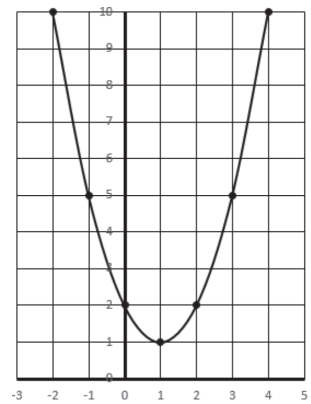
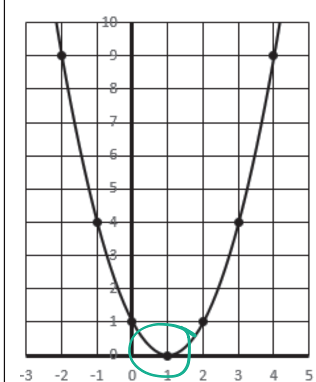
$\sqrt{0}$

To better understand the "number and nature" of the solutions, we'll look at a few examples.

b^2-4ac

Determine the discriminant for the equations below.

$y = ax^2+bx+c$

	4. $x^2 - 1 = y$	5. $x^2 - 2x + 2 = y$	6. $x^2 - 2x + 1 = y$
Values of a, b and c	$a =$ 1; $b =$ 0; $c =$ -1	$a =$ 1; $b =$ -2; $c =$ 2	$a =$ 1; $b =$ -2; $c =$ 1
Discriminant: $b^2 - 4ac$	$0^2-4(1)(-1)=4$	$(-2)^2-4(1)(2)=-4$ $4-8$	$(-2)^2-4(1)(1)=0$ $4-4$
Number of solutions (x-intercepts)	2	None	1
Graph of the function			

7. How many solutions does each equation have? How does this relate to the graphs of the functions? What pattern do you notice?

8. Fill in the blanks for the rules for discriminants.

A. As we saw in Exercise 4, when the discriminant is positive, then we have $\pm\sqrt{(\text{positive number})}$, which yields 2 real solutions. If the discriminant is a perfect square number, then we get two integer solutions.

B. When the discriminant is a negative number, as in Exercise 5, then we have $\pm\sqrt{(\text{negative number})}$, which gives us No solutions.

C. When the discriminant equals zero, as it did in Exercise 6, then we have $\pm\sqrt{0}$, which yields only one solution, $\frac{-b}{2a}$.

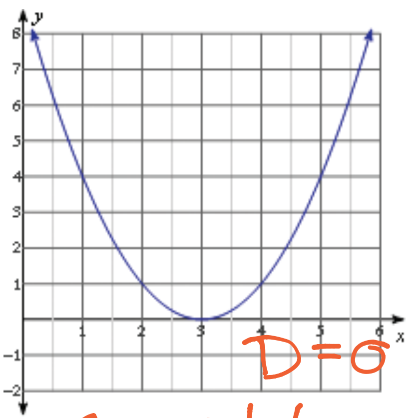
$D = b^2 - 4ac$

$D > 0$ Two solutions

$D = 0$ one solution

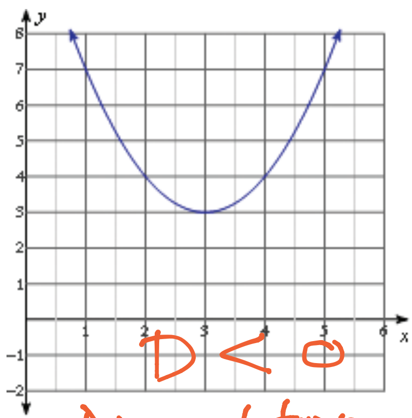
$D < 0$ no solution

9. What are the differences among these three graphs? Which of these graphs belongs to a quadratic equation with a positive discriminant? Which belongs to a quadratic equation with a negative discriminant? Which graph has a discriminant equal to zero?



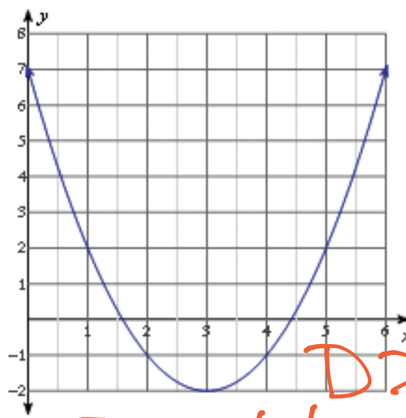
$D = 0$

One solution



$D < 0$

No solution



$D > 0$

Two solutions

Practice Exercises—Finding the Discriminant

$b^2 - 4ac$

For Exercises 10–13, determine the number of real solutions for each quadratic equation **without** solving. Notice that the variable does not need to be *x*.

10. $p^2 + 7p + 33 = 8 - 3p$

$+3p \quad -8 \quad -8 \quad +3p$

$p^2 + 10p + 25 = 0$

$a = 1 \quad b = 10 \quad c = 25$

$b^2 - 4ac$

$(10)^2 - 4(1)(25)$

$100 - 100 = 0$

one solution

11. $7x^2 + 2x + 5 = 0$

$a = 7 \quad b = 2 \quad c = 5$

$(2)^2 - 4(7)(5)$

$4 - 140$

-136

No Solution

12. $2y^2 + 10y = y^2 + 4y - 3$

$-y^2 - 4y \quad -y^2 - 4y \quad +3$

$+3$

$y^2 + 6y + 3 = 0$

$(6)^2 - 4(1)(3)$

$36 - 12$

24 Two solutions

13. $4z^2 + 9 = -4z$

$+4z \quad +4z$

$4z^2 + 4z + 9 = 0$

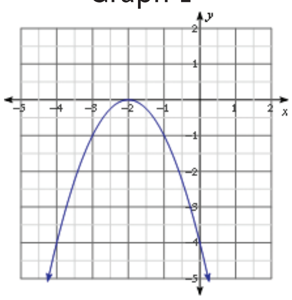
$(4)^2 - 4(4)(9)$

$16 - 144$

-128 No solution.

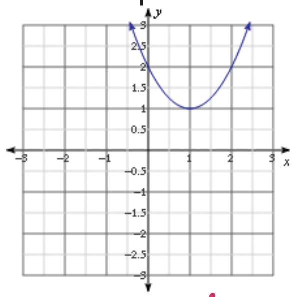
14. On the line below each graph, state whether the discriminant of each quadratic equation is positive, negative, or equal to zero. Then, identify which graph matches the discriminants below.

Graph 1



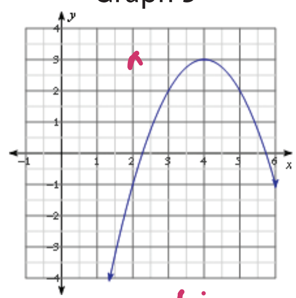
0

Graph 2



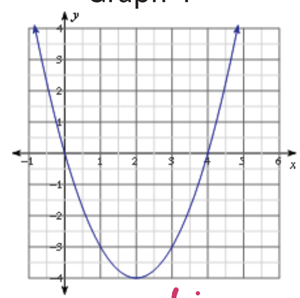
Negative

Graph 3



positive

Graph 4



positive

Discriminant A:	Discriminant B:	Discriminant C:	Discriminant D:
$(-2)^2 - 4(1)(2) = -4$	$(-4)^2 - 4(-1)(-4) = 0$	$\overset{b}{(-4)^2} - 4(\overset{a}{1})(\overset{c}{0}) = 16$	$(-8)^2 - 4(\overset{a}{-1})(-13) = 12$
Graph: 2	Graph: 1	Graph: 4	Graph: 3

15. Consider the quadratic function $f(x) = x^2 - 6$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

A. Use the quadratic formula to find the x-intercepts of the graph of the function.

$a = 1 \quad b = 0 \quad c = -6$

$$x = \frac{-0 \pm \sqrt{0^2 - 4(1)(-6)}}{2(1)} = \frac{0 \pm \sqrt{24}}{2} = \frac{\pm\sqrt{24}}{2} = \frac{\pm \cancel{2}\sqrt{6}}{\cancel{2}}$$

$= \pm\sqrt{6}$

B. Use the x-intercepts to write the quadratic function in factored form.

$2, \; -3$

$(x-2)(x+3)$

$\sqrt{6}, \; -\sqrt{6}$

$(x - \sqrt{6})(x + \sqrt{6})$

C. Show that the function from Part B written in factored form is equivalent to the original function.

$y = (x - \sqrt{6})(x + \sqrt{6})$

$y = x^2 + \cancel{x\sqrt{6}} - \cancel{x\sqrt{6}} - \sqrt{36}$

$y = x^2 - 6$

16. **Challenge** Consider the quadratic equation $ax^2 + bx + c = 0$.

A. Write the equation in factored form, $a(x - m)(x - n) = 0$, where m and n are the solutions to the equation.

B. Show that the equation from Part A is equivalent to the original equation.

Lesson Summary

The quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, is one way to solve a quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$.

The formula can be used to solve any quadratic equation, and is especially useful for those that are not easily solved using any other method (i.e., by factoring or completing the square).

You can use the sign of the discriminant, $b^2 - 4ac$, to determine the number of real solutions to a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$.

- If the equation has a positive discriminant, there are two real solutions.
- A negative discriminant yields no real solutions.
- A discriminant equal to zero yields only one real solution.

NAME: ______________________ PERIOD: __________ DATE: __________

Homework Problem Set

Without solving, determine the number of real solutions for each quadratic equation.

1. $b^2 - 4b + 3 = 0$

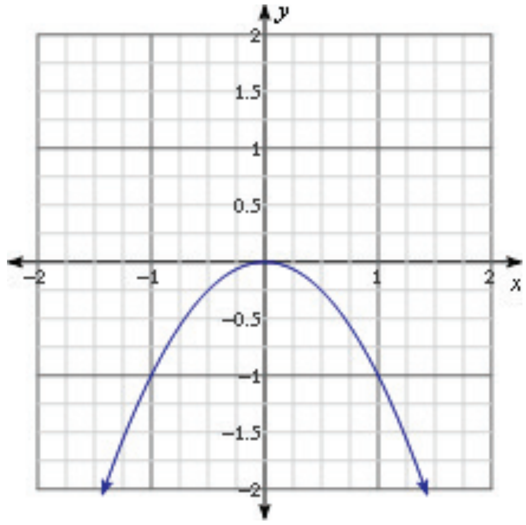
2. $2n^2 + 7 = -4n + 5$

3. $x - 3x^2 = 5 + 2x - x^2$

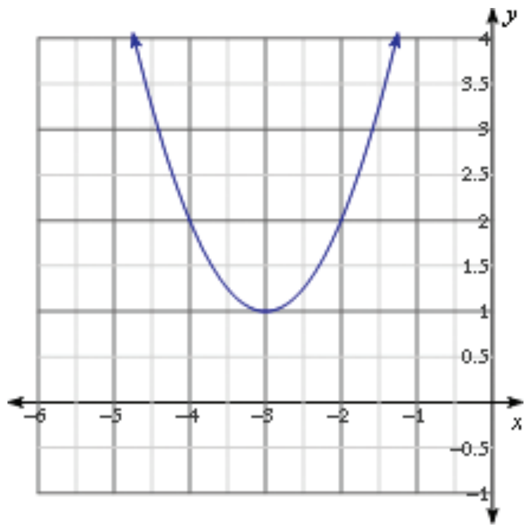
4. $4q + 7 = q^2 - 5q + 1$

Based on the graph of each quadratic function, $y = f(x)$, determine the number of real solutions for each corresponding quadratic equation, $f(x) = 0$.

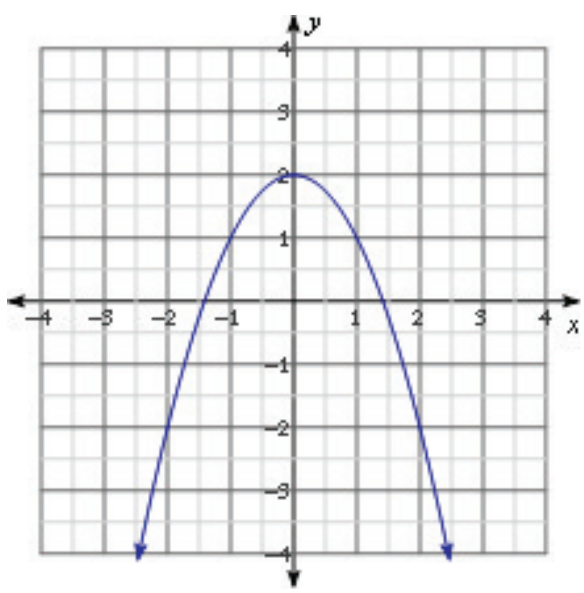
5.



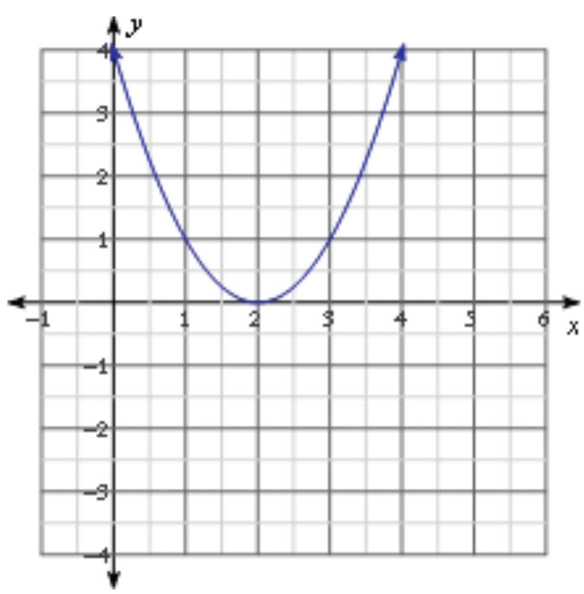
6.



7.



8.



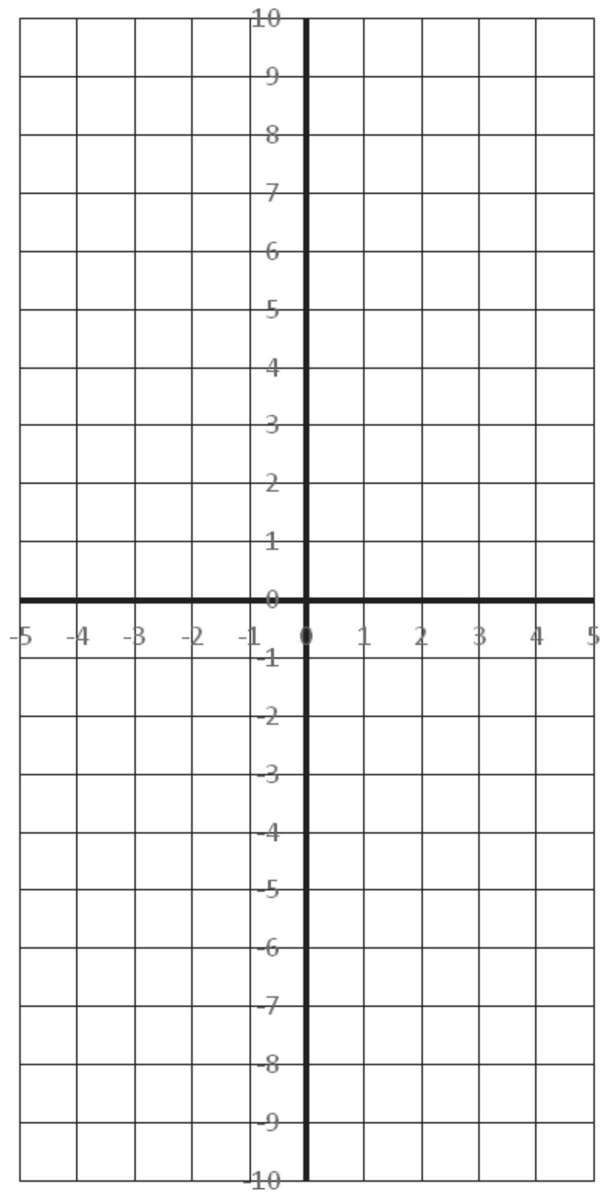
9. Consider the quadratic function $f(x) = x^2 - 7$.

A. Find the x-intercepts of the graph of the function.

B. Use the x-intercepts to write the quadratic function in factored form.

C. Show that the function from Part B written in factored form is equivalent to the original function.

D. Graph the equation.



10. **Challenge** Consider the quadratic function $f(x) = -2x^2 + x + 5$.

A. Find the x-intercepts of the graph of the function.

B. Use the x-intercepts to write the quadratic function in factored form.

C. Graph the equation.