LESSON 27

Deriving the Quadratic Formula

LEARNING OBJECTIVES

- Today I am: using the quadratic formula.
- So that I can: see how this formula was derived.
- I'll know I have it when I can: determine which method is most effective for solving for *x*.

Opening Exercise

1. Solve for *x* without using the quadratic formula.

$x^2 + 2x = 8$	$x^2 - 12x + 5 = 0$

Discussion

2. Which of these problems makes more sense to solve by completing the square? Which makes more sense to solve by factoring? How could you tell early in the problem solving process which strategy to use?

3. How would you solve this equation for x: $ax + b = 0$, where a and b could be replaced with any numbers?

4. Can we say that $x = -\frac{b}{a}$ is a *formula* for solving any equation in the form $ax + b = 0$? Explain your thinking.

5. What would happen if we tried to come up with a way to use just the values of a, b, and c to solve a quadratic equation? Can we solve the general quadratic equation $ax^2 + bx + c = 0$? Is this even possible? Which method would make more sense to use, factoring or completing the square?

Let's use the Quadratic Formula with the problems from the Opening Exercise.

6. Solve for x, using the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ for } ax^2 + bx + c = 0$$

$x^2 + 2x = 8$	$7x^2 - 12x + 4 = 0$
$a =$ ______; $b =$ ______; $c =$ ______	$a =$ ______; $b =$ ______; $c =$ ______

7. Did your answers in Exercise 6 agree with your answers in Exercise 1?

8. In the space below, solve the equation $3x^2 - 24x + 3 = 0$ for x on the left. Do NOT use the quadratic formula. Then solve $ax^2 + bx + c = 0$ for x on the right. Hint: Think about completing the square as you did in earlier lessons.

$\cancel{a}\cdot\frac{b^2}{4a^{\cancel{2}}}$

$(3x^2 - 24x) + 3 = 0$

$3(x^2-8x+16)+3^{-48}=0$

$3(x-4)^2-45=0$

$\frac{3(x-4)^2}{3}=\frac{45}{3}$

$\sqrt{(x-4)^2}=\pm\sqrt{15}$

$x-4=\pm\sqrt{15}$

$x=4\pm\sqrt{15}$

$ax^2 + bx + c = 0$

$a\left(x^2+\frac{b}{a}x+\frac{b^2}{4a^2}\right)+c^{-\frac{b^2}{4a}}=0$

$\frac{b}{a}\cdot\frac{1}{2}=\left(\frac{b}{2a}\right)^2$

$a\left(x+\frac{b}{2a}\right)^2+c-\frac{b^2}{4a}=0$

$a\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a}-\frac{c\cdot 4a}{1\cdot 4a}$

$\frac{1}{a}\cdot a\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a}\cdot\frac{1}{a}$

$\sqrt{\left(x+\frac{b}{2a}\right)^2}=\sqrt{\frac{b^2-4ac}{4a^2}}$

$x+\frac{b}{2a}=\frac{\pm\sqrt{b^2-4ac}}{2a}$

$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

9. Let's look a little more closely at the Quadratic Formula. Notice that the whole expression can be split into two separate expressions as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

This part is the axis of symmetry!

This part describes the location of the x-intercept in relation to the axis of symmetry!

Solve these quadratic equations, using a different method for each: solve by factoring, solve by completing the square, and solve using the quadratic formula. Before starting, indicate which method you will use for each.

10. Method long factoring	11. Method Quadratic formula	12. Method Square root completing the squar
$2x^2 + 5x - 3 = 0$ × −6 6 −1 5 + $(2x^2+6x)-(x-3)=0$ $2x(x+3)-1(x+3)=0$ $(2x-1)(x+3)=0$ $2x-1=0$ $x+3=0$ $2x=1$ $x=-3$ $x=\frac{1}{2}$	$x^2 + 3x - 5 = 0$ $a=1$ $b=3$ $c=-5$ $x=\frac{-3\pm\sqrt{(3)^2-4(1)(-5)}}{2(1)}$ $x=\frac{-3\pm\sqrt{9+20}}{2}$ $x=\frac{-3\pm\sqrt{29}}{2}$	$2\left(\frac{1}{2}x^2 - x - 4 = 0\right)$ $x^2-2x-8=0$ $(x^2-2x+1)\overset{-1}{-8}=0$ $\left(\frac{-2}{2}\right)^2$ $(x-1)^2-9=0$ $\sqrt{(x-1)^2}=\pm\sqrt{9}$ $x-1=\pm3$ $+1$ $+1$ $x=1\pm3$ $x=4,-2$

Lesson Summary

The quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, is derived by completing the square on the general form of a quadratic equation: $ax^2 + bx + c = 0$, where $a \neq 0$.

The formula can be used to solve any quadratic equation, and is especially useful for those that are not easily solved using any other method (i.e., by factoring or completing the square).

NAME: ______________________ PERIOD: __________ DATE: __________

Homework Problem Set

Use the quadratic formula to solve each equation.

1. Solve for z: $z^2 - 3z - 8 = 0$.

2. Solve for q: $2q^2 - 8 = 3q$

Completing

3. Solve for m: $3\left(\frac{1}{3}m^2 + 2m + 8 = 5.\right)$

$$m^2+6m+24=15$$
$$-15 \quad -15$$
$$(m^2+6m+9)+9-9=0$$
$$\sqrt{(m+3)^2}=\sqrt{0}$$
$$m+3=0$$
$$m=-3$$

4. Determine the error in Sergio's work below. Then determine the correct answers.

$$0 = 3x^2 - 4x - 5$$

$$x = \frac{-4 \pm \sqrt{(-4)^2 - 4(3)(-5)}}{2(3)}$$

$$x = \frac{-4 \pm \sqrt{16 + 60}}{6}$$

$$x = \frac{-4 \pm \sqrt{76}}{6} = \frac{-4 \pm 2\sqrt{19}}{6} = \frac{-2 \pm \sqrt{19}}{3}$$

$$x = \frac{-2 + \sqrt{19}}{3} \text{ or } x = \frac{-2 - \sqrt{19}}{3}$$

Solve these quadratic equations, using a different method for each: solve by factoring, solve by completing the square, and solve using the quadratic formula. Before starting, indicate which method you will use for each.

5. Method ________________

$3x^2 + 13x - 10 = 0$

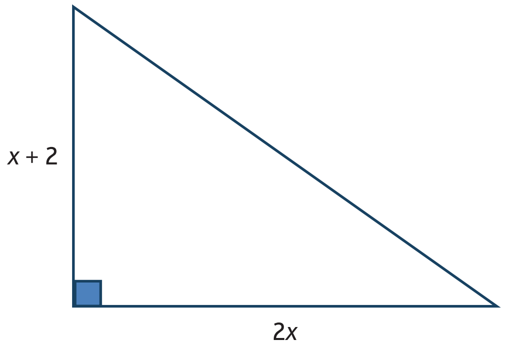
6. Method ________________

$x^2 - 12x + 28 = 0$

7. Method ________________

$2x^2 - 9x - 9 = 0$

8. **Geometry Connection** Determine the base and height of the right triangle below. Its area is 8 cm^2.



Practice Exercises

Use the quadratic formula to solve each equation.

9. $x^2 - 2x = 12 \longrightarrow a =$ ________, $b =$ ________, $c =$ ________ (Watch the negatives.)

10. $\frac{1}{2}r^2 - 6r = 2 \longrightarrow a =$ ________, $b =$ ________, $c =$ ________ (Did you remember the negative?)

11. $2p^2 + 8p = 7 \longrightarrow a =$ ________, $b =$ ________, $c =$ ________

12. $2y^2 + 3y - 5 = 4 \longrightarrow a =$ ________, $b =$ ________, $c =$ ________

Spiral REVIEW—Exponents Rules

Simplify each expression. Your resulting expression should have no negative exponents.

13. $x^3 \cdot x^8 =$ ________

14. $2^4 \cdot 2^5 =$ ________

15. $12^2 \cdot 12^{15} =$ ________

16. $\frac{x^5}{x^2} =$ ________

17. $\frac{2^4}{2^5} =$ ________

18. $\frac{x^2y^3}{xy^5} =$ ________

19. $\left(\frac{x^5}{x^2}\right)^0 =$ ________

20. $(xy^0z^2)^2 =$ ________

21. $\frac{x^0y^3}{xy^0} =$ ________

22. $(x^3)^4 =$ ________

23. $(3^5 2^7)^3 =$ ________

24. $(x^2y^3)^5 =$ ________

25. $\frac{x^{-3} \cdot x^{8}}{x^{5} \cdot x^{-4}} =$ ________

26. $\left(\frac{x^{2}}{y}\right)^{-3} =$ ________

27. $-5x^{-2}y^{3} =$ ________

28. $\left(\frac{4x^{5}y}{16xy^{-2}}\right)^{-1} =$ ________

29. $5x^{2}y^{3}\,(2x^{-3}y^{4}z) =$ ________

30. $6x^{0} \cdot 7x^{3} =$ ________

31. $\frac{4x^{7}}{2x^{-2}} =$ ________

32. $6(xyz)^{0} =$ ________

33. $\frac{ab^{3}c^{4}}{a^{0}b^{11}c^{-4}} =$ ________